Non-minimal coupling in inflation and inflating with the Higgs boson

F. L. Bezrukov*
Max-Planck-Institut für Kernphysik
PO Box 103980, 69029 Heidelberg, Germany
Institute for Nuclear Research of Russian Academy of Sciences
Prospect 60-letiya Oktyabrya 7a, Moscow 117312, Russia

Abstract

We analyse the effect of the non-minimal coupling of the form $\xi\phi^2 R/2$ on the single field inflation. If the non-minimal coupling is large, it relaxes the constraint on the field self coupling, making it possible to use the Standard Model Higgs field as the inflaton. At the same time, even small non-minimal coupling constant, $\xi \gtrsim 10^{-3}$, brings the usual inflaton with quartic potential in agreement with the WMAP5 observations.

1 Introduction

It is now widely accepted that the initial phase of the Universe was that of the exponential expansion, called inflation [1, 2, 3, 4, 5, 6]. It explains the extreme flatness of the Universe, and predicts a nearly scale invariant spectrum of the initial density perturbations, which is now confirmed by experimental observations of the Cosmic Microwave Background (CMB) [7]. Simplest realization of inflation can be made in a theory of a scalar field. However, the observed amplitude of the perturbations (COBE normalization) requires extremely flat potential for this field, $\lambda \sim 10^{-13}$ for the quartic coupling constant [8]. Moreover, simplest models of inflation (with monomial potentials) predict large amount of tensor modes generated during inflation. For the case of quartic interaction this is already strongly disfavoured by observations.

It was noticed, that non-minimal coupling to gravity relaxes the required fine-tuning of the coupling constant [9, 10, 11, 12, 13, 14]. Here I will argue, that taking into account non-minimal interaction of the scalar field with gravity (which is quite natural and is in fact even required by quantum corrections to the action [15]) allows to incorporate inflation in the Standard Model (SM) without introduction of any new particles. Or, if the new scalar inflaton is introduced, non-minimal coupling largely reduces the amount of tensor modes produced, which will be bounded by future experiments.

2 Generic description

If we write the action for the scalar field and gravity with all operators of dimension not greater then 4 with no more then two derivatives[1] we get

$$S_J = \int d^4x\sqrt{-g}\left\{-\frac{M^2+\xi\phi^2}{2}R + \frac{\partial_\mu\phi\partial^\mu\phi}{2} - V(\phi)\right\} , \tag{1}$$

* **e-mail**: fedor@ms2.inr.ac.ru

[1] Terms with more derivatives produce additional degrees of freedom in the theory, and presumable need special analysis.

with the potential

$$V(\phi) = \frac{\lambda}{4}\phi^4 + \frac{m^2}{2}\phi^2 \ , \qquad (2)$$

In the following we suppose that quadratic term is irrelevant during inflationary regime (it is true if $m \ll M_P$). The simplest way to analyse this action (see, eg. [11, 13, 16]) is to make the conformal transformation

$$\hat{g}_{\mu\nu} = \Omega^2 g_{\mu\nu} \ , \quad \Omega(\phi)^2 = \frac{M^2 + \xi\phi^2}{M_P^2} \ , \qquad (3)$$

where $M_P \equiv 1/\sqrt{8\pi G_N} = 2.44 \times 10^{18}$ GeV is the reduced Planck mass. This transformation leads to a non-minimal kinetic term for the scalar field, which can be removed by changing to the new scalar field χ

$$\frac{d\chi}{d\phi} = \sqrt{\frac{\Omega^2 + 6\xi^2\phi^2/M_P^2}{\Omega^4}} \ . \qquad (4)$$

Finally, the action (called the Einstein frame action, opposed to the original Jordan frame action S_J)

$$S_E = \int d^4x\sqrt{-\hat{g}}\left\{ -\frac{M_P^2}{2}\hat{R} + \frac{\partial_\mu\chi\partial^\mu\chi}{2} - U(\chi)\right\} \ , \qquad (5)$$

where $\hat{R}$ is calculated using the metric $\hat{g}_{\mu\nu}$ and the potential is rescaled with the conformal factor

$$U(\chi) = \frac{V(\phi(\chi))}{\Omega(\phi(\chi))^4} \ . \qquad (6)$$

Already here one can hope that the situation is better, than without the non-minimal coupling: for large field values $\Omega \propto \phi$, and the Einstein frame potential U becomes flat.

In the following two sections I will describe two cases, corresponding to large and small non-minimal coupling ξ.

3 Large non-minimal coupling, or inflation with the Higgs

The case of large non-minimal coupling ξ is particularly simple [17, 18, 10, 16]. We have the following change of variables

$$\chi \simeq \begin{cases} \phi & \text{for} \quad \phi < \sqrt{\frac{2}{3}}\frac{M_P}{\xi} \ , \\ \sqrt{\frac{3}{2}}M_P \log\Omega(\phi) & \text{for} \quad \sqrt{\frac{2}{3}}\frac{M_P}{\xi} < \phi \ , \end{cases} \qquad (7)$$

and the potential

$$U(\chi) \simeq \begin{cases} \frac{\lambda}{4}\chi^4 & \text{for } \chi < \sqrt{\frac{2}{3}}\frac{M_P}{\xi} \ , \\ \frac{\lambda M_P^4}{4\xi^2}\left(1 - \mathrm{e}^{-\frac{2\chi}{\sqrt{6}M_P}}\right)^2 & \text{for } \sqrt{\frac{2}{3}}\frac{M_P}{\xi} < \chi \, . \end{cases} \qquad (8)$$

Thus, at large χ the potential is exponentially flat for *any* value of the self coupling λ. The ratio λ/ξ^2 defines the energy density at high fields now, and thus it is possible to satisfy the COBE normalization for any λ by choosing sufficiently large value of ξ,

$$\xi \simeq \sqrt{\frac{\lambda}{3}\frac{N_{\mathrm{COBE}}}{0.027^2}} \simeq 49000\sqrt{\lambda} = 49000\frac{m_H}{\sqrt{2}v} \ , \qquad (9)$$

where $N_{\mathrm{COBE}} \simeq 60$.

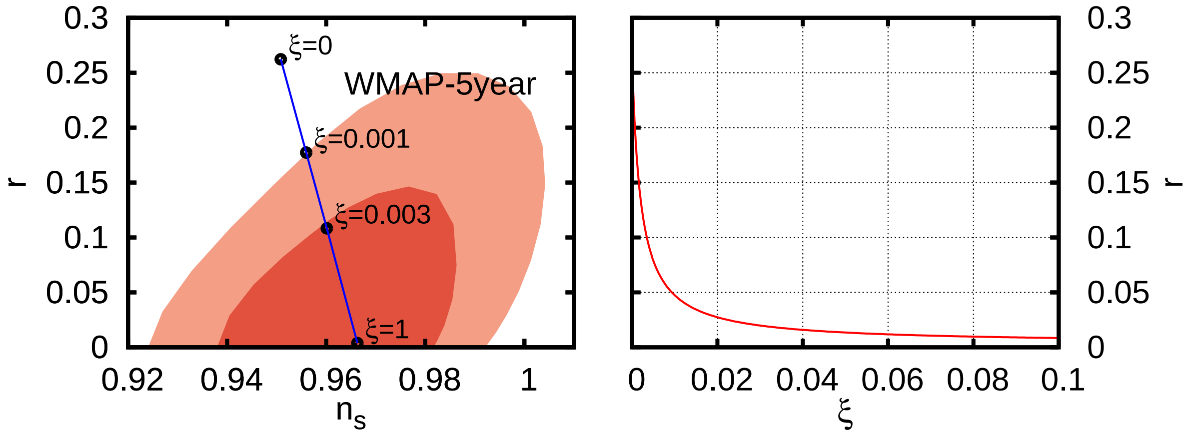

Figure 1: Left: WMAP5 preferred region for the spectral index n_s and tensor-to-scalar ratio r with the predictions form the non-minimally coupled inflation for several ξ. Note, that the points for $\xi > 1$ are indistinguishable from the one with $\xi = 1$. Right: scalar-ro-tensor ratio dependance on the non-minimal coupling constant ξ.

The spectral index and tensor-to-scalar ratio are independent of the coupling constants in the limit of large ξ, $n_s \simeq 1 - 8(4N+9)/(4N+3)^2 \simeq 0.97$, $r \simeq 192/(4N+3)^2 \simeq 0.0033$. The predicted values are well within one sigma of the current WMAP measurements [7], see Fig. 1.

A very interesting candidate for the inflaton in this case is just the SM Higgs field. Though its self interaction is of the order of unity, sufficiently large $\xi \sim 10^4$, (9), allows it to drive inflation, and no new fields have to be added to the SM.

The interesting problem here is the radiative corrections to the potential form the other fields of the SM, like the top quark and the gauge bosons. There are several ways of addressing this problem, that differ by the frame used to define he cut off energy (or normalization point). In [17, 18] the cut off effectively corresponds to constant Planck mass in the Einstein frame, leading to corrections that do not spoil the flatness of the potential (8), while [19] advocates that constant Plank mass cut off should be taken in the Jordan frame, leading to larger corrections, but, interestingly, still allowing for successful inflation, only for rather heavy Higgs mass. The proper way of taking these corrections into account is an important question open for further discussion.

4 Small non-minimal coupling

However, large non-minimal coupling ξ is not the only possibility. As far as for zero ξ the amount of tensor modes predicted is too large, and it is very small for large ξ, one can ask a question what is the minimal value of ξ that suppresses the tensor mode generation just enough to put them into the WMAP5 preferred region. To answer this question, let us write the expressions for the spectral index and tensor-to-scalar ratio in the limit of small non-minimal coupling constant ξ.

The slow roll parameters ϵ and η are [13, 16]

$$\epsilon = \frac{8M_P^4}{\phi^2(M_P^2 + \xi\phi^2(1+6\xi))}\,, \tag{10}$$

$$\eta = \frac{4M_P^2(3M_P^4 + \xi M_P^2\phi^2(1+12\xi) - 2\xi^2\phi^4(1+6\xi))}{\phi^2(M_P^2+\xi\phi^2(1+6\xi))^2}\,. \tag{11}$$

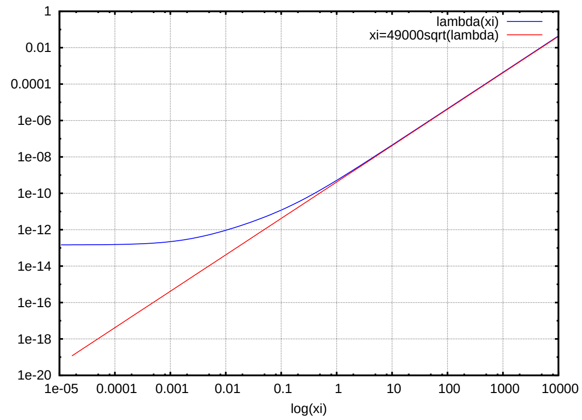

Figure 2: Dependence of the quartic self-coupling λ on the non-minimal coupling parameter ξ, deduced from requirement of the correct normalization of the density perturbations.

The end of the slow-roll regime ($\epsilon = 1$) corresponds to the field value ϕ_e

$$\begin{aligned} \frac{\xi\phi_{\rm e}^2}{M_P^2} =& \frac{1}{2(1+6\xi)}\left(\sqrt{192\xi^2+32\xi+1}-1\right) \\ &\approx 8\xi + \mathcal{O}(\xi^2),\ (\xi \ll 1)\ . \end{aligned} \tag{12}$$

The number of e-foldings that happened when the field changed its value from ϕ_N to ϕ_e is

$$\begin{aligned} N =& \frac{1}{M_P^2}\int_{\phi_{\rm e}}^{\phi_N} \frac{V}{(dV/d\phi)}\left(\frac{d\hat{\phi}}{d\phi}\right)^2 d\phi \\ =& \frac{1}{8}\left[\frac{\phi_N^2-\phi_{\rm e}^2}{M_P^2}(1+6\xi) - 6\ln\left(\frac{M_P^2+\xi\phi_N^2}{M_P^2+\xi\phi_{\rm e}^2}\right)\right]\ . \end{aligned} \tag{13}$$

For small $\xi \ll 1$ it reduces to $\phi_N \simeq \sqrt{8(N+1)}M_P$. Then it is easy to calculate the tensor-to scalar ratio [7]

$$\begin{aligned} r = 16\epsilon =& \frac{128 M_P^4}{\phi_N^2(M_P^2+\xi\phi_N^2(1+6\xi))} \\ \simeq& \frac{16}{(N+1)(1+8(N+1)\xi(1+6\xi))}\ . \end{aligned} \tag{14}$$

The spectral index can be obtained as $n_s - 1 = -6\epsilon + 2\eta$. The exact formulas are not hard to get by combining (10), (11), (12), and (13), but they are slightly cumbersome and not very instructive, though they were used to plot the figures. Fig. 1 gives the results for several values of ξ together with the WMAP5 preferred region [7]. On can see, that for $\xi > 0.001$ and $\xi > 0.003$ the predictions enters the 2σ and 1σ contours, respectively. The quartic coupling constant λ should still be extremely small for these values to satisfy the COBE normalization $U/\epsilon = (0.027 M_P)^4$, see Fig. 2.

One can even argue, that for "natural" value for the non-minimal coupling constant of the order of 1, the amount of tensor perturbations generated is extremely small, so it is natural *not* to expect the large tensor modes form the inflation, contrary to the usual conclusion for large single field inflation.

5 Conclusions

We have demonstrated that the non-minimal coupling of the scalar field to gravity loosens the bounds on the field self-coupling constant required for successful inflation, and reduces the amount of tensor modes produced. Large non-minimal coupling (of the order of 10^4) allows to use the SM model Higgs field for inflation. At the same time even very small coupling $\xi \gtrsim 10^{-3}$ makes inflaton with quartic potential compatible with the CMB observations. For the coupling constant $\xi \sim 1$ very small amount of tensor modes is expected for the quartic inflation. Interestingly, that at the same time the non-minimally coupled inflation with quadratic potential and $\xi \sim 1$ is no longer compatible with observations, generating too small spectral index, or even leading potential with runaway behaviour (see [16]).

As a summary, adding non-minimal coupling constant changes the usual expectations from inflation a lot, and lead to interesting predictions. One interesting application to the inflation in the νMSM model was made in [20].

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