Extraction of angle α from the data on tree dominated $B_d \to \rho^\pm \pi^\mp$ decays and direct CPV in $B \to K\pi$ decays

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Abstract

The current most precise determination of UT triangle angle α is performed. Recent results on direct CP asymmetries in $B \to K\pi$ decays are discussed as well.

1 Introduction

In papers [1, 2] we developed an approach to the calculation of the strong interaction phases of the amplitudes of B-meson decays into two light mesons. Our approach is based on the accounting of the final state rescattering of the produced in B-decays two particle states to which branching ratios of B-decays are maximal. In papers [1, 2] $B \to \pi\pi$ and $B \to \rho\rho$ decays were analyzed and we get understanding of the very special picture of the branching ratios of B-decays to final states with different electric charges ($\pi^+\pi^-$, $\pi^\pm\pi^0$, $\pi^0\pi^0$, $\rho^+\rho^-$, $\rho^\pm\rho^0$, $\rho^0\rho^0$).

Inspired by this success we made the next steps in the present paper.

In Sect. 2 we analyze experimental data on $B_d \to \rho^\pm\pi^\mp$ decays and find, in particular, a value of CKM triangle angle α which follows from the data. It nicely coincides with values of α extracted in [2] from data on $B \to \pi\pi$ and $B \to \rho\rho$ decays and has the best accuracy. Since penguin contribution shifts value of α extracted from $B_d \to \pi^+\pi^-$ decay by 20^o, while that extracted from $B_d \to \rho^+\rho^-$ and $B_d \to \pi^\pm\rho^\mp$ decays by 5^o - 6^o, their coincidence signal in favor of small (if any) contributions of New (non MFV) Physics in $b \to sg$ and $b \to dg$ penguin amplitudes.[1]

Experimental data on direct CP violation in $B \to K\pi$ decays confirmed recently by Belle get natural explanation in our approach (Sect. 3).

2 Analysis of $B_d(\bar{B}_d) \to \rho^\pm\pi^\mp$ decays

The time dependence of these decays are given by the following formula [4]:

$$\frac{dN(B_d(\bar{B}_d) \to \rho^\pm\pi^\mp)}{d\Delta t} = \left(1 \pm A_{\rm CP}^{\rho\pi}\right) e^{-t/\tau} \times$$
$$\times \quad [1 - q(C_{\rho\pi} \pm \Delta C_{\rho\pi})\cos(\Delta m t) + q(S_{\rho\pi} \pm \Delta S_{\rho\pi})\sin(\Delta m t)] \quad , \qquad (1)$$

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[1] The latter was calculated from the former with the help of $s \leftrightarrow d$ interchange symmetry of strong interactions, accuracy of which as well as that of experimental data on CP asymmetries S bound contribution of New non Minimal Flavor Violating (MFV) [3] Physics in penguin amplitudes to be smaller than that of SM.

where $q = -1$ corresponds to the decay of a particle which was B_d at $t = 0$, while $q = 1$ corresponds to the decay of a particle which was $\bar{B}_d$ at $t = 0$. According to [4]

$$A_{\rm CP}^{\rho\pi} = \frac{|A^{+-}|^2 - |\bar{A}^{-+}|^2 + |\bar{A}^{+-}|^2 - |A^{-+}|^2}{|A^{+-}|^2 + |\bar{A}^{-+}|^2 + |\bar{A}^{+-}|^2 + |A^{-+}|^2} \ , \tag{2}$$

where $A^{\pm\mp}$ are the amplitudes of $B_d \to \rho^\pm\pi^\mp$ decays, while $\bar{A}^{\pm\mp}$ are the amplitudes of $\bar{B}_d \to \rho^\pm\pi^\mp$ decays. Introducing the ratios of decay amplitudes

$$\lambda^{\pm\mp} = \frac{q}{p}\frac{\bar{A}^{\pm\mp}}{A^{\pm\mp}} \ , \tag{3}$$

where $q/p = e^{-2i\beta}$ comes from $B_d - \bar{B}_d$ mixing and β is an angle of unitarity triangle, we obtain the expressions for remaining parameters entering Eq. (1):

$$C_{\rho\pi} \pm \Delta C_{\rho\pi} = \frac{1 - |\lambda^{\pm\mp}|^2}{1 + |\lambda^{\pm\mp}|^2} \ , \ \ S_{\rho\pi} \pm \Delta S_{\rho\pi} = \frac{2Im\lambda^{\pm\mp}}{1 + |\lambda^{\pm\mp}|^2} \ , \tag{4}$$

where $C_{\rho\pi}$ and $S_{\rho\pi}$ (as well as $A_{\rm CP}^{\rho\pi}$) are CP-odd observables, while $\Delta C_{\rho\pi}$ and $\Delta S_{\rho\pi}$ are CP-even. Experimental data for observables entering Eq. (1) accompanied by averaged branching fraction are presented in Table 1 [5].

Table 1

$Br B_d(\bar{B}_d) \to$ $\to \rho^\pm\pi^\mp$	$A_{\rm CP}^{\rho\pi}$	$C_{\rho\pi}$	$\Delta C_{\rho\pi}$	$S_{\rho\pi}$	$\Delta S_{\rho\pi}$
$(23.1 \pm 2.7)10^{-6}$	-0.13 ± 0.04	0.01 ± 0.07	0.37 ± 0.08	0.01 ± 0.09	-0.04 ± 0.10

Experimental values of observables which describe $B_d(\bar{B}_d) \to \rho^\pm\pi^\mp$ decays.

Decay amplitudes $\bar{A}^{\pm\mp}$ are described by shown in Fig. 1 Feynman diagrams. Analogous diagrams describe amplitudes $A^{\pm\mp}$. The corresponding formulas look like:

$$\begin{aligned} \bar{A}^{-+} &= A_1 e^{-i\gamma} + P_1 e^{i(\beta+\delta_1)} \ , \\ A^{-+} &= A_2 e^{i\gamma} + P_2 e^{-i(\beta-\delta_2)} \ , \\ \bar{A}^{+-} &= A_2 e^{-i\gamma} + P_2 e^{i(\beta+\delta_2)} \ , \\ A^{+-} &= A_1 e^{i\gamma} + P_1 e^{-i(\beta-\delta_1)} \ , \\ A_1/A_2 &\equiv a_1/a_2 e^{i\tilde{\delta}} \ , \\ P_1/P_2 &\equiv p_1/p_2 e^{i\tilde{\delta}} \ , \end{aligned} \tag{5}$$

where γ and β are angles of unitarity triangle, while δ_1 and δ_2 are the difference of FSI strong phases between penguin and tree amplitudes (for penguin amplitudes we use the so-called t-convention, subtracting charm quark contribution to penguin amplitudes).

In all we have seven parameters in Eq.(5) specific for $\rho\pi$ final states ($a_1, a_2, p_1, p_2, \delta_1, \delta_2$ and $\tilde{\delta}$) plus UT angle $\alpha = \pi - \beta - \gamma$, while the number of experimental observables in Table 1 is six. To go further we should involve additional theoretical information in order to reduce the number of parameters. If we find the values of p_1 and p_2 even with considerable uncertainties it will be very helpful for determination of UT angle α, since penguin amplitudes shift α by small amount proportional to p_i/a_i, and even large uncertainty in this shift lead to few degrees (theoretical) uncertainty in α (see below).

The most straightforward way is to calculate matrix elements of the corresponding weak interactions lagrangian with the help of factorization, as it was done in [6]. However the accuracy of such a calculation can not be determined theoretically. From the experimental data for direct CP-asymmetry in $B_d(\bar{B}_d) \to \pi^+\pi^-$ decays we know that factorization strongly underestimate the contribution of penguin diagram into the decay amplitude [1, 2]. Another approach is to extract penguin amplitudes from the branching ratios of the $B^- \to \bar{K}^{0*}\pi^+$ and $B^- \to \bar{K}^0\rho^+$ decays in which penguin dominates with the help of $s \leftrightarrow d$ quark interchange symmetry, analogously to what was done for penguins in $B \to \pi\pi$ [7] and $B \to \rho\rho$ [8] decays.

Feynman diagrams responsible for these decays are drawn in Fig. 2.

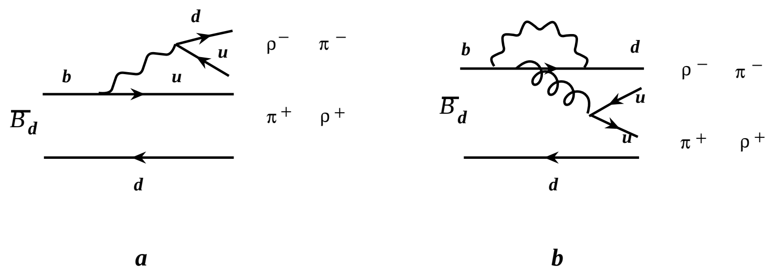

Figure 1: *Diagrams for the amplitudes A_1 and A_2 (a) and P_1 and P_2 (b).*

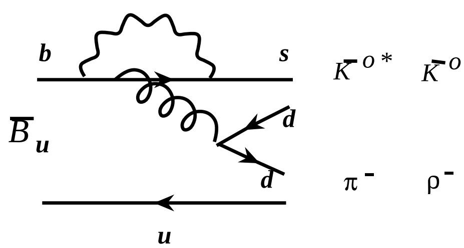

Figure 2: B^- *decays in which penguin diagram dominates.*

Comparing Fig. 2 with Fig. 1 (b) we readily get the following relations:

$$
\begin{aligned}
Br(\bar{B}_d \to \pi^+\rho^-)_{P_1} &= \frac{\tau_{B_d}}{\tau_{B_u}} Br(B^- \to \bar{K}^{0*}\pi^-)\left|\frac{V_{td}}{V_{ts}}\right|^2 = \\
&= \frac{1}{1.071}(10.7 \pm 0.8)\cdot 10^{-6}\cdot(0.20)^2 = 0.40(4)\cdot 10^{-6} \quad ,
\end{aligned} \tag{6}
$$

$$
\begin{aligned}
Br(\bar{B}_d \to \rho^+\pi^-)_{P_2} &= \frac{\tau_{B_d}}{\tau_{B_u}} Br(B^- \to \bar{K}^{0}\rho^-)\left|\frac{V_{td}}{V_{ts}}\right|^2 = \\
&= \frac{1}{1.071}(8.0 \pm 1.5)\cdot 10^{-6}\cdot(0.20)^2 = 0.30(6)\cdot 10^{-6} \quad ,
\end{aligned} \tag{7}
$$

from which values of p_1 and p_2 follow:

$$
p_1^2 = 0.40(4)\cdot 10^{-6} \ , \ \ p_2^2 = 0.30(6)\cdot 10^{-6} \quad , \tag{8}
$$

where here and below we neglect common factor $16\pi m_B \Gamma_{B_d}$, to which squares of amplitudes are proportional. Remaining $8 - 2 = 6$ parameters entering Eq.(5) we will determine from the six experimental numbers presented in Table 1.

From Eq. (5) we get the following relation for the averaged branching ratio of $B_d(\bar{B}_d)$ decays to $\rho^\pm\pi^\mp$:

$$\frac{a_1^2+a_2^2}{2}+\frac{p_1^2+p_2^2}{2}=23.1(2.7)\cdot 10^{-6} \ , \tag{9}$$

where penguin-tree interference terms are omitted (being proportional to $\cos(\pi-\beta-\gamma)=\cos\alpha$ they are very small since UT is almost rectangular, $\alpha\approx\pi/2$).

To determine values of a_i an equation for $\Delta C_{\rho\pi}$ is helpful:

$$\Delta C_{\rho\pi}=\frac{a_1^2-a_2^2}{a_1^2+a_2^2}+O\left(\frac{p_i^2}{a_i^2}\right) \ , \tag{10}$$

and from (8) - (10) and experimental value for $\Delta C_{\rho\pi}$ from the Table 1 we get:

$$a_1^2=31(3)\cdot 10^{-6} \ , \ \ a_2^2=14(3)\cdot 10^{-6} \ . \tag{11}$$

Now from the equations for $C_{\rho\pi}$ and $A_{\rm CP}^{\rho\pi}$ using experimental data from Table 1 we are able to extract FSI phases δ_1 and δ_2:

$$\begin{aligned} C_{\rho\pi} &= \frac{2p_1a_1\sin\delta_1+2p_2a_2\sin\delta_2}{a_1^2+a_2^2}+\frac{a_1^2-a_2^2}{(a_1^2+a_2^2)^2}[2p_2a_2\sin\delta_2-2p_1a_1\sin\delta_1] \ , \\ A_{\rm CP}^{\rho\pi} &= \frac{2p_1a_1\sin\delta_1-2p_2a_2\sin\delta_2}{a_1^2+a_2^2} \ , \end{aligned} \tag{12}$$

$$\sin\delta_1=-0.55(30) \ , \ \ \sin\delta_2=0.51(40) \tag{13}$$

and we see that large experimental errors of $C_{\rho\pi}$ and $A_{\rm CP}^{\rho\pi}$ do not allow accurate determination of the values of FSI phases.

From equations for S and ΔS we will determine the values of α and $\tilde{\delta}$:

$$S_{\rho\pi}+\Delta S_{\rho\pi}= \tag{14}$$

$$=2\frac{a_1a_2\sin(2\alpha-\tilde{\delta})-p_1a_2\cos(\delta_1-\tilde{\delta})-p_2a_1\cos(\delta_2-\tilde{\delta})+2p_1a_2\sin\delta_1\sin\tilde{\delta}}{a_1^2+a_2^2+2p_1a_1\sin\delta_1-2p_2a_2\sin\delta_2} \ ,$$

$$S_{\rho\pi}-\Delta S_{\rho\pi}= \tag{15}$$

$$=2\frac{a_1a_2\sin(2\alpha+\tilde{\delta})-p_2a_1\cos(\delta_2+\tilde{\delta})-p_1a_2\cos(\delta_1+\tilde{\delta})-2p_2a_1\sin\delta_2\sin\tilde{\delta}}{a_1^2+a_2^2+2p_2a_2\sin\delta_2-2p_1a_1\sin\delta_1} \ ,$$

where in (small) terms proportional to p_i we have substituted $\alpha=\pi/2$. Substituting numerical values for parameters in denominators we get:

$$[(6\pm 4)S_{\rho\pi}-(45\pm 4)\Delta S_{\rho\pi}]10^{-6}=2a_1a_2\sin\tilde{\delta}\cos 2\alpha \ , \tag{16}$$

$$\begin{aligned} [(45\pm 4)S_{\rho\pi}-(6\pm 4)\Delta S_{\rho\pi}]10^{-6} &= 2a_1a_2\sin 2\alpha\cos\tilde{\delta}- \\ &-2p_2a_1\cos(\tilde{\delta}-\delta_2)-2p_1a_2\cos(\tilde{\delta}+\delta_1) \ . \end{aligned} \tag{17}$$

From the first equation we see that $\tilde{\delta}$ equals zero or π with $\pm 5^0$ accuracy. For UT angle α from the second equation neglecting penguin contributions we obtain:

$$\alpha_{\rho\pi}^T=90^o\pm 3^o({\rm exp}) \ , \tag{18}$$

while taking penguins into account we get:

$$\alpha_{\rho\pi}=84^o\pm 3^o({\rm exp}) \ , \tag{19}$$

where $\delta_1 \approx -30^o$ and $\delta_2 \approx 30^o$ were used.

Thus penguins shift α by 6^o and even assuming only 50% accuracy of $d \leftrightarrow s$ symmetry which was used to determine numerical values of p_i allows us to determine $\alpha_{\rho\pi}$ with theoretical accuracy which equals the experimental one, originating from that in $S_{\rho\pi}$ and pointed out in (19):

$$\alpha_{\rho\pi} = 84^o \pm 3^o(\mathrm{exp}) \pm 3^o(\mathrm{theor}) \ . \tag{20}$$

Consideration of $B_d(\bar{B}_d) \to \pi\pi$ decays performed in [2] leads to the following result:

$$\alpha_{\pi\pi} = 88^o \pm 4^o(\mathrm{exp}) \pm 10^o(\mathrm{theor}) \ , \tag{21}$$

where relatively large theoretical error is due to big (20^o) shift of tree level value of $\alpha_{\pi\pi}$ by poorely known penguins and this time (unlike in [2]) we suppose 50% theoretical uncertainty in the value of penguin amplitude.

In the case of $B_d(\bar{B}_d) \to \rho^+\rho^-$ decays penguin shifts the value of α by the same amount as in considered in this paper $B_d(\bar{B}_d) \to \rho^\pm\pi^\mp$ decays, so theoretical uncertainty is the same:

$$\alpha_{\rho\rho} = 87^o \pm 5^o(\mathrm{exp}) \pm 3^o(\mathrm{theor}) \ , \tag{22}$$

while larger experimental uncertainty is due to that is $S_{\rho\rho}$,

$$S_{\rho\rho} = -0.06 \pm 0.18 \ , \tag{23}$$

which is twice that in $S_{\rho\pi}$.

It is interesting to compare numerical values (19), (21), (22) with the recent results of the fit of Unitarity Triangle [9, 10]:

$$\alpha^{\mathrm{CKMfitter}} = 88^o \pm 6^o \ , \tag{24}$$

$$\alpha^{\mathrm{UTfit}} = 91^o \pm 6^o \ . \tag{25}$$

Large New Physics contribution in $b \to dg$ penguin could help to avoid large FSI phases since now enhancement of direct CPV seen in $A_{\mathrm{CP}}^{\rho\pi}$ will originate from closeness of tree level and penguin amplitudes. Also puzzle of large $Br B_d(\bar{B}_d) \to \pi^0\pi^0$ can be resolved by NP contribution into $b \to dg$ penguin comparable with SM one recalculated from $B_u \to K^0\pi^+$ decay. The bound on such contribution comes from the close values of α extracted from $B \to \pi\pi$, $\rho\pi$ and $\rho\rho$ decays, where penguin contributions are very different.[2]

These are strong arguments in favor of measurements of the parameters of $B \to \pi\pi$, $\rho\pi$ and $\rho\rho$ decays with better accuracy, which can be performed at LHCb and Super B factory. A search of NP manifestation by different values of UT angle α extracted from $B \to \pi\pi$ and $B \to \pi\rho, \rho\rho$ decays is analogous to one suggested in [11] through the difference of α extracted from the penguin polluted $B \to \pi\pi$ decay and from UT analysis based on tree dominated observables V_{cb}, γ and V_{ub}.

At the end of this section let us note that results (21) and (22) were obtained in analysis based on isotopic invariance of strong interactions from violation of which the additional uncertainty in α could follow [12]. Fortunately since in the absence of penguin amplitudes relation $S_{\pi\pi,\rho\rho} = \sin 2\alpha^T$ is free from this type of uncertainty, it manifests only as several percent correction to an induced by penguin shift of α which is negligible even for $B \to \pi\pi$ decays.

[2]The same argument can be applied against large NP contributions to $b \to sg$ penguin: if the same NP does not enhance $b \to dg$ penguin the value of α from $B \to \pi\pi$ data will be closer to $\alpha_{\pi\pi}^T = 109^o$ and disagree with that from $\alpha_{\pi\rho}$ and $\alpha_{\rho\rho}$.

3 Direct CPV in $B \to \pi K$ decays

Recently Belle published new results of the measurement of CP asymmetries in $B_d(\bar{B}_d) \to K^+\pi^-(K^-\pi^+)$ and $B^+(B^-) \to K^+\pi^0(K^-\pi^0)$ decays [13]:

$$A_{CP}(K^+\pi^-) \equiv \frac{\Gamma(\bar{B}_d \to K^-\pi^+) - \Gamma(B_d \to K^+\pi^-)}{\Gamma(\bar{B}_d \to K^-\pi^+) + \Gamma(B_d \to K^+\pi^-)} = -0.094(18)(8) \ , \tag{26}$$

$$A_{CP}(K^+\pi^0) \equiv \frac{\Gamma(B^- \to K^-\pi^0) - \Gamma(B^+ \to K^+\pi^0)}{\Gamma(B^- \to K^-\pi^0) + \Gamma(B^+ \to K^+\pi^0)} = 0.07(3)(1) \ . \tag{27}$$

In [13] the 4.5 standard deviations difference of these asymmetries was considered as a paradox in the framework of the Standard Model which it really would be IF one neglects color suppressed tree quark amplitude. Taking into account QCD penguin diagram and tree diagrams one easily gets the following relation between CP asymmetries [14]:

$$A_{CP}(K^+\pi^-) = A_{CP}(K^+\pi^0) + A_{CP}(K^0\pi^0) \ , \tag{28}$$

where $A_{CP}(K^0\pi^0)$ is proportional to color suppressed amplitude C. Experimental value of $A_{CP}(K^0\pi^0)$ has large uncertainty:

$$A_{CP}(K^0\pi^0) = -0.14 \pm 0.11 \ , \tag{29}$$

however with the help of $d \leftrightarrow s$ interchange symmetry it can be related with CP asymmetry C_{00} of $B_d(\bar{B}_d) \to \pi^0\pi^0$ decays:

$$A_{CP}(K^0\pi^0) = \frac{\Gamma(B_d \to \pi^0\pi^0) + \Gamma(\bar{B}_d \to \pi^0\pi^0)}{\Gamma(B_d \to K^0\pi^0) + \Gamma(\bar{B}_d \to \bar{K}^0\pi^0)} \left| \frac{V_{us}V_{ts}}{V_{td}} \right| \frac{\sin\gamma}{\sin\alpha} C_{00} \ , \tag{30}$$

where opposite signs in definitions of A_{CP} and C_{00} are compensated by negative sign of V_{ts}. Experimental uncertainty of C_{00} is also very large, that is why we use for numerical estimate obtained in [2] result:

$$C_{00} \approx -0.6 \ . \tag{31}$$

Substituting (31) in (30) and (26), (27) and (30) in (28) we finally obtain:

$$-0.094(20) = 0.07(3) - 0.07 \ , \tag{32}$$

resolving in this way paradox noted in [13] (remaining $\approx 2\sigma$ difference can be safely attributed to statistical fluctuation). Concluding this Section let us remind that the absence of color suppression of the tree amplitude C in $B_d \to \pi^0\pi^0$ decay is explained in [2] by large FSI phases difference of tree amplitudes with isospin zero and two.

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