Diffusion of UltraHigh Energy Particles in Expanding Universe

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Abstract

Presence of sizeable (up to 10 nG) turbulent magnetic field in intergalactic space may essentially change the observed UltraHigh Energy Cosmic Ray *(UHECR)* spectrum. We have developed a quasi-analytical approach allowing to account for magnetic fields. The method is based on generalization of Syrovatsky solution. It is shown that the problem of superluminal propagation may be solved using an appropriate generalization of Jüttner propagator. This allows a smooth interpolation between high-energy rectilinear and low-energy diffusive propagation modes.

1 Introduction

UltraHigh Energy *(UHE)* Cosmic Ray *(CR)* particles hit the atmosphere at energies up to $E \sim 3\times10^{20}$ eV, producing Extended Air Showers *(EAS)* [1, 2, 3, 4, 5, 6]. To observe these rare events one needs extremely large (3000 km^2 in case of Pierre Auger Observatory *(PAO)* [7]) arrays of surface scintillator or water Cherenkov detectors. Due to excitation of nitrogen high in the atmosphere, the energy of an event may be measured by detection of the concomitant fluorescent light, provided the conditions (moonless cloudless night) were appropriate. Fluorescence method allows to calibrate the surface array in case of hybrid detector like PAO.

In spite of great efforts, neither type nor origin of UHECR particles are known after more than 50 years of research. The observed isotropy in the arrival directions and lack of reliable candidates among known Galactic sources hint that these particles are of extragalactic origin. Since photons and neutrinos are practically ruled out by observations, the conservative point of view is to assume UHECR are protons or nuclei accelerated in distant extremely powerful sources like AGN or GRB. At the moment there is no good source model allowing to accelerate CR up to such high energies. Typically one assumes that protons or nuclei are accelerated at shocks, though too large magnetic field and shock radius in a source are needed.

In the following only protons will be discussed as UHECR particles. Clear observations [4, 8, 6] of the predicted [9] dip due to e^+e^--pair production in measured spectra of all experiments at $E \sim (10^{18} - 4\times10^{19})$ eV and the spectrum suppression (most probably indicating the beginning of the GZK cutoff [10, 11] due to pion photoproduction) at $E \gtrsim 5\times10^{19}$ eV are in favor of such a model.

An alternative point of view is that dip in the spectrum is due to intersection of rapidly decreasing Galactic CR spectrum (which extends up to $E \sim 10^{19}$ eV in this case) and a flatter extragalactic one, the latter being a mixture of protons and heavier, up to iron, nuclei (see e.g. [12]). The PAO observation [13] of heavy nuclei fraction enhancement at $E \gtrsim 10^{19}$ eV is

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actually in favor of such an approach. However, low accuracy of depth of shower maximum X_{max} measurements and large systematic errors make this result uncertain. The HiRes measurements of X_{max}, on the contrary, indicate the pure proton composition at all energies [14].

Since sources of UHECR, most likely AGN, are unknown, one may guess them to be distributed isotropically in the universe with average spacing $d \sim (30-60)$ Mpc, in accordance with MC simulations [15, 16]. All sources are assumed to emit identical power-law decreasing with energy fluxes of protons,

$$Q_g(E,z) = Q_0 \times (1+z)^m \times (E/E_0)^{-\gamma_g} \ , \tag{1}$$

with spectrum indexes $\gamma_g = 2.2 - 2.7$, $m = 0 - 3$ accounting for a possible evolution of sources, z being a redshift of the epoch. Q_0 is the free normalization parameter relating to the total luminosity of sources. Additional parameters are E_{max} and z_{max}, the maximum energy of acceleration and time when sources have been "switched on", respectively. We shall concentrate on the propagation of CR through CMBR, with special emphasis on influence of turbulent InterGalactic Magnetic Fields *(IGMF)* on the observed spectra.

2 Universal spectrum

As the first step, let us neglect the point-like nature of sources, assuming they are homogeneously spread over the volume. In this case the "universal spectrum" arises as a solution of the equation for density n_p of UHE protons,

$$\frac{\partial}{\partial t} n_p(E,t) - \frac{\partial}{\partial E}\left[\tilde{b}(E,t) n_p(E,t)\right] = Q_g(E,t) \ , \tag{2}$$

with $\tilde{b}(E,t) = EH(t) + b(E,t)$, where $H(z) = H_0\,\sqrt{\Omega_m(1+z)^3 + \Omega_\Lambda}$ is the Hubble constant at epoch z with $\Omega_m = 0.3$ and $\Omega_\Lambda = 0.7$. Here $EH(t)$ describes adiabatic energy loss and $b(E,t) = dE/dt$ – those due interaction with CMB, i.e. $p+\gamma \to e^+ + e^- + p$, $p+\gamma \to \pi + X$ [9].

Written in the equivalent form

$$\frac{\partial}{\partial t} n_p(E,t) - \tilde{b}(E,t)\frac{\partial}{\partial E} n_p(E,t) - n_p(E,t)\frac{\partial \tilde{b}(E,t)}{\partial E} = Q_g(E,t) \ , \tag{3}$$

this first order partial differential equation reduces to an ordinary differential equation on characteristic $\mathcal{E}(t) = E_g(E,t)$, the solution of

$$dE/dt = -\tilde{b}(E,t) \tag{4}$$

with the initial condition $\mathcal{E}(0) = E$. The solution of obtained ODE reduces to the quadrature [9]:

$$n_p(E) \;=\; \int\limits_0^{z_{\max}} \frac{dz'}{(1+z')\,H(z')}\; Q_g\left[\mathcal{E}(z')\right] \times \frac{dE_g}{dE}[\mathcal{E}(z')] \ ; \tag{5}$$

$$\frac{dE_g}{dE}[\mathcal{E}(z')] \;=\; (1+z') \times \exp \int\limits_0^{z'} dz''\; \frac{(1+z'')^2}{H(z'')} \times \frac{\partial b[(1+z'')\,\mathcal{E}(z''),0]}{\partial E} \ . \tag{6}$$

Fitting actually parameters of the model to experimental data one get the universal spectrum which shows both dip and GZK-cutoff.

The analysis is especially convenient in terms of "modification factor" $\eta(E) = J_p(E)/J_p^{unm}(E)$. Here $J_p(E) = cn_p(E)$ is calculated according to (5) and $J_p^{unm}(E)$ is the spectrum calculated under an assumption that CMB energy losses are absent.

3 UHECR spectrum in case of grid of sources

Assume now sources are located in vertices of some cubic grid with comoving spacing d. The average luminosity of the comoving volume remains equal to that in case of universal spectrum. For rectilinear propagation, UHECR flux at Earth, lying in the origin of the 3D coordinate system, is a sum of contributions from point-like sources with coordinates $\{id, jd, kd\}$:

$$J_p(E) = \frac{c}{4\pi d^2} \sum_{i,j,k} \frac{Q_g(x_{ijk}, E)}{[(i+1/2)^2 + (j+1/2)^2 + (k+1/2)^2](1+z_{ijk})} \frac{dE_g(x_{ijk}, E)}{dE} , \tag{7}$$

where

$$x_{ijk} = d\sqrt{(i+1/2)^2 + (j+1/2)^2 + (k+1/2)^2} \tag{8}$$

is the distance to the source and dE_g/dE is given by Eq. (6).

The difference between these two approaches is significant only at very high energies. The lower is the spacing, the closer calculated spectrum is to the universal one. Calculated spectra [9] are shown in Fig. 1 in comparison with Akeno-AGASA spectrum.

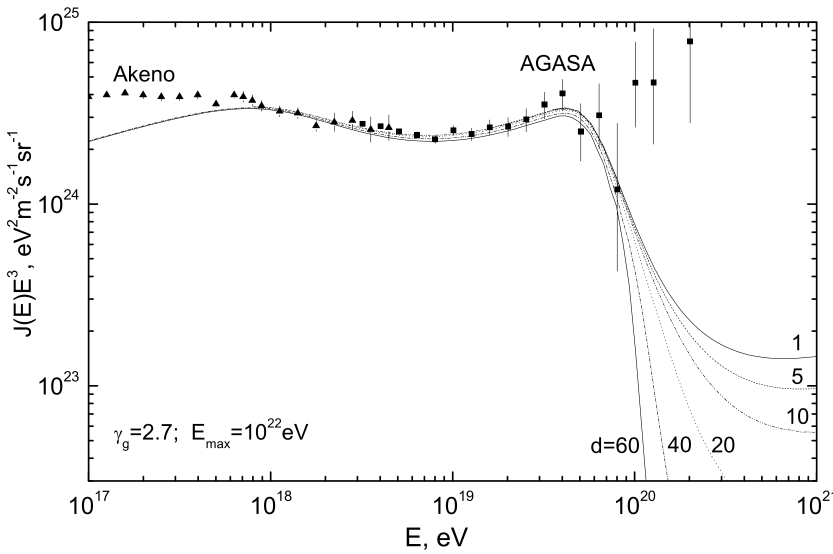

Figure 1: Proton spectra for rectilinear propagation from discrete sources. Sources are located in vertices of 3D cubic grid with spacing $d = 60, 40, 20, 10, 5$ and 1 Mpc. The calculations are performed for $z_{max} = 4$, $E_{max} = 1 \times 10^{22}$ eV and $\gamma_g = 2.7$.

It should be noted that more accurate calculations based on MC simulations, kinetic and Fokker-Plank equations practically coincide with the found spectrum. A small difference, the so called second dip, is discussed in [17].

4 Diffusion in magnetic field

It is quite natural to assume the intergalactic space to be filled with weak magnetic fields (for review see e.g. [18]). The scale and distribution of these fields are practically unknown. Hydrodynamical MC simulations of large scale structure formation, with magnetic field amplitude in the end rescaled to those observed in clusters of galaxies [15, 19], give different results: strong average magnetic field in calculations of [15] and weak in [19] simulation. Hence $< |\vec{B}| >$ may vary in the range $(10^{-3} - 10)$ nG.

Propagating through magnetized plasma particles do not loose their energy. However the diffusive path from a source to the detector gets longer as compared to the rectilinear one. Energy losses on CMB shift the particle to low energies, suppressing the spectrum. Since diffusive propagation proceeds as consecutive scatterings off different turbulent scales, it is important to set the highest coherent scale in magnetized plasma, l_c. This assumption determines the diffusion coefficient $D(E)$ at the highest energies when the proton Larmor radius, $r_L(E) \gg l_c$:

$$D(E) = \frac{c}{3} \frac{r_L^2(E)}{l_c} \; . \tag{9}$$

At "low" energies, when $r_L(E) \lesssim l_c$ the dependence of diffusion coefficient on energy is unknown. One can discuss two possibilities:

- the Kolmogorov diffusion coefficient

$$D_K(E) = \frac{c\; l_c}{3} \left[\frac{r_L(E)}{l_c} \right]^{1/3} , \tag{10}$$

- and the Bohm diffusion coefficient

$$D_B(E) = \frac{c}{3}\, r_L(E) \; . \tag{11}$$

The characteristic energy E_c of the transition between the 'high'- and 'low'-energy regimes is determined by the condition $r_L(E) = l_c$:

$$E_c = 0.93 \left(\frac{B}{1 \text{ nG}} \right) \left(\frac{l_c}{Mpc} \right) \text{ EeV} \; . \tag{12}$$

5 Syrovatsky solution in Galaxy

The diffusive propagation of high energy protons in turbulent Galactic magnetic fields was first discussed by S. I. Syrovatsky in 1959 [20]. For the case of energy dependent energy losses, $b(E)$, and a diffusion coefficient, $D(E)$, and a source located at distance $\vec{r}$ from a detector the diffusion equation reads

$$\frac{\partial}{\partial t} n_p(E, \vec{r}, t) - \text{div}\left[D(E) \nabla n_p\right] - \frac{\partial}{\partial E}\left[b(E) n_p\right] = Q(E, \vec{r}) \delta^3(\vec{r} - \vec{r}_g) \; . \tag{13}$$

Here $n_p(E, \vec{r}, t)$ is the space density of particles p with energy E at time t at the point $\vec{r}$, $Q(E, \vec{r}, t)$ is the source generation function. Syrovatsky found an exact analytic solution, the Green function, to this diffusion equation. For a single-source spherically-symmetric case (13) it can be presented [21] as

$$n_p(E, r) = \frac{1}{b(E)} \int_E^\infty dE_g \; Q(E_g) \, \frac{exp\left[- \frac{r^2}{4\lambda(E, E_g)} \right]}{\left[4\pi \lambda(E, E_g)\right]^{3/2}} \; , \tag{14}$$

where $r = |\vec{r}|$ and

$$\lambda(E, E_g) = \int_E^{E_g} dE' \; \frac{D(E')}{b(E')} \tag{15}$$

is the Syrovatsky variable which has the meaning of the squared distance traversed by a particle in the observer direction, while its energy diminishes from E_g to E.

6 Diffusion in intergalactic space

The problem of proton diffusion in intergalactic space is more complicated. Both diffusion coefficient and energy losses now depend on time; the adiabatic energy losses are to be taken into account as well. An unknown complicated space distribution of magnetic fields (in general low in voids and high in filaments and clusters of galaxies) makes the problem especially difficult.

Many authors studied the propagation of UHECR in extragalactic space using different methods. It is difficult to cite all papers with correct chronology, and we do not intend to make a complete list of them. For example, a MC simulation approach was undertaken in Ref. [22]. However, due to a rapid increase of the consumed processor time with energy decreasing, the study was limited by $E \gtrsim 3\times10^{19}$ eV. Actually, this is a common problem for all MC simulations: it is slow and expensive to vary parameters so that to see the dependence of spectra on them. On the other hand, uncertainties in input parameters make the account for fluctuation (an otherwise great advantage of MC method) useless in this case.

An analytical analysis of propagation in magnetic fields was made in Refs. [23, 24]. It was shown that due to the limited age of universe and to the rapid GZK energy losses increase, there arises a 'magnetic horizon' for sources yielding to the observed spectrum. In fact, such a horizon was already present in the paper of Syrovatsky [20]. Really, due to presence of upper limit E_{max} to acceleration energy in any source, the maximum squared distance traversed by a particle in the observer direction, $\lambda(E, E_g)$ (15), is limited. This parameter actually is the squared size of magnetic horizon. At small energies the horizon is defined by the age of universe, $t_0 \sim H_0^{-1}$.

In paper [25] the analytical approach of Syrovatsky was extended from Galaxy to the whole universe. Since Syrovatsky solution exists just for the time-independent diffusion coefficient and energy losses, it was done under an assumption of 'static universe'. The dependence of the coefficients just on energy was assumed, and additionally the adiabatic energy losses were included in the model. Such an approach was valid at high energies, especially when the diffusion equation solution was taken in a combination with the rectilinear one at highest energies. The problem of transition from the rectilinear near the source or at extremely high energy propagation to the diffusion description was solved there by interpolation between these solutions. An important 'propagation theorem' has been proved in this paper. It stated that *if distance between sources is much less than all propagation distances, such as energy-attenuation length, $l_{\rm att}$, and diffusion length $l_{\rm diff}$, the spectrum is not distorted and has a universal (standard) shape.* So, the universal spectrum could be regarded as an upper limit to all solutions taking into account the IGMF.

An important problem of a low-energy ($E \lesssim 1$ EeV) flattening of the extragalactic proton spectrum due to diffusion in IGMF was solved in Ref. [26]. Using a technique similar to that developed in Ref. [25], it was shown that the account for IGMF do allows so solve the problem of transition from the Galactic to extragalactic CR spectrum at $E \sim 1$ EeV.

The solution of diffusion equation

$$\frac{\partial}{\partial t}n_p(E,\vec{r},t) - \mathrm{div}\,[D(E,t)\nabla n_p] - \frac{\partial}{\partial E}\,[b(E,t)n_p] = Q(E,\vec{r},t)\delta^3(\vec{r}-\vec{r}_g)\ , \tag{16}$$

with time dependent coefficients $D(E,t)$ and $b(E,t)$ in the expanding universe has been obtained in paper [27]. It generalized the Syrovatsky solution by introducing new variable $\lambda(E,t,t')$, which was analogous to (15)

$$\lambda(E,t,t') = \int_{t'}^{t} dt'' \frac{D[\mathcal{E}(t''),t'']}{a^2(t'')}\ , \tag{17}$$

where $a(t)$ is the scale factor of the expanding universe and $\mathcal{E}(t') = E(E,t,t')$ is the characteristic trajectory, the solution of differential equation

$$\frac{dE}{dt} = -b(E,t)\ , \tag{18}$$

with an initial condition $\mathcal{E}(0) = E$. The main idea was to change the integration with respect to energy (15) by integration along the curved characteristic lines, given by solutions of Eq. (18). The derivation of this solution was given in the paper using Fourier transformation in the comoving space.

The found solution, similar to the Syrovatsky one, was

$$\begin{aligned} n(\vec{x}, E) &= \int_0^{z_g} dz \left|\frac{dt}{dz}\right| (1+z)\, Q(\mathcal{E}_g, z)\, \exp\left[\int_0^{z} dz' \left|\frac{dt'}{dz'}\right| \frac{\partial b_{int}(\mathcal{E}', z')}{\partial \mathcal{E}'}\right] \times \\ & \quad \frac{\exp[-(\vec{x}-\vec{x}_g)^2/4\lambda(E,z)]}{[4\pi\lambda(E,z)]^{3/2}} \end{aligned} \tag{19}$$

with $\vec{x}$ being the comoving system coordinate of observation and $\vec{x_g}$ being the coordinate of a source; $b_{int}(E,t)$ is the energy loss due to interactions with CMB.

The method was tested by checking the convergence of solutions to universal spectrum and to rectilinear solution in limiting cases.

With the help of solution (19) proton spectra were recalculated in papers [28, 29]. In spirit of approach of paper [30], sources have been distributed in vertices of 3D cubic grid with equal spacing $d = (20 - 100)$ Mpc in the comoving system. Again, at very high energies, when the diffusion length $l_d = 3D(E)/c$ is higher that distance to a source, the rectilinear solution (7) was applied. At lower energies the solution (19) was used. In fact, these two solutions did not meet smoothly each other, signaling the problem. Different interpolation have been used to treat the problem, but it was unavoidable and revealed itself in the proton spectrum at intermediate energies after summation over grid.

One more severe problem was the presence of unphysical superluminal propagation signal in the diffusion solution, which is quite natural for the non-covariant diffusion equation. This is the fundamental problem, well known for diffusion equations.

However, the spectra calculated in paper [28] were correct in low- and high-energy limits and they converged to the universal spectrum, as it has been expected. In this paper the probable evolution of average magnitude of IGMF was also studied.

7 Jüttner propagator

The problem of superluminal propagation in non-relativistic diffusion is well-known for a long time. Many authors tried to solve it, e.g. using the telegraph equation

$$\tau_d \frac{\partial^2}{\partial t^2} n + \frac{\partial}{\partial t} n - D\nabla^2 n = Q\,, \tag{20}$$

by adding one more small parameter τ_d; in the limit $\tau_d \to 0$ Eq. (20) reduces to the ordinary diffusion equation. Nonetheless, the satisfactory solution of the problem is still not found.

In our case the problem of superluminal propagation shows itself by particle immediate arriving from a source to the detector, so that no energy loss occurs; it must be considered as an unphysical signal. In terms of Syrovatsky solution (14) the problem is that integration starts at $E_{min} = E$, while it is forbidden for any distant source due to finite energy losses. It implies the velocity of a particle $v \to \infty$.

The *physical* value of the minimal generation energy $E_g^{\min}(E, r)$ is given by the rectilinear propagation and it can be easily calculated if $\tilde{b}(E) = -dE/dt$ (4) is known. But with this $E_g^{\min}$ Eq. (14) is not any more the solution of Eq. (13). In the left panel of Fig. 2 the unphysical region of the solution (14) with superluminal velocity is shown as the hatched area.

In the work [31] it was observed that the problem of relativization of the Maxwell distribution is similar to the relativization of diffusion propagator (the Green function). The normalized

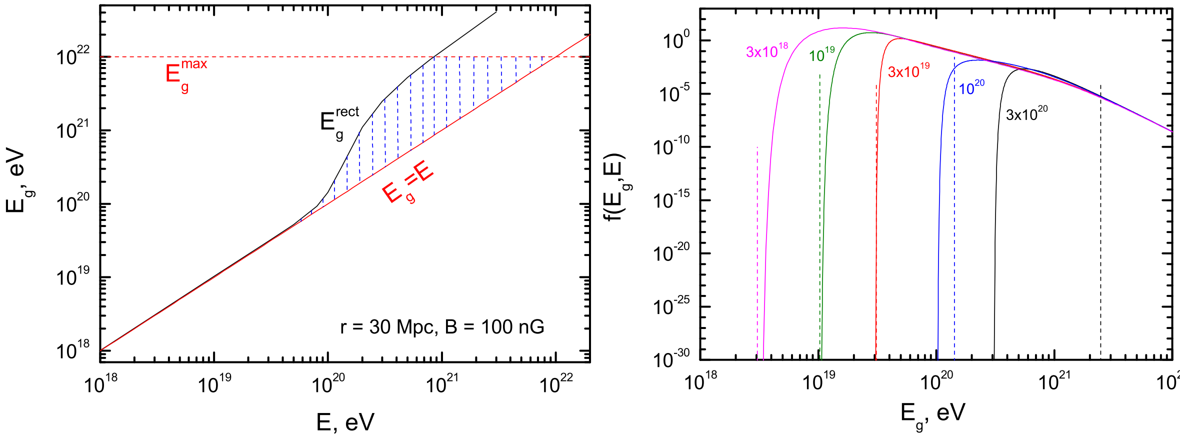

Figure 2: The superluminal problem in the Syrovatsky solution caused by energy losses of UHE protons for $B_c = 100$ nG and distance to the source $r = 30$ Mpc. In the left panel is shown the region in $(E_g\ , E)$ plane (above $E_g = E$ line) allowed by Eq. (14). The line $E_g = E_g^{\rm rect}(E, r)$ corresponds to rectilinear propagation and to the physical lower limit in the solution (14). The hatched region between $E_g = E$ and $E_g = E_g^{\rm rect}$ corresponds to superluminal velocities. For $E \lesssim 1\times 10^{20}$ eV the solutions practically do not have superluminal velocities. In the right panel the integrand of Eq. (14) is shown as the function of E_g for fixed E with values indicated by the numbers, given in eV. The dashed lines show the rectilinear physical lower limits $E_g^{\rm min}$. At $E \lesssim 1 \times 10^{20}$ eV the regions below these limits, i.e. with $v > c$ are negligibly small.

(per unit phase volume) probability density function of the Maxwell distribution is given by

$$P_M(v) = \left(\frac{m}{2\pi kT}\right)^{3/2} \exp\left(-\frac{mv^2}{2kT}\right). \tag{21}$$

Changing $v \to x$ and $kT/m \to 2Dt$, one obtains the Green function of the 3D diffusion equation

$$P_{\rm diff}(r,t) = \frac{1}{(4\pi Dt)^{3/2}} \exp\left(-\frac{r^2}{4Dt}\right) , \tag{22}$$

where r is a distance to the source and D is time-independent diffusion coefficient. Therefore, it is reasonable to use the Jüttner function [32] for the diffusive Green function, where superluminal velocities are absent.

Using the propagator $P(E,t,r)$, where E is the observed energy, t is the propagation time and r is the distance to a source, the yield to the observed space density is

$$n(E,r) = \int_0^\infty dt\; Q[E_g(E,t),t]\; P(E,t,r)\; \frac{dE_g}{dE}(E,t), \tag{23}$$

where $E_g(E,t)$ is the energy of a particle at time t, analytic expression for dE_g/dE is given by Eq. (6), and $Q(E,t)$ is the source generation function. $P(E,t,r)$ can be thought of as a Green function of an unknown relativistic equation of propagation.

For rectilinear propagation of ultrarelativistic particles with $v \approx c$ the propagator is given by

$$P_{\rm rect}(E,t,r) = \frac{1}{4\pi c^3 t^2}\; \delta(t - \frac{r}{c}) , \tag{24}$$

and Eq. (23) results in

$$n(E,r) = \frac{q[E_g(E,r/c), r/c]}{4\pi c r^2}\; \frac{dE_g}{dE}, \tag{25}$$

which coincides with expression obtained from conservation of number of particles. For the case of diffusion the propagator Eq. (19) was

$$P_{\text{diff}}(r,t) = \frac{1}{[4\pi\lambda(E,t)]^{3/2}} \exp\left(-\frac{r^2}{4\lambda(E,t)}\right), \tag{26}$$

Both propagation functions $P_{\text{rec}}(E,t,r)$ and $P_{\text{diff}}(E,t,r)$ are normalized by unity

$$\int dV P(E,t,r) = 1 \tag{27}$$

and thus they have a meaning of probability to find a particle in a unit volume at distance r from a source at time t after emission.

The modified Jüttner distribution in terms of $r = v$ and $2Dt = kT/m$, is given in [31] as

$$P_J(E,t,r) = \frac{\theta(ct-r)}{(ct)^3 Z(c^2t/2D)\,[1-r^2/(c^2t^2)]^2} \exp\left[-\frac{c^2t/2D}{[1-r^2/(ct)^2]^{1/2}}\right], \tag{28}$$

where

$$Z(y) = 4\pi K_1(y)/y \tag{29}$$

with $K_1(y)$ being the modified Bessel function. The superluminal propagation with $r > ct$ is forbidden for this propagator.

In paper [33] we introduced instead of the Jüttner function (28) the generalized Jüttner function $P_{gJ}(E,t,r)$, imposing to it two limiting conditions of transition to rectilinear propagator (24) and the Syrovatsky propagator (22), and keeping the condition of subluminal velocities $r \le ct$. For this aim we substituted in Eq. (28)

$$\frac{c^2t}{2D} \to \frac{c^2t^2}{2\lambda[E_g(E,t)]} \equiv \alpha(t)\ , \tag{30}$$

($\lambda[E_g(E,t)]$ is given by Eq. (17)) and introduced instead of t a new variable

$$\xi(t) = r/ct\ . \tag{31}$$

Both new quantities are dimensionless.

The generalization imposed by Eq. (30) is motivated by time-dependent diffusion coefficient $D[E_g(t)]$ and by the presence of energy losses $b(E) = -dE/dt$. In this case we generalize the quantity $D \cdot t$ in the Jüttner distribution (28) to $\int D(t)dt = \lambda(E,t)$ given by Eq. (17). As a result we have

$$\frac{c^2t}{2D} = \frac{c^2t^2}{2Dt} \to \frac{c^2t^2}{2\int D(E,t)dt} = \frac{c^2t^2}{2\lambda(E,t)} \equiv \alpha(E,t)\ . \tag{32}$$

In terms of ξ and $\alpha(E,t)$ the generalized Jüttner function $P_{gJ}(E,t,x)$ and density of particles $n(E,r)$ are given by:

$$P_{gJ}(E,t,r) = \frac{\theta(1-\xi)}{4\pi(ct)^3}\frac{1}{(1-\xi^2)^2}\frac{\alpha(E,\xi)}{K_1[\alpha(E,\xi)]}\exp\left[-\frac{\alpha(E,\xi)}{\sqrt{1-\xi^2}}\right]\ , \tag{33}$$

$$n(E,r) = \frac{1}{4\pi c r^2}\int\limits_{\xi_{\min}}^{1} d\xi \frac{Q[E_g(E,\xi)]}{(1-\xi^2)^2}\xi \frac{\alpha(E,\xi)}{K_1[\alpha(E,\xi)]}\exp\left[-\frac{\alpha(E,\xi)}{\sqrt{1-\xi^2}}\right]\frac{dE_g}{dE}\ . \tag{34}$$

Various regimes of propagation are defined mostly by parameter α. In particular, $\alpha \ll 1$ corresponds to rectilinear propagation and $\alpha \gg 1$ to diffusion. In Fig. 3 the evolution of $\alpha(E,z)$ is presented as function of redshift z, along the energy trajectories $E_g = \mathcal{E}(E,z)$, where E is

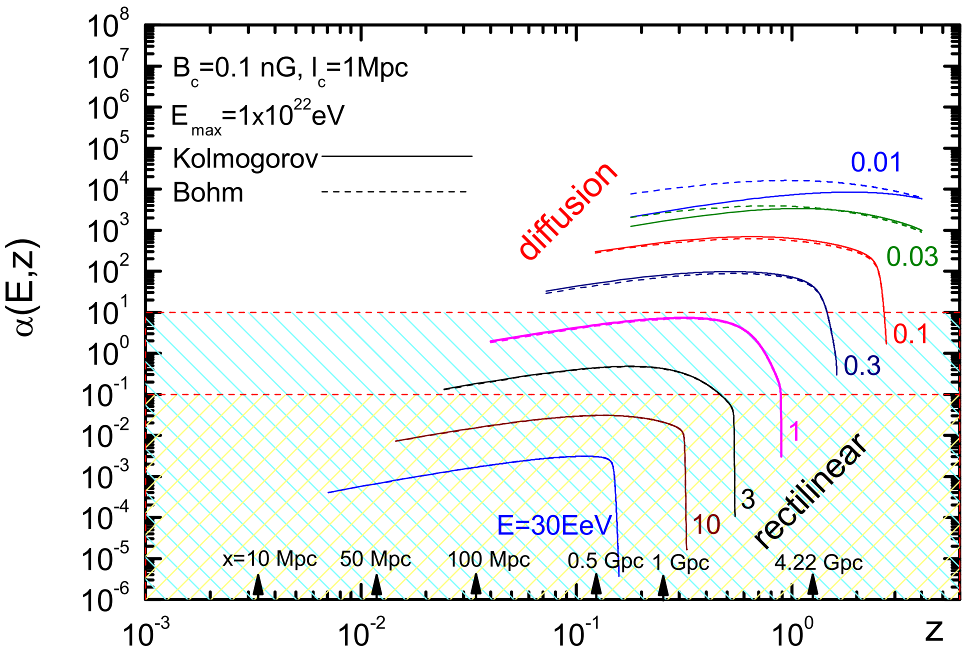

Figure 3: Parameter $\alpha(E,z)$ as function of observed energy E and redshift z for magnetic field configuration $(B_c, l_c) = (0.1$ nG, 1 Mpc$)$. Parameter α changes along the particle energy trajectory $E_g = \mathcal{E}(E,z)$. The mode of space propagation (diffusive, intermediate, rectilinear) is changing accordingly. The regions of propagation correspond to diffusion ($\alpha \geq 10$), rectilinear ($\alpha \leq 0.1$) and intermediate ($0.1 \leq \alpha \leq 10$). The observed energies in EeV are shown at the evolutionary curves. Along each trajectory the energy E_g increases and α typically decreases due to increasing of $\lambda(E, E_g)$.

the observed energy. The evolution is shown for different E and magnetic field configuration (B_c, l_c) =(0.1 nG, 1 Mpc).

It is easy to prove that Eqs. (33) and (34) have the correct rectilinear and diffusive asymptotic behavior (see [33]).

Since generalized Jüttner propagator (33) is a smooth differentiable function of energy and has the correct asymptotic behaviors (rectilinear at high energy and diffusive at low energies), it provides the smooth interpolation between these solutions with superluminal propagation being excluded. Thus it solves simultaneously both problems of non-relativistic diffusion equation.

Using the generalized Jüttner propagator (33) the proton spectra have been calculated in [33] for several sets of parameters, values of average magnetic fields $B(E,t)$, types of diffusion at low energies (Kolmogorov or Bohm), largest coherent scale in magnetized plasma l_c and different spacing between sources according to a formula:

$$J_p(E) = \frac{c}{4\pi H_0}\,\frac{q_0(\gamma_g-2)}{E_0^2}\sum_s \frac{1}{4\pi c x_s^2}\int\limits_{\xi_{\rm min}}^{1} \frac{\xi_s d\xi_s}{1+z(\xi_s)}\,\frac{[E_g(E,\xi_s)]^{-\gamma_g}}{(1-\xi_s^2)^2}\,\frac{\alpha}{K_1(\alpha)}\exp\left(-\frac{\alpha}{\sqrt{1-\xi_s^2}}\right)\frac{dE_g}{dE}. \tag{35}$$

In Figs. 4, 5 spectra calculated using interpolation of [28] and with the help of generalized Jüttner propagator are compared. The advantage of the latter approach is clearly seen. It does not allow the superluminal propagation of particles, smoothly interpolates between known asymptotics and provides an efficient definition of the type of propagation in terms of one dimensionless parameter $\alpha(E,z)$ (32).

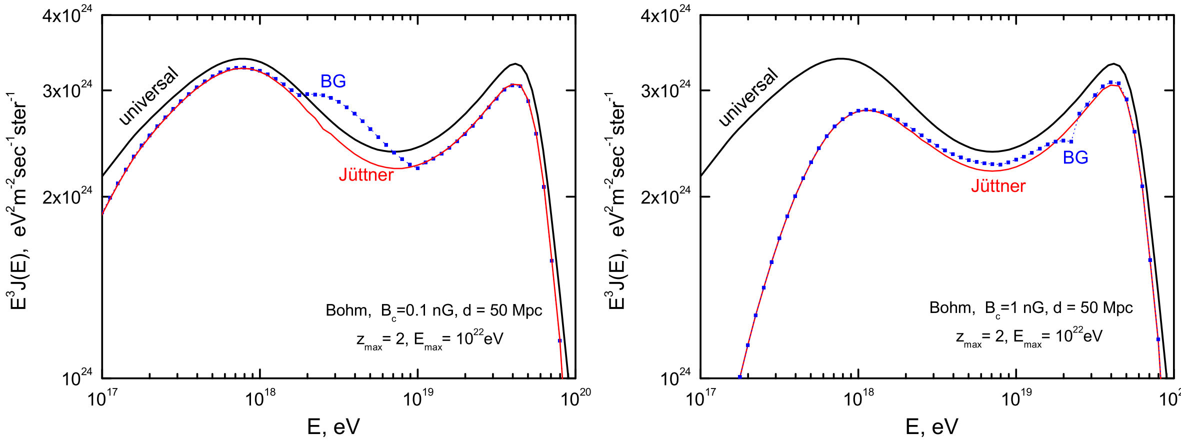

Figure 4: Comparison of the Jüttner solution with the combined diffusive and rectilinear solution (BG dotted curves) from work [28]. The left panel shows the case $B_c = 0.1$ nG, and the right panel $B_c = 1$ nG, the distance between sources $d = 50$ Mpc in both cases and $\gamma_g = 2.7$. The universal spectrum is also presented for $\gamma_g = 2.7$. The features seen in the BG spectra are artifacts produced by assumption about transition from diffusive to rectilinear propagation (see text). These features are small: note the large scale on the ordinate axis.

8 Conclusions

The present state of the problem of propagation of UHECR, presumably mostly protons, through intergalactic turbulent magnetic fields was discussed. Simultaneous analytical description of both rectilinear and diffusive parts of a particle trajectory from a source to a detector remains a serious problem. We have traced the development of ideas in this field starting from the fundamental Syrovatsky solution applied to Galaxy to the description with help of generalized Jüttner propagator in the expanding universe. With certain set of parameters, the found solution allows to satisfactorily explain the measured UHECR spectra, conserving such predicted features as dip at $E \sim 1 - 40$ EeV and GZK cutoff at $E \gtrsim 50$ EeV.

It should be noted that the alternative mixed composition model [34] also allows the account for IGMF. In paper [35] an interesting study of UHECR, including also the pure proton fluxes, propagation in IGMF was performed. A combination of MC simulations and analytical approach

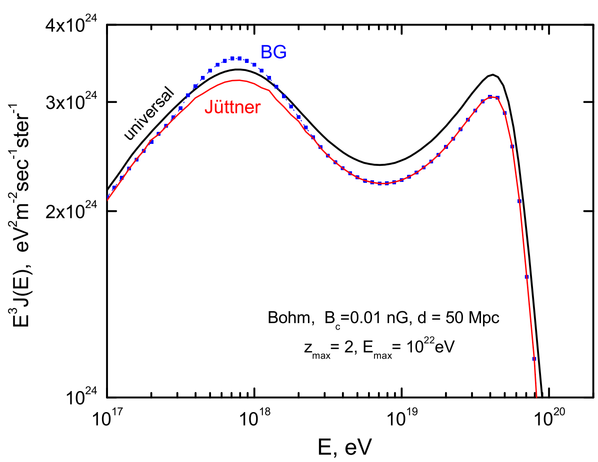

Figure 5: The same as in Fig. 4 for $B_c = 0.01$ nG.

for simultaneous diffusive and 'ballistic' parts of trajectory was used.

Different methods of description of UHECR propagation in the universe are to be developed in future. A careful measurement of chemical composition of UHECR flux and of accompanying cosmogenic ultrahigh energy neutrino fluxes will also allow to better understand the IGMF.

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