Warm Dark Matter, Phase Space Density and the LHC

A. Khmelnitsky[a]*, D. Gorbunov and V. Rubakov
[a] *Institute for Nuclear Research of the Russian Academy of Sciences*
60th October Anniversary Prospect, 7a, 117312 Moscow, Russia

Abstract

Some problems concerning small scale structure recently emerged in Cold Dark Matter scenario: missing satellites problem, cusped density profiles and lack of galactic angular momentum. All these problems seem to have solution in scenarios with Warm Dark Matter. We make use of the phase space density approach to discuss keV gravitino as a warm dark matter candidate. Barring the fine tuning between reheat temperature in the Universe and superpaticle masses, we find that warm gravitinos have both appropriate total mass density and suitable primordial phase space density at low momentum provided that their mass is in the range 1 keV $\lesssim m \lesssim$ 15 keV, reheat temperature in the Universe is low, $T_R \lesssim$ 10 TeV, and masses of some of the superpartners are sufficiently small, $M \lesssim$ 350 GeV. The latter property implies that the gravitino warm dark matter scenario will be either ruled out or supported by the LHC experiments.

1 Motivation and Recipe

The predictions of the ΛCDM model are in outstanding consistency with the bulk of cosmological observations (see Ref. [1] and references therein). Yet there are clouds above the collisionless cold dark matter scenario, which have to do with cosmic structure at subgalactic scales. Three most notable of them are missing satellites, cuspy galactic density profiles and too low angular momenta of spiral galaxies. All these suggest that CDM may be too cold, i.e. that the vanishing primordial velocity dispersion of dark matter particles may be problematic. Hence, one is naturally lead to consider warm dark matter (WDM) scenarios [2]. In this work we present the ways to quantify the notion of Warm Dark Matter and consider the particular WDM candidate — light gravitino[1].

There are several ways to describe the difference between WDM and CDM scenarios. The simplest one is to say that warm dark matter particles have larger free streaming length l_{fs}. Density fluctuations on scales smaller than l_{fs} do not grow. Thus free streaming length on the moment of matter-radiation equality (since this time density fluctuations begin to grow rapidly) defines the scale of the smallest objects formed in WDM cosmology. For non-interacting particles it is possible to estimate $l_{fs}(t_{eq})$ as

$$l_{fs}(t_{eq}) \sim v \cdot t_{eq} = \frac{p}{T}\frac{T_{eq}t_{eq}}{m},$$

with v, p, m being typical velocity and momentum and the mass of dark matter particles correspondingly. For thermal-like distribution $p/T \sim 3$, and present size of suppression scale is given by

$$l_0 \sim 200 \text{ kpc}\frac{1 \text{ kev}}{m}.$$

*e-mail: khmeln@inr.ac.ru

[1] The talk is largely based on papers [3], where one can find additional details and references.

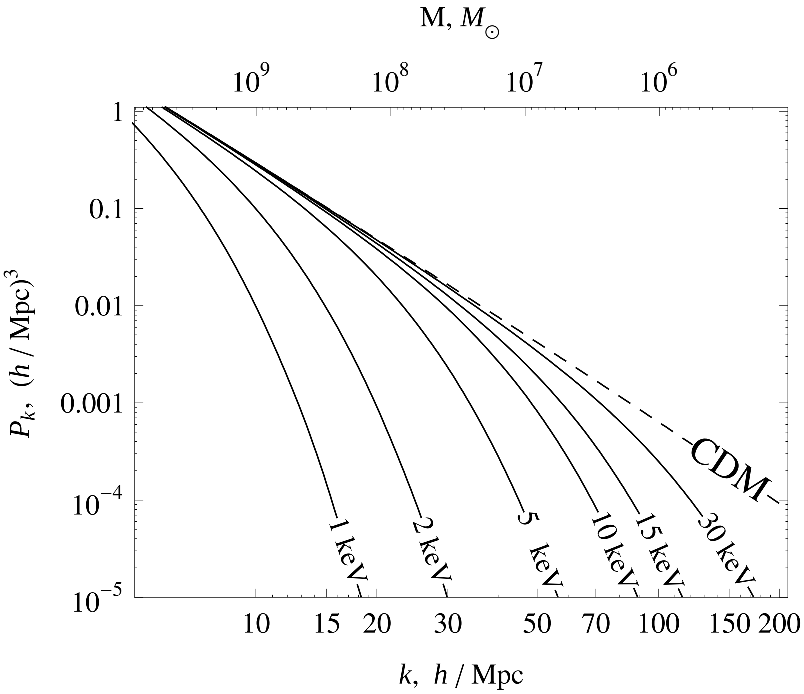

Figure 1: Linear matter power spectrum for standard ΛCDM cosmology (dashed line) and ΛWDM (solid lines) assuming the normalized Fermi–Dirac distribution of WDM particles with masses $m = 1, 5, 10, 15, 20$ and 30 keV and $g_* = g_{\mathrm{MSSM}}$.

Perturbations on this scale correspond to the objects with mass

$$M \simeq \rho_{DM} \cdot \frac{4\pi}{3} l_0^3 \sim 10^9 M_\odot \left(\frac{1\ \mathrm{keV}}{m}\right)^3 .$$

The scale of missing satellites is believed to be of order $10^7 - 10^8 M_\odot$ [4], which suggest that m should be in the keV range.

To be more precise one can calculate the linear power spectrum of density perturbations by solving numerically the Boltzmann evolution equations. Warm particles filter primordial power spectrum on small scales, and thus the formation of small halos is suppressed. The filtering scale must be small enough, since the power spectrum shows no significant deviations from the CDM prediction on scales within reach of current observations. This leads to constraints on the primordial velocity dispersion of WDM particles(cf. [5] and references therein). On the other hand, in order to improve on structure formation, the filtering scale must be of the order of the scale of missing satellites.

We have calculated linear matter power spectrum in ΛWDM cosmology assuming that dark matter particles have the Fermi–Dirac primordial distribution function, normalized to correct present total density. To this end we have modified the Boltzmann evolution equations implemented in the Code for Anisotropies in the Microwave Background (CAMB) [6]. Figure 1 presents the resulting ΛWDM power spectrum for $m = 1, 2, 5, 10, 15$ and 30 keV (solid) in comparison with ΛCDM (dashed). One concludes that the power spectrum is suppressed by about an order of magnitude on the scales corresponding to $10^8 M_\odot$ and smaller provided the WDM particle mass is about $10 - 15$ keV. Of course, this is an indicative figure.

Alternative way to quantify the notion of warm dark matter is to make use of the phase space density approach [7]. Its key ingredient is the ratio between the mass density and the cube of the one-dimensional velocity dispersion in a given volume, $Q \equiv \rho/\sigma^3$. On the one hand, this quantity is measurable in galactic halos; on the other hand, it can be used as an estimator for coarse-grained distribution function of halo particles. Namely, for non-relativistic dark matter particles

$$Q \simeq m^4 \cdot \frac{n}{\langle \frac{1}{3} p^2 \rangle^{3/2}} ,$$

where m is the mass of these particles and n is their average number density in a halo. Assuming that the coarse-grained distribution of halo particles is isotropic, $f_{halo}(\mathbf{p}, \mathbf{r}) = f_{halo}(p, r)$, one estimates

$$\frac{n}{\langle p^2 \rangle^{3/2}} = \frac{\left[\int f_{halo}(\mathbf{p}, \mathbf{r}) d^3\mathbf{p}\right]^{5/2}}{\left[\int f_{halo}(\mathbf{p}, \mathbf{r}) \mathbf{p}^2 d^3\mathbf{p}\right]^{3/2}} \sim f_{halo}(p_*, r) ,$$

where p_* is a typical momentum of the dark matter particles. In this way the magnitude of the coarse-grained distribution function in galactic halos is estimated as

$$f_{halo} \simeq \frac{Q}{3^{3/2}\, m^4} . \qquad (1)$$

Coarse-grained distribution function is known to decrease during violent relaxation in collisionless systems [8]. Hence, the primordial phase space density of dark matter particles cannot be lower than that observed in dark halos. This leads to the Tremaine–Gunn-like constraints on dark matter models [7]. The strongest among these constraints are obtained by making use of the highest phase space densities observed in dark halos, namely those of dwarf spheroidal galaxies (dSph) [4, 7]. dSph's are the most dark matter dominated compact objects, and seem to be hosted by the smallest halos containing dark matter [4]. In recently discovered objects Coma Berenices, Leo IV and Canes Venaciti II, the value of Q ranges from $5 \cdot 10^{-3} \frac{M_\odot/\text{pc}^3}{(\text{km/s})^3}$ to $2 \cdot 10^{-2} \frac{M_\odot/\text{pc}^3}{(\text{km/s})^3}$ [9]. In what follows we use the first, more conservative value,

$$Q = 5 \cdot 10^{-3} \frac{M_\odot/\text{pc}^3}{(\text{km/s})^3} . \qquad (2)$$

By requiring that the primordial distribution function exceeds the coarse-grained one, $f > f_{halo}$, one arrives at the constraint

$$3^{3/2} m^4 f > Q . \qquad (3)$$

This constraint gives rise to a reasonably well defined lower bound on m in a given model.

If the primordial distribution is such that (3) is barely satisfied, the formation of high-Q objects like dSph's is suppressed. In fact, it may be suppressed even for larger f, since the coarse-grained distribution function may decrease considerably during the evolution. The parameter

$$\Delta \equiv \frac{3^{3/2} m^4 f}{Q}$$

shows how strongly the coarse-grained distribution function f must be diluted due to relaxation processes in order that the formation of dense compact dark matter halos be suppressed. It is known from simulations that the phase space density decreases during the structure formation. In particular, during the nonlinear stage it decreases by a factor of 10^2 to 10^3 [10], or possibly higher. Hence, the primordial distribution function of WDM particles should be such that $\Delta \gtrsim 10^2 - 10^3$. At least naively, obtaining the dilution factor in a given model in the ballpark $\Delta = 1 - 10^3$ would indicate that the primordial phase space density is just right to make dwarf galaxies but not even more compact objects. Interestingly, we will find that Δ is indeed in this ballpark for WDM gravitinos.

2 Results

We make use of this phase space density criterion to examine light ($m \lesssim 15$ keV) gravitino as a warm dark matter candidate, assuming that R-parity is conserved and hence gravitino is stable. We find that gravitino mass should be in the range

$$1\,\mathrm{keV} \lesssim m_{\tilde{G}} \lesssim 15\,\mathrm{keV}\ .$$

In the early Universe, light gravitinos are produced in decays of superparticles and in scattering processes [12]. For so light gravitinos, their production in decays of superparticles plays an important role [13]. This process is most efficient at temperatures of the order of decaying particles mass. At lower temperatures number density of decaying particles is suppressed by Boltzmann factor and at higher temperatures expansion rate of the Universe is higher so the gravitino production rate is lower. Most notably, gravitinos serve as warm dark matter candidates only if other superparticles are rather light. We find that superparticles whose mass M is below the reheat temperature should obey

$$M \lesssim 350\ \mathrm{GeV}\ , \tag{4}$$

otherwise gravitinos are overproduced in their decays and in scattering and/or relic gravitinos are too cold. Barring fine tuning between the reheat temperature in the Universe and superparticle masses, this means that gravitino as warm dark matter candidate will soon be either ruled out or supported by the LHC experiments.

Gravitino production in scattering processes operates most efficiently at the highest possible temperatures in the early Universe, so the requirement that gravitinos are not overproduced restricts severely the reheat temperature T_R, cf. [13, 14]; we find that T_R must be at most in the TeV range.

The bound (4) is to be compared to the experimental bounds on masses of gluino and quarks of the 1st and 2nd generations, $M_{\tilde{q},\tilde{g}} \geq 250 - 325$ GeV [1]. Given the narrow interval between these bounds, we find it disfavored that squarks and gluinos participate in gravitino production processes. Hence, we elaborate also on a scenario with relatively light colorless superparticles whose masses M obey (4), heavy squarks and gluinos, and reheat temperature in between,

$$M \lesssim T_R \ll M_{\tilde{q},\tilde{g}}\ . \tag{5}$$

In this scenario, squarks and gluinos do not play any role in gravitino production, while the important production processes are decays and collisions of sleptons, charginos and neutralinos. We find that in this case, the overall picture is consistent in rather wide range of parameters, with the reheat temperature extending up to 10 TeV.

We conclude that unlike in the WIMP case, gravitino WDM does not automatically have the present mass density in the right ballpark. If the heaviest superparticles are squarks and gluinos, and they were relativistic in the cosmic plasma (the first scenario), the allowed range of parameters is rather narrow. We consider least contrived the possibility that the masses of sleptons, charginos and neutralinos are in the range $M = 150-300$ GeV, the reheat temperature is $T_R = 200$ GeV-10 TeV and the masses of gluinos and squarks are higher, $M_{\tilde{g},\tilde{q}} \gg T_R$ (second scenario). Then for masses $m_{\tilde{G}} = 1 - 15$ keV, gravitinos can indeed serve as warm dark matter particles. In any case, gravitino as warm dark matter candidate will be either ruled out or supported by the LHC experiments.

Acknowledgments

AKh gratefully acknowledges the conference organizers for support and hospitality. We are indebted to F. Bezrukov, A. Boyarsky, S. Demidov, V. Lukash, O. Ruchayskiy, M. Shaposhnikov and I. Tkachev for the interest to this work and useful discussions. This work was supported in part by the grant of the President of the Russian Federation NS-1616.2008.2, by the RFBR grant 08-02-00473-a and by the Russian Science Support Foundation (DG).

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