Two-Higgs doublet potential of the MSSM at finite temperature and Higgs boson masses

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Abstract

In the framework of MSSM the one-loop corrections to the parameters of effective two-doublet Higgs potential at finite temperature are calculated. The scalar quarks mass splitting influences strongly on the effective parameters of temperature potential, providing interesting possibilities for the phase transition evidence. The physical masses vanish, when the temperature increases up to the critical values, which corresponds to the phase transition. In the limiting case, when the temperature is equal to zero and all mass parameters of the soft SUSY breaking sector are degenerated, the predictions for observables from two-doublet potential coincide with known previous results.

The baryon asymmetry and an extremely small observed ratio of the baryon number to entropy could be understood on the basis of Sakharov conditions which are respected at the electroweak phase transition. Necessary requirements for the baryogenesis are the baryon number violation, CP violation and heat non-equilibrium. The most scenarios of baryon asymmetry generation demand the presence of strong first order phase transition, differently the baryon asymmetry generated at the electroweak phase transition subsequently disappears [1]. The phase transition is described by the temperature Higgs boson potential [2-7]. The theoretical predictions should be coordinated with LEP2 restriction on the lightest neutral Higgs boson mass m_H >115 GeV. And the effective connection of baryogenesis constrains with nondirect future LHC data analysis on the definition of trilinear coupling constant λ_{hhh} [8] could be possible in consistence with the first order phase transition requirement on the light Higgs boson mass. One of the most perspective scenarios is the one with small mass parameter of the right $\tilde{t}$-squark in the minimal supersymmetric model (MSSM) [7]. For the description of finite temperature potential evolution and for the concrete definition of phase transition type it is necessary to calculate the "scalar quarks - Higgs boson" sector contributions to the parameters λ_i of effective potential, using the decomposition method on degrees of $1/M^2_{SUSY}$ [9, 2] or a high-temperature decoupling limit [8, 2], and also the diagrammatic method [6].

In this article in the framework of MSSM with explicit *CP*-violation the one-loop effective potential of two-Higgs doublet model (THDM) at finite temperature is considered. The calculations of scalar integrals in the temperature field theory are carried out as the calculations of one-loop Feynman diagrams. This types of integrals are reduced to the generalized Gurvits's zeta-functions. For the special case of MSSM two-doublet Higgs sector, when CP invariance is violated by the Higgs boson interaction with the scalar quarks of third generation, we calculate the Higgs boson masses.

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1 The method of integration and summation for the calculation of effective potential parameters

The extended Higgs sector of MSSM [10] with real parameters $\lambda_{5,6,7}$ of the effective Higgs potential includes five scalar particles, two CP-even h,H, a pseudoscalar A, and two charged $H^{\pm}$ scalar bosons. The radiatively corrected mass of the lightest Higgs boson h must be less than 135 GeV [11], which is accessible at the next generation colliders. This picture becomes more complicated in the more general case of complex-valued parameters $\lambda_{5,6,7}$ in the two-doublet Higgs potential [12], which are naturally induced radiatively in the MSSM by the Higgs bosons interaction with the third generation scalar quarks [13, 14, 15].

In THDM there are two $SU(2)$ doublets of complex scalar fields with nonzero vacuum expectation values. The effective two-doublet Higgs potential of MSSM at the energy scale m_{top} has four real parameters $\lambda_1 \div \lambda_4$ and three complex-valued parameters $\lambda_5, \lambda_6, \lambda_7$, which explicitly violates CP invariance in the Higgs sector. The parameters $\lambda_1 \div \lambda_7$ can be calculated and expressed through the parameters of MSSM sector of scalar quarks-Higgs boson interaction. In this sense the MSSM Higgs sector can be embedded in a general THDM, providing the possibilities to interpret some special MSSM features in the language of THDM parameters space.

The parameters of effective potential of any model depend on the energy scale where they are fixed or measured. The dependence is described by the renormalization group equations. At the energies smaller than the supersymmetry breaking scale the physical parameters are affected by the large quantum corrections where the main contribution is coming from the Higgs boson interactions with the third generation of quarks and scalar quarks.

In the imaginary time formalism of finite temperature quantum field theory Feynman diagrams with the boson propagators are represented as one with Matsubara frequencies, $\omega_n = 2\pi n T$ $(n = 0, \pm 1, \pm 2, ...)$, T – temperature, which lead to calculation of the following objects

$$I[m_1, m_2, ..., m_b] = T \sum_{n=-\infty}^{\infty} \int \frac{d\mathbf{k}}{(2\pi)^3} \prod_{j=1}^{b} \frac{(-1)^b}{(\mathbf{k}^2 + \omega_n^2 + m_j^2)}, \tag{1}$$

where the summation and integration can be made in any order. Here $\mathbf{k}$ is the three-dimensional momentum. First we integrate over the three-dimensional momentum and then take the sum over frequencies. Such approach is the reduction to three-dimensional theories at the high-temperature limit (for zero Matsubara frequencies). In our case at $n \neq 0$ we obtain [16]:

$$I[m_1, m_2, ..., m_b] = 2T\,(2\pi T)^{3-2b}\ \frac{(-1)^b \pi^{3/2}}{(2\pi)^3}\ \frac{\Gamma(b-3/2)}{\Gamma(b)}\ S(M, b-3/2), \tag{2}$$

where

$$S(M, b-3/2) = \int \{x\} \sum_{n=1}^{\infty} \frac{1}{(n^2+M^2)^{b-3/2}}, \qquad M^2 \equiv \left(\frac{m}{2\pi T}\right)^2. \tag{3}$$

At $b > 1$ the parameter m^2 is a linear function of m_j^2 and Feynman parametrization variables $\{x\}$. At the integer values of b the sum $S(M, b-3/2)$ represents the integral over generalized Hurwits zeta-function. For calculation of the leading threshold corrections to the effective potential parameters $b > 2$, therefore the wave-function renormalization appears in connection with divergency at $b = 2$ (which are suppressed by vertex factors [9]).

The first method of integral calculation consists of the using the Feynman parametrization for simplification of momentum integrals, and then using the dimensional regularization. At the final stage the summation by Matsubara frequencies is carried out. Here the dividing of components with zero and nonzero Matsubara frequencies is convenient. The calculations by means of the first method are very consuming. In the other method we present analytically the result of integration from two propagators in the basic case, and receive corresponding integrals with more propagators by the differentiation of basic case. Using the basic integrals, it is possible to calculate easily the particles contributions to Feynman diagrams for effective potential. In the considered case of scalar quarks contributions the combinations of propagators depend on two or three mass parameters. Both methods lead to identical results.

In the limiting case $T \to 0$ the values of effective parameters coincide numerically with those at zero temperature field theory. At high-temperature limit the zero boson mode gives the defining contribution, that corresponds to the well known conclusion about the suppression of quantum effects at high temperature.

The effective potential parameters at finite temperature, received by the diagram method, contain the threshold corrections $\Delta\lambda_i^{thr}$ from diagrams with three and four propagators, the logarithmic (also from 'fish'-type diagrams) and the field renormalization corrections $\Delta\lambda_i^{field}$ (see fig. 1). The threshold corrections $\Delta\lambda_i^{thr}$ to effective parameters [9] at finite temperature: $\lambda_i = \lambda^{SUSY} - \Delta\lambda_i^{thr}$.

The generalization of effective potential method is developed, where the logarithmic contributions corresponding to the summation of high order corrections in the case of non-degenerated mass parameters arise. The logarithmic corrections for unequal squark masses are considered by generalization of the results for degenerated mass parameters (corresponding to remarks, offered in [17] and [18], and also [9]), replacing $\ln\left(\frac{M_{SUSY}}{m_t^2}\right)$ on $\ln\left(\frac{m_Q m_U}{m_t^2}\right)$. Also we considered logarithms which are not connected with the renormalization group, and arising in the integration for non-degenerate mass parameters.

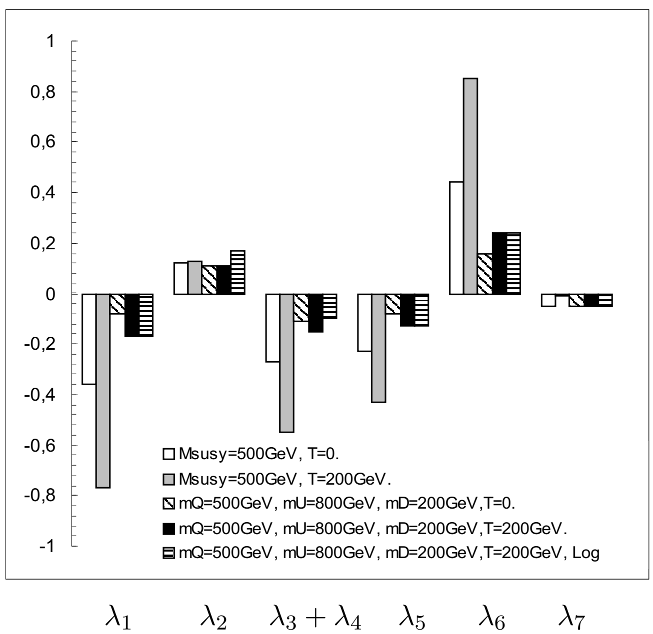

Figure 1: The bars for the parameters of effective potential λ_1, λ_2, $\lambda_3+\lambda_4$, λ_5, λ_6, λ_7 with various corrections (threshold, logarithmic (summarised by the effective potential method)), depending on temperature in CPX-scenarios (A =1000 GeV, μ =2000 GeV) in following cases: equal scalar quark mass parameters at zero temperature ($m_Q = m_t = m_b = M_{SUSY}$ =500 GeV), equal scalar quark mass parameters at finite temperature ($m_Q = m_t = m_b = M_{SUSY}$ =500 GeV, T =200 GeV), unequal scalar quark mass parameters at zero temperature (m_Q =500 GeV; m_t =800 GeV; m_b =200 GeV), unequal scalar quark mass parameters at finite temperature (m_Q =500 GeV; m_t =800 GeV; m_b =200 GeV, T =200 GeV)

2 Higgs boson masses at finite temperature in the case of non-degenerate squarks mass parameters

Using the expressions for effective parameters in the case of non-degenerated mass parameters of squark sector, it is possible to receive the physical states of Higgs bosons and to calculate their masses at finite temperature. The diagonalization procedure is similar to the one with degenerated scalar quark mass parameters [9].

Two opposite scenarios for the MSSM Higgs sector has been discussed in many details. In the decoupling regime of the CP-conserving MSSM Higgs sector (see e. g. the appendix of [9]) the Higgs bosons H,A and $H^{\pm}$ are very heavy and almost degenerate in mass, while the lightest scalar h has the properties of the Standard Model (SM) Higgs boson. In the decoupling regime it is possible the large CP violation due to the mixing in H and A states. It was shown by detailed phenomenological analysis that the experimental detection of h is straightforward, but other scalars with masses $\sim$1 TeV are not accessible, and it is questionable whether one could distinguish between the SM and the MSSM in the decoupling regime. Much more interesting situation takes place in the CP-conserving MSSM when the three neutral scalars h,H and A have very close masses of the order of maximal value allowed for $m_h \sim 135$ GeV with the charged scalar mass less than 150 GeV [19]. This case is particularly interesting if the $\tan\beta$ parameter of the MSSM is large, leading to large couplings of the h, H to the b-quark and τ-lepton together with maximal h, HZ and $h, H^{\pm}W$ couplings. In this intense-coupling scenario all five scalars would be accessible at the next generation colliders. The purpose here is to perform a study of the intense-coupling scenario in the MSSM with complex $\lambda_{5,6,7}$ parameters of the effective two-doublet Higgs potential. In the following we are using the so-called CPX scenario [20] of strong CP-violation in the Higgs sector, when the imaginary parts of $\lambda_{5,6,7}$ parameters are large leading to strong mixing of CP-even/CP-odd states, most interesting from the phenomenological point of view. Decoupling regime take place for the large values of charged Higgs boson mass (more then 250 GeV). The intensive regime is observed for the small mass of charged Higgs boson and $\tan\beta \approx$30.

On fig. 2 the lightest Higgs boson mass is presented depending on CP violating phase φ for the decoupling regime of heavy Higgs bosons. Green (horizontal) lines are for the CP-conserving case, blue curves correspond to mediate state $m_h(\varphi)$, red curves correspond to physical state m_{h_1}. The left plot corresponds to a degenerated case, the right plot corresponds to the unequal mass parameters. Figure 3 corresponds to the intensive coupling regime when $m_{H^{\pm}}$ =190 GeV. All results are obtained for CPX scenario. In CPX scenario there are maximum effects of CP mixing. The mass of the lightest Higgs boson in MSSM with CP violation can deviate from the CP conserving case up to 30 GeV and get values up to 50 GeV down.

For the consideration of non-degenerated case the behavior of lightest Higgs boson mass differs essentially from the case of equal mass parameters of squark sector. Similar plots are constructed for the case of strong mixing, and also the behavior of heavy Higgs boson masses is investigated depending on the parameters of model.

3 Summary

In the framework of MSSM the one-loop corrections to the parameters of effective two-doublet Higgs potential at finite temperature are calculated. The scalar quarks mass splitting influences strongly on the effective parameters of temperature potential. In the limiting case, when the temperature is equal to zero and three mass parameters of the soft SUSY breaking sector are equal $M_U = M_D = M_Q$, the predictions for observables from two-doublet potential coincide with known previous results.

The lightest Higgs boson mass is arisen at low temperature, and the mass m_{h_2} is arisen at larger temperature (see fig. 4). At lower temperatures $m_{h_2} \approx$100 GeV, which is rather usual for

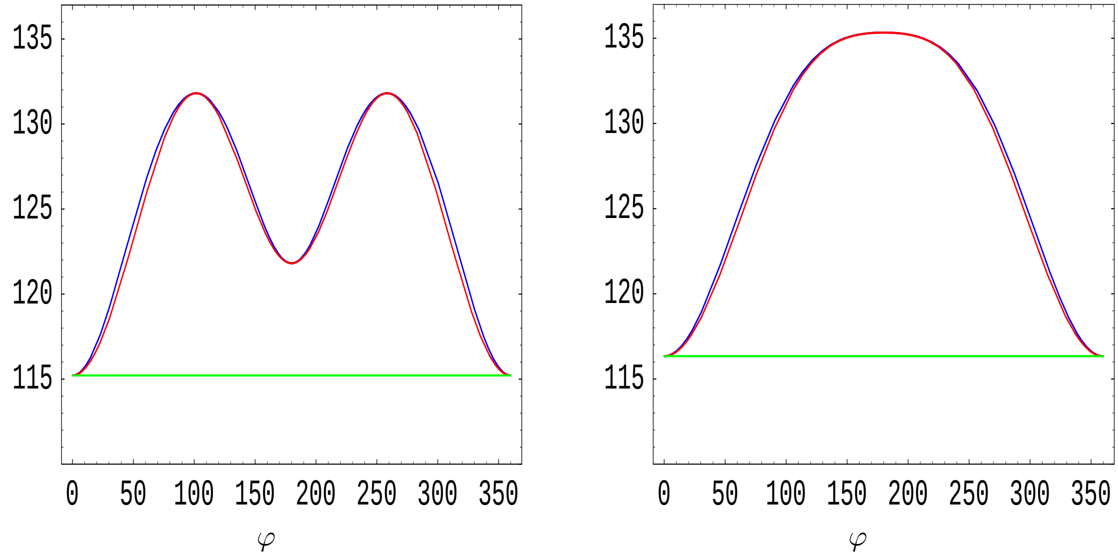

Figure 2: The neutral Higgs boson masses m_{h_1} (red curve) and m_h (green line) (in GeV) vs on phases φ in CPX-scenarios at $m_{H^\pm}$ =300 GeV. Left plot – for M_{SUSY} =500 GeV, right plot – for the case of non-degenerated mass parameters of scalar sector: m_Q =500 GeV, m_t =800 GeV, m_b =200 GeV

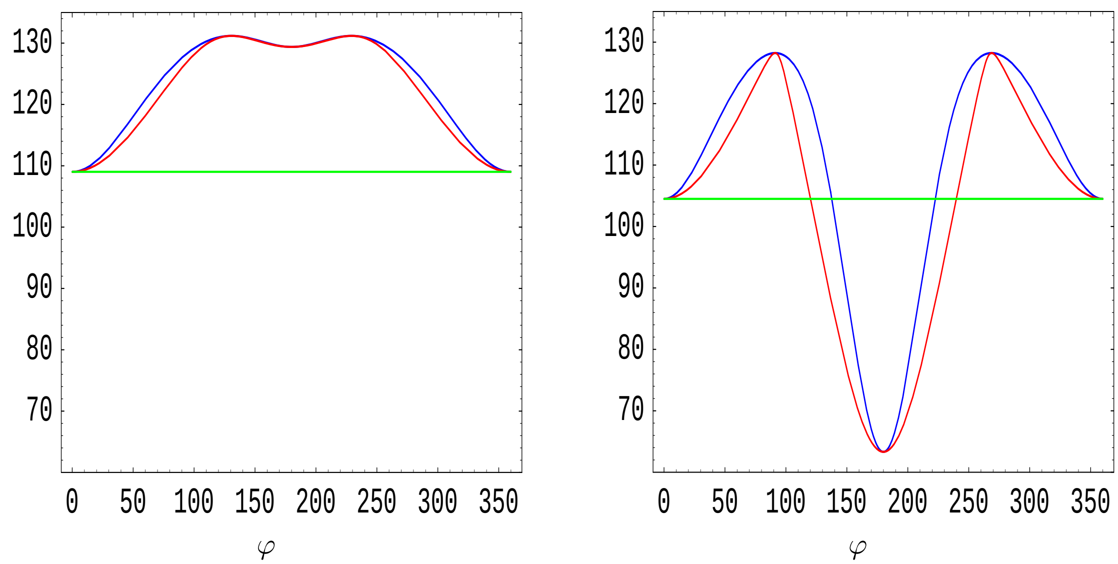

Figure 3: The neutral Higgs boson masses m_{h_1} (red curve) and m_h (green line) (in GeV) vs on phases φ in CPX-scenarios at $m_{H^\pm}$ =190 GeV. Left plot – for M_{SUSY} =500 GeV, right plot – for the case of non-degenerated mass parameters of scalar sector: m_Q =500 GeV, m_t =800 GeV, m_b =200 GeV

$m_{H^\pm}$ =100 GeV. Such situation takes place at $\tan\beta \approx$40, usual for ration between the masses of t- and b-quarks or the vacuum expectation values v_1 and v_2. There is a reduction separately of masses m_{h_1} and m_{h_2} to zero at temperature growth up to two different values, at which and $v \to$0. And at the same time it is natural as the reference critical condition in a zero of a gradient from a combination of mass terms of effective temperature potential was demanded. Therefore it is necessary to find only the area of parameters where such extremum will exist,

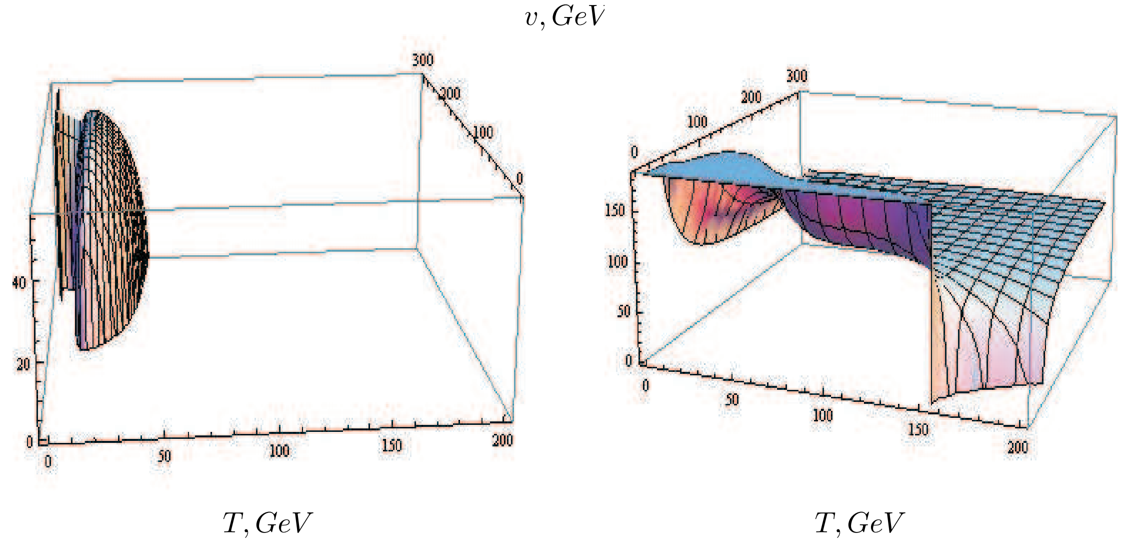

Figure 4: The neutral Higgs boson masses m_{h_1} (left plot) and m_{h_2} (right plot)

and this area will be reasonable from the point of view of possible restrictions. Thus we also consider the minimum conditions at a finding of mass states. The physical masses vanish, when the temperature increases up to the critical values, which corresponds to the phase transition. And at heats thus symmetry is restored.

References

[1] V.A. Rubakov, M.E. Shaposhnikov, Usp.Fiz.Nauk 166: 493-537, 1996, Phys.Usp. 39: 461-502, 1996

[2] Dolan L., Jackiw R., Phys. Rev. D, 9 3320 (1974)

[3] Weinberg S., Phys. Rev. D, 9 3357 (1974)

[4] M. Dine, R. Leigh, P. Huet, A. Linde, D. Linde, Phys. Lett. B 283, 319 (1992); Phys. Rev. D 46, 550 (1992). A. Megevand, A. D. Sanchez Phys. Rev. D, 77 063519 (2008)

[5] A. Brignole, J.R. Espinosa, M. Quiros, F. Zwirner, Phys. Lett. B 324 94, 181-191 (1994)

[6] P. Arnold, O. Espinosa, Phys. Rev. D 47, 3546 (1993), Err. 50, 6662 (1994)

[7] M. Carena, M. Quiros, C.E.M. Wagner // Phys. Lett. B. 380, 81 (1996)

[8] Y. Okada et al. (Shinya Kanemura, Yasuhiro Okada and Eibun Senaha) In: Proc. of CERN Workshop on CP studies and nonstandard Higgs physics, ed. by S.Kraml, G.Azuelos, D.Dominici, J.Ellis, G.Grenier, H.Haber, J.S.Lee, D.Miller, A.Pilaftsis, W.Porod, CERN Yellow Report 2006–009, 2006. hep-ph/0608079. (`http://kraml.home.cern.ch/kraml/cpnsh/`). Andrew Noble and Maxim Perelstein hep-ph 0711.3018

[9] E.N. Akhmetzyanova, M.V. Dolgopolov, M.N. Dubinin // Phys.Part.Nucl.37:677-734,2006; E.N. Akhmetzyanova, M.V. Dolgopolov, M.N. Dubinin // Phys. Rev. D 71, 075008 (2005). E.N. Akhmetzyanova, M.V. Dolgopolov, M.N. Dubinin Phys.Atom.Nucl.68:1851-1865,2005, Yad.Fiz.68:1913-1927,2005 **68**, 569 (2005)

[10] J.F.Gunion, H.E.Haber, G.L.Kane, S.Dawson, "The Higgs Huter's Guide", Addison-Wesley, Reading, 1990

[11] Y. Okada, M. Yamaguchi, T. Yanagida, Phys. Lett. B262, 54 (1991).
J. Ellis, G. Ridolfi, F. Zwirner, Phys. Lett. B257, 83 (1991).
H.E. Haber, R. Hempfling, Phys. Rev. Lett. 66, 1815 (1991).
R. Barbieri, M. Frigeni, F. Caravaglios, Phys. Lett. B258, 167. (1991)

[12] for a rewiew, see H.Haber, M.Krawczyk, in: Proc.of the Workshop on CP Studies and Non-Standard Higgs Physics, ed. by S. Kraml, G.Azuelos, D.Dominici, J.Ellis, G.Grenier, H.Haber, J.S.Lee, D.Miller, A.Pilaftsis, W.Porod, CERN Yellow Report 2006–009, 2006

[13] Pilaftsis A., Wagner C.E.M. Higgs bosons in the minimal supersymmetric standard model with explicit CP violation // Nucl. Phys. B553. 1999. P. 3–42

[14] Dubinin M.N., Semenov A.V. Triple and quartic interactions of Higgs bosons in the two Higgs doublet model with CP violation // Eur. J. Phys. C28. 2003. P. 223–236

[15] Akhmetzyanova E.N., Dolgopolov M.V., Dubinin M.N. Higgs bosons in the two-doublet model with CP violation // Phys. Rev. D71. 2005. P. 075008; Higgs bosons in the two-doublet model involving CP violation // Phys. of Atom. Nucl. Vol. 68. No. 11. 2005. P. 1851–1865

[16] L. Vergara, Journal of Physics A 30, 6977 (1997); P.Amore, Journal of Physics A 38, 6463-6472 (2005)

[17] H. E. Haber, R.Hempfling. A.H. Hoang, Z.Phys.C 75, 539-554, 1997

[18] S.Y. Choi, Manuel Drees, Jae Sik Lee. Phys.Lett.B481:57-66, 2000

[19] E.Boos, A.Djouadi, M.Muhlletner, A.Vologdin, The MSSM Higgs bosons in the intense-coupling regime, Phys.Rev. D66 (2002) 055004; E. Boos, A. Djouadi, A. Nikitenko Phys.Lett.B578:384, 2004

[20] Carena M. et al. CP-violating MSSM Higgs bosons in the light of LEP 2 / M. Carena, J.R. Ellis, A. Pilaftsis, C.E.M. Wagner // Phys. Lett. B495. 2000. P. 155–163