Hybrid Models of Supersymmetry Breaking

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Abstract

We study a model in which supersymmetry breaking is mediated mainly by gauge mediation, via a direct coupling between the messengers and the adjoint Higgs that breaks SU(5). This gives rise to an interesting phenomenology where a neutralino mostly bino is naturally the LSP. Moreover, the gravitational effects are strong enough to generate the μ and B μ term of the MSSM.

1 Introduction and motivations

Supersymmetry (SUSY) breaking is the central open question in supersymmetric extensions of the Standard Model. There are two major transmission mechanisms, each having its own advantages and disadvantages:

- gravity mediation [1] easily generates all soft terms needed at low energy in the Minimal Supersymmetric Standard Model (MSSM), including the μ and $B\mu$ terms of the Higgs sector [2], all being of the order of the gravitino mass at high energy. A traditional problem is that the flavor universality needed in order to avoid flavor changing neutral current (FCNC) transitions is not automatic. The lightest supersymmetric particle (LSP) in gravity mediation is generically the lightest neutralino.

- gauge mediation (GMSB) [3, 4, 5] uses Standard Model gauge loops, and therefore successfully addresses the flavor problem of supersymmetric models. The soft terms are typically of the order of a scale determined by the SUSY breaking times a loop factor, which we call M_{GM} in the following. There is however a serious problem in generating μ and $B\mu$ of the right size [6]. The gravitino, whose mass $m_{3/2}$ is much smaller than M_{GM}, is the LSP. Its lightness is the main signature of gauge mediation.

An obvious way of combining the advantages and possibly reducing the disadvantages of both mechanisms is to assume

$$m_{3/2} \sim (0.01 - 0.1)\, M_{GM} \ , \qquad M_{GM} \sim 1\,\text{TeV} \ . \tag{1}$$

In this case, the FCNC amplitudes induced by the non-universal gravity contributions to soft scalar masses are suppressed by a factor of order $m_{3/2}^2/M_{GM}^2$. Concerning the $\mu/B\mu$ problem, an option would be to generate $\mu \sim B \sim m_{3/2}$, through the Giudice-Masiero mechanism [2]. However, since $M_{GM} \gg m_{3/2}$, the squark and gluino masses are much larger than $m_{3/2}$, and therefore electroweak symmetry breaking requires $\mu \gg m_{3/2}$. As we will see explicitly later on, there is a way of generating $\mu \sim M_{GM}$ in the scenario considered in this paper, namely through Planck-suppressed non-renormalizable operators.

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Combining the gauge and gravity mediation mechanisms is an obvious possibility, which has been considered in the past or more recently from various perspectives [7]. It is easy to see that such a hybrid scenario arises for messenger masses close to the GUT scale. Indeed, consider a set of messenger fields generically denoted by $(\Phi, \tilde{\Phi})$ coupling to a set of SUSY breaking fields, generically denoted by X:

$$W_m = \Phi\,(\lambda_X X + m)\,\tilde{\Phi}\ , \tag{2}$$

with $\langle X\rangle = X_0 + F_X\theta^2$. The gauge-mediated contributions to the MSSM soft terms are proportional to

$$M_{GM} = \frac{g^2}{16\pi^2}\,\frac{\lambda_X F_X}{M}\ , \tag{3}$$

where $M = \lambda_X X_0 + m$, and $g^2/16\pi^2$ is the loop suppression of gauge mediation. Since the gravitino mass is given by $m_{3/2} \sim F_X/M_P$ (numerical factors are omitted in this introductory part), the ratio of the gauge to the gravity contribution reads

$$\frac{M_{GM}}{m_{3/2}} \sim \frac{g^2}{16\pi^2}\lambda_X\frac{M_P}{M}\ , \tag{4}$$

which shows that gravity mediation is subdominant for $M \lesssim \frac{g^2}{16\pi^2}\,\lambda_X M_P \sim \lambda_X M_{GUT}$, but not completely negligible if M lies within a few orders of magnitude of $\lambda_X M_{GUT}$.

In the case where messengers come into vector-like pairs of complete $SU(5)$ multiplets, such as $(5, \bar{5})$ or $(10, \overline{10})$, and ignoring for simplicity a possible “flavor” structure in the messenger indices, the messenger mass matrix can be written as

$$M(X) = \lambda_X X + m\ , \qquad m = m_0\, I + \lambda_\Sigma\,\langle\Sigma\rangle + \dots\ , \tag{5}$$

where Σ is the $SU(5)$ adjoint Higgs field. Indeed, any vector-like pair of complete SU(5) multiplets, besides having an $SU(5)$ symmetric mass m_0, can also couple to Σ and get an $SU(5)$-breaking mass term from its vev. Depending on the messenger representation, m could also receive contributions from other operators, denoted by dots in Eq. (5): operators involving other $SU(5)$ Higgs representations than Σ, or higher-dimensional operators such as $\Phi\Sigma^2\tilde{\Phi}/M_P$.

From a model-building perspective, the main novelty of the present work [8] is to consider the case where the messenger mass matrix is mostly given by the second term in m, i.e. we assume

$$M(X) = \lambda_X X + \lambda_\Sigma\langle\Sigma\rangle\ , \quad \text{with} \quad \lambda_X X_0 \ll \lambda_\Sigma\langle\Sigma\rangle\ . \tag{6}$$

As we shall see in Section 3.2, the latter condition is naturally satisfied when X is identified with the SUSY breaking field of a hidden sector, e.g. when X is the meson field of the ISS model [9]. Since[1] $\langle\Sigma\rangle = 6vY$, where $v \approx 10^{16}$ GeV and Y is the hypercharge generator embedded in $SU(5)$, Eq. (6) implies

$$M = 6\lambda_\Sigma v\, Y\ , \tag{7}$$

up to small corrections of order $\lambda_X X_0$. Eq. (7) has a significant impact on the structure of the GMSB-induced soft terms in the visible (MSSM) sector. Most notably, since the gaugino masses M_a (where a refers to the SM gauge group factor $G_a = SU(3)_C$, $SU(2)_L$ or $U(1)_Y$) are proportional to $\mathrm{Tr}\left(Q_a^2/M\right)$, where the Q_a's stand for the charges of the messenger fields under G_a, it is readily seen that the gauge-mediated contribution to the bino mass vanishes in the limit $X_0 = 0$:

$$M_1|_{GMSB,\,X_0=0} \propto \mathrm{Tr}\left(Y^2 M^{-1}\right) \propto \mathrm{Tr}\,Y = 0\ . \tag{8}$$

This result holds independently of the $SU(5)$ representation of the messengers. A nonzero bino mass is generated from gravity mediation, from $X_0 \neq 0$ and from possible other terms in m,

[1] In the following, we define the $SU(5)$ breaking vev v by $\langle\Sigma\rangle = v\,\mathrm{Diag}\,(2,2,2,-3,-3)$. By identifying the mass of the superheavy $SU(5)$ gauge bosons with the scale M_{GUT} at which gauge couplings unify, we obtain $v = \sqrt{2/25}\,M_{GUT}/g_{GUT} \approx 10^{16}$ GeV.

but it is expected to be much smaller than the other gaugino masses, which are of order M_{GM}. The resulting mass hierarchy,

$$M_1 \ll M_2 \sim M_3 \sim \mu \ , \tag{9}$$

leads to a light mostly-bino neutralino, which is therefore the LSP (unless $M_1 \gtrsim 2m_{3/2}$ at the messenger scale, in which case the LSP is the gravitino). In addition to being theoretically well motivated, this scheme provides a natural realization of the light neutralino scenarios occasionally considered in the literature [10, 11, 12, 13, 14], and invoked more recently [15] in connection with the new DAMA/LIBRA data [16].

The plan of this proceedings, based on [8] is the following. In Section 2, we present the MSSM soft terms induced by the messenger mass matrix (6), which breaks the $SU(5)$ symmetry in a well-defined manner. In Section 3, we couple the messenger sector to an explicit (ISS) supersymmetry breaking sector. We study the stability of the phenomenologically viable vacuum after including quantum corrections, and discuss the generation of the μ and $B\mu$ terms by Planck-suppressed operators. In Section 4, we discuss the low-energy phenomenology of the scenario, paying particular attention to the dark matter constraint. Finally, we present our conclusions in Section 5. The appendices contain technical details about the computation of the MSSM soft terms and the quantum corrections to the scalar potential.

2 Gauge mediation with GUT-induced messenger mass splitting

The main difference between minimal gauge mediation and the scenario considered in this paper[2] lies in the messenger mass matrix (6). The messenger mass splitting depends on the $SU(5)$ representation of the messenger fields. Denoting by $(\phi_i, \tilde{\phi}_i)$ the component messenger fields belonging to definite SM gauge representations and by Y_i their hypercharge, one has

$$\mathrm{Tr}\,(\Phi\langle\Sigma\rangle\tilde{\Phi}) \ = \ 6v \sum_i Y_i\, \phi_i\tilde{\phi}_i \ , \tag{10}$$

yielding a mass $M_i = 6\lambda_\Sigma v Y_i$ for $(\phi_i, \tilde{\phi}_i)$ (again $X_0 = 0$ is assumed). In the cases of $(\bar{5}, 5)$ and $(10, \overline{10})$ messengers, the component fields and their masses are, respectively,

$$\Phi(\bar{5}) \ = \ \left\{\phi_{\bar{3},1,1/3}\,,\, \phi_{1,2,-1/2}\right\}, \qquad M \ = \ \{2\lambda_\Sigma v\,,\, -3\lambda_\Sigma v\}\,, \tag{11}$$

$$\Phi(10) \ = \ \left\{\phi_{3,2,1/6}\,,\, \phi_{\bar{3},1,-2/3}\,,\, \phi_{1,1,1}\right\}, \quad M \ = \ \{\lambda_\Sigma v\,,\, -4\lambda_\Sigma v\,,\, 6\lambda_\Sigma v\}\,, \tag{12}$$

where the subscripts denote the $SU(3)_C \times SU(2)_L \times U(1)_Y$ quantum numbers, and the components $\tilde{\phi}_i$ of $\tilde{\Phi}$ are in the complex conjugate representations.

The one-loop GMSB-induced gaugino masses are given by (see Appendix A)

$$M_a(\mu) \ = \ \frac{\alpha_a(\mu)}{4\pi} \sum_i 2T_a(R_i)\, \frac{\partial \ln(\det M_i)}{\partial \ln X}\, \frac{F_X}{X}\bigg|_{X=X_0} \ , \tag{13}$$

where the sum runs over the component messenger fields $(\phi_i, \tilde{\phi}_i)$, and $T_a(R_i)$ is the Dynkin index of the representation R_i of ϕ_i. As noted in the introduction, with the messenger mass matrix (6), the gauge-mediated contribution to the bino mass vanishes irrespective of the $SU(5)$ representation of the messengers, up to a correction proportional to $\lambda_X X_0$ which will turn out to be negligible (see Section 3.2). Then M_1 is mainly of gravitational origin:

$$M_1 \ \sim \ m_{3/2} \ . \tag{14}$$

[2] For recent analyses of general messenger masses, see e.g. Refs. [17].

As the messenger masses are not $SU(5)$ symmetric, the running between the different messenger scales should be taken into account in the computation of the soft scalar masses. The corresponding formulae are given in Appendix A. For simplicity, we write below the simpler expressions obtained when the effect of this running is neglected. The two-loop MSSM soft scalar mass parameter m^2_χ, induced by N_1 messengers of mass M_1 and N_2 messengers of mass M_2 and evaluated at the messenger scale, reads

$$m^2_\chi \;=\; 2\sum_a C^a_\chi \left(\frac{\alpha_a}{4\pi}\right)^2 \left\{ 2N_2 T_a(R_2) \left|\frac{\partial \ln M_2}{\partial \ln X}\right|^2 + 2N_1 T_a(R_1) \left|\frac{\partial \ln M_1}{\partial \ln X}\right|^2 \right\} \left|\frac{F_X}{X}\right|^2 . \qquad (15)$$

In Eq. (15), C^a_χ are the second Casimir coefficients for the superfield χ, normalized to $C(N) = (N^2-1)/2N$ for the fundamental representation of $SU(N)$ and to $C^1_\chi = 3Y^2_\chi/5$ for $U(1)$, and $T_a(R_i)$ are the Dynkin indices for the messenger fields.

While the vanishing of the GMSB contribution to the bino mass is a simple consequence of the underlying hypercharge embedding in a simple gauge group and of the structure of the mass matrix (6) (i.e. it is independent of the representation of the messengers), the ratios of the other superpartner masses, including the ratio of the gluino to wino masses M_3/M_2, do depend on the representation of the messengers. This is to be compared with minimal gauge mediation [5], in which the ratios of gaugino masses (namely, $M_1 : M_2 : M_3 = \alpha_1 : \alpha_2 : \alpha_3$) as well as the ratios of the different scalar masses is independent of the representation of the messengers [18]. Leaving a more extensive discussion of the mass spectrum to Section 4, we exemplify this point below with the computation of the gaugino and scalar masses in the cases of $(5,\bar{5})$ and $(10,\overline{10})$ messengers:

i) $(5,\bar{5})$ **messenger pairs:** in this case the gluino and $SU(2)_L$ gaugino masses are given by

$$M_3 \;=\; \frac{1}{2}\, N_m\, \frac{\alpha_3}{4\pi}\, \frac{\lambda_X F_X}{\lambda_\Sigma v}\ , \qquad M_2 \;=\; -\frac{1}{3}\, N_m\, \frac{\alpha_2}{4\pi}\, \frac{\lambda_X F_X}{\lambda_\Sigma v}\ , \qquad (16)$$

where N_m is the number of messenger pairs, leading to the ratio $|M_3/M_2| = 3\alpha_3/2\alpha_2$ (≈ 4 at $\mu = 1$ TeV). The complete expressions for the scalar masses can be found in Appendix A. For illustration, we give below the sfermion soft masses at a messenger scale of 10^{13} GeV, neglecting the running between the different messenger mass scales as in Eq. (15):

$$m^2_Q : m^2_{U^c} : m^2_{D^c} : m^2_L : m^2_{E^c} \;\approx\; 0.79 : 0.70 : 0.68 : 0.14 : 0.08\ , \qquad (17)$$

in units of $N_m M^2_{GM}$, with $M_{GM} \equiv (\alpha_3/4\pi)(\lambda_X F_X/\lambda_\Sigma v)$. In Eq. (17) as well as in Eq. (19) below, we used $(\alpha_1/\alpha_3)(10^{13}\ \text{GeV}) = 0.65$ and $(\alpha_2/\alpha_3)(10^{13}\ \text{GeV}) = 0.85$.

ii) $(10,\overline{10})$ **messenger pairs:** in this case the gluino and $SU(2)_L$ gaugino masses are given by

$$M_3 \;=\; \frac{7}{4}\, N_m\, \frac{\alpha_3}{4\pi}\, \frac{\lambda_X F_X}{\lambda_\Sigma v}\ , \qquad M_2 \;=\; 3\, N_m\, \frac{\alpha_2}{4\pi}\, \frac{\lambda_X F_X}{\lambda_\Sigma v}\ , \qquad (18)$$

leading to the ratio $M_3/M_2 = 7\alpha_3/12\alpha_2$ (≈ 1.5 at $\mu = 1$ TeV). In this case too, we give the sfermion soft masses at a messenger scale of 10^{13} GeV for illustration:

$$m^2_Q : m^2_{U^c} : m^2_{D^c} : m^2_L : m^2_{E^c} \;\approx\; 8.8 : 5.6 : 5.5 : 3.3 : 0.17\ , \qquad (19)$$

again in units of $N_m M^2_{GM}$. For the Higgs soft masses, one has $m^2_{H_u} = m^2_{H_d} = m^2_L$ irrespective of the messenger representation.

In contrast to minimal gauge mediation with $SU(5)$ symmetric messenger masses, in which the ratios of gaugino masses are independent of the messenger representation, in our scenario the gaugino mass ratios and more generally the detailed MSSM mass spectrum are representation dependent. There is however one clear-cut prediction, which distinguishes it from both minimal gauge mediation and minimal gravity mediation, namely the vanishing of the one-loop GMSB contribution to the bino mass. Notice also the lightness of the scalar partners of the right-handed leptons for $(10, \overline{10})$ messengers, which arises from the correlation between the hypercharge and the mass of the different component messenger fields (the lightest components have the smallest hypercharge). Finally, we would like to point out that, due to the fact that messengers carrying different SM gauge quantum numbers have different masses, gauge coupling unification is slightly modified compared to the MSSM. Since the messengers are heavy and their mass splitting is not very important, however, this effect is numerically small.

In the above discussion, the higher-dimensional operator $\lambda'_\Sigma\, \Phi \Sigma^2 \tilde{\Phi} / M_P$ was assumed to be absent. Before closing this section, let us briefly discuss what relaxing this assumption would imply. If $\lambda'_\Sigma \neq 0$, the messenger mass matrix (7) receives an additional contribution, which affects the gauge-mediated MSSM soft terms. In particular, M_1 no longer vanishes:

$$M_1|_{GMSB} \;=\; -\frac{6}{5}\, d\, \mathrm{Tr}\, Y^2\, \frac{\lambda'_\Sigma v}{\lambda_\Sigma M_P}\, \frac{\alpha_1}{\alpha_3}\, N_m M_{GM}\ , \tag{20}$$

where d is the dimension of the messenger representation, and the trace is taken over the representation. Eq. (20) was derived under the assumption that the λ'_Σ-induced corrections to the messenger masses are small, so that to a good approximation, the scalar and electroweak gaugino masses are still given by Eqs. (16) to (19). It is easy to show that this implies

$$M_1|_{GMSB} \;\ll\; 0.2\, N_m M_{GM}\ , \tag{21}$$

for both $(5, \bar{5})$ and $(10, \overline{10})$ messengers. In the rest of the paper, we shall therefore neglect the contribution of $\lambda'_\Sigma \neq 0$ and assume that M_1 is generated by gravity mediation.

3 A complete model

The computation of the MSSM soft terms performed in the previous section is to a large extent insensitive to the details of the sector that breaks supersymmetry. The generation of the μ and $B\mu$ terms, on the other hand, depends on its details. The goal of the present section is to consider an explicit SUSY breaking sector, to couple it to the messenger sector, and to check that the following constraints are satisfied: (i) nonperturbative instabilities towards possible color-breaking vacua are sufficiently suppressed; (ii) μ and $B\mu$ parameters of the appropriate size can be generated.

The model can be described by a superpotential of the form:

$$W \;=\; W_{MSSM} \;+\; W_{SB}(X, \ldots) \;+\; W_m(\Phi, \tilde{\Phi}, X, \Sigma) \;+\; W_{GUT}(\Sigma)\ , \tag{22}$$

where $W_{SB}(X, \ldots)$ describes the SUSY breaking sector, $W_m(\Phi, \tilde{\Phi}, X, \Sigma)$ the couplings of the messengers fields $(\Phi, \tilde{\Phi})$ to the SUSY breaking fields X and to the $SU(5)$ adjoint Higgs field Σ, and $W_{GUT}(\Sigma)$ describes the breaking of the unified gauge symmetry, $SU(5) \to SU(3)_C \times SU(2)_L \times U(1)_Y$. In this paper, we consider the case where $W_m(\Phi, \tilde{\Phi}, X, \Sigma) = \Phi\, (\lambda_X X + \lambda_\Sigma \Sigma)\, \tilde{\Phi}$. The details of the GUT sector are irrelevant for our purposes and will not be further discussed in the following. The implicit assumption here is that the SUSY breaking sector and the GUT sector only couple via gravity and via the messenger fields. It is therefore reasonable to expect that they do not influence significantly their respective dynamics.

3.1 The SUSY breaking sector

A generic dynamical supersymmetry breaking sector [19] coupled to the messenger sector is enough for our purposes. For concreteness and simplicity, we consider here the ISS model [9], namely $\mathcal{N} = 1$ SUSY QCD with N_f quark flavors and gauge group $SU(N_c)$ in the regime $N_c < N_f < \frac{3}{2} N_c$. In the IR, the theory is strongly coupled, giving rise to a low-energy physics that is better described by a dual "magnetic" theory with gauge group $SU(N_f - N_c)$, N_f flavors of quarks q_a^i and antiquarks $\tilde{q}_i^a$, and meson (gauge singlet) fields X_i^j ($i, j = 1 \dots N_f$, $a = 1 \dots N$, with $N \equiv N_f - N_c$). The magnetic theory is IR free and can be analyzed perturbatively.

The superpotential of the magnetic theory,

$$W_{ISS} \;=\; h\, q_a^i X_i^j \tilde{q}_j^a \;-\; h f^2\, \mathrm{Tr} X \;, \tag{23}$$

leads to supersymmetry breaking a la O'Raifeartaigh, since the auxiliary fields $(-F_X^\star)^i_j = h q_a^i \tilde{q}_j^a - h f^2 \delta_j^i$ cannot all be set to zero. Indeed, the matrix $q_a^i \tilde{q}_j^a$ is at most of rank N, whereas the second term $h f^2 \delta_j^i$ has rank $N_f > N$. The supersymmetry-breaking ISS vacuum is defined by $\langle q_a^i \rangle = \langle \tilde{q}_i^a \rangle = f \delta_i^a$, $\langle X \rangle = 0$. At tree level, there are flat directions along which the components $i, j = (N+1) \dots N_f$ of X_i^j are non-vanishing; quantum corrections lift them and impose $\langle X \rangle = 0$ [9]. This means that the R-symmetry under which X is charged is not spontaneously broken, which in turn implies that no gaugino masses are generated in the minimal ISS model. Another important feature of the ISS vacuum is that it is metastable. Indeed, according to the Witten index, the theory possesses N_c supersymmetric vacua. These vacua are obtained in the magnetic description by going along the branch with nonzero meson vev's, $\langle X \rangle \neq 0$, where magnetic quarks become massive and decouple, so that the low-energy theory becomes strongly coupled again. In order to ensure that the lifetime of the ISS vacuum is larger than the age of the universe, one requires $f \ll \Lambda_m$, where Λ_m is the scale above which the magnetic theory is strongly coupled.

3.2 Coupling the SUSY breaking sector to messengers

Let us now couple the SUSY breaking sector to the messenger sector by switching on the superpotential term $\lambda_X \Phi X \tilde{\Phi}$, and address the following two questions:

- How is the vacuum structure of the model affected, in particular is the ISS vacuum still metastable and long lived?
- Is it possible to generate μ and $B\mu$ of the appropriate size?

The first question has been investigated in several works [20] in the case of $SU(5)$ symmetric messenger masses. We reanalyze it in our scenario and come to a similar conclusion: the messenger fields induce a lower minimum which breaks the SM gauge symmetries, a rather common feature of gauge mediation models. To our knowledge, the solution we propose for the second issue has not been discussed in the literature[3]. We now proceed to address the above two questions in detail.

3.2.1 Stability of the phenomenologically viable vacuum

It is well known that coupling a SUSY breaking sector to a messenger sector generally introduces lower minima in which the messenger fields have nonzero vev's. Since the messengers carry SM gauge quantum numbers, these vacua are phenomenologically unacceptable. Such minima also appear in our scenario. Summarizing the analysis done in Appendix B, we indeed find two types of local supersymmetry-breaking minima at tree level:

- the ISS vacuum with no messenger vev's and energy $V(\phi\tilde{\phi} = 0) = (N_f - N) h^2 f^4$;

[3]For recent approaches to the $\mu / B\mu$ problem of gauge mediation, see Refs. [21].

- lower minima with messenger vev's $\phi\tilde{\phi} = \frac{-\sum_{i=N+1}^{N_f} \lambda^i_{X,i} h f^2}{\sum_{(i,j)\notin\{i=j=1\dots N\}} \left|\lambda^i_{X,j}\right|^2}$ and energy

$$V(\phi\tilde{\phi} \neq 0) = h^2 f^4 \left(N_f - N - \left| \sum_{i=N+1}^{N_f} \lambda^i_{X,i} \right|^2 \Big/ \sum_{(i,j)\notin\{i=j=1\dots N\}} \left|\lambda^i_{X,j}\right|^2 \right).$$

Transitions from the phenomenologically viable ISS minimum to the second class of minima, in which the SM gauge symmetry is broken by the messenger vev's, must be suppressed. An estimate of the lifetime of the ISS vacuum in the triangular approximation gives $\tau \sim \exp(\frac{(\Delta\phi)^4}{\Delta V})$, with

$$\frac{\Delta V}{(\Delta\phi)^4} = \sum_{(i,j)\notin\{i=j=1\dots N\}} |\lambda^i_{X,j}|^2 \equiv \overline{\lambda^2} . \tag{24}$$

The lifetime of the phenomenologically viable vacuum is therefore proportional to $e^{1/\overline{\lambda^2}}$. To ensure that it is larger than the age of the universe, it is enough to have $\overline{\lambda^2} \lesssim 10^{-3}$.

We conclude that, as anticipated, the superpotential coupling $\lambda_X \Phi X \tilde{\Phi}$ induces new minima with a lower energy than the ISS vacuum, in which the messenger fields acquire vev's that break the SM gauge symmetry. In order to ensure that the ISS vacuum is sufficiently long lived, the coupling between the ISS sector and the messenger sector, λ_X, has to be small. We believe that this result is quite generic.

Let us now discuss the stability of the phenomenologically viable vacuum under quantum corrections. As shown in Ref. [9], the ISS model possesses tree-level flat directions that are lifted by quantum corrections. The novelty of our analysis with respect to Ref. [9] is that we include messenger loops in the computation of the one-loop effective potential, and we find that these corrections result in a nonzero vev for X. The detailed analysis is given in Appendix B; here we just notice that since the messenger fields do not respect the R-symmetry of the ISS sector, it is not surprising that coupling the two sectors induces a nonzero vev for X (which otherwise would be forbidden by the R-symmetry). Indeed, the one-loop effective potential for the meson fields reads, keeping only the leading terms relevant for the minimization procedure (see Appendix B for details):

$$\begin{aligned} V_{1-\text{loop}}(X_0, Y_0) &= 2Nh^2 f^2 |Y_0|^2 + \frac{1}{64\pi^2} \Big\{ 8h^4 f^2 (\ln 4 - 1) N (N_f - N) |X_0|^2 \\ &+ \frac{10 N_m h^2 f^4 |\text{Tr}'\lambda|^2}{3\lambda_\Sigma v} \Big[(\text{Tr}'\lambda)\, X_0 + (\text{Tr}''\lambda)\, Y_0 + \text{h.c.} \Big] \Big\} , \end{aligned} \tag{25}$$

where we have set $\tilde{X} = X_0\, I_{N_f - N}$, $\tilde{Y} = Y_0\, I_N$ and defined $\text{Tr}'\lambda \equiv \sum_{i=N+1}^{N_f} \lambda^i_{X,i}$, $\text{Tr}''\lambda \equiv \sum_{i=1}^{N} \lambda^i_{X,i}$. In Eq. (25), the first line contains the tree-level potential for X and the one-loop corrections computed in Ref. [9], whereas the linear terms in the second line are generated by messenger loops. The latter induce vev's for the meson fields:

$$\langle X_0 \rangle \simeq - \frac{5 N_m\, |\text{Tr}'\lambda|^2\, (\text{Tr}'\lambda)^\star}{12 (\ln 4 - 1) h^2 N (N_f - N)}\, \frac{f^2}{\lambda_\Sigma v} , \tag{26}$$

$$\langle Y_0 \rangle \simeq - \frac{5 N_m\, |\text{Tr}'\lambda|^2\, (\text{Tr}''\lambda)^\star}{192 \pi^2 N}\, \frac{f^2}{\lambda_\Sigma v} . \tag{27}$$

Notice that, due to $\langle Y_0 \rangle \neq 0$, magnetic quarks (and antiquarks) do contribute to supersymmetry breaking: $F_q \sim \tilde{q} X \neq 0$ ($F_{\tilde{q}} \sim q X \neq 0$), while $F_q = F_{\tilde{q}} = 0$ in the ISS model as a consequence of the R-symmetry. Here instead, the R-symmetry is broken by the coupling of the ISS sector to the messengers fields, and the F-terms of the magnetic (anti-)quarks no longer vanish. We have checked that, in the messenger direction, $\phi = \tilde{\phi} = 0$ is still a local minimum. We have also checked that the nonzero vev's (26) and (27) resulting from quantum corrections do not affect the discussion about the lifetime of the phenomenologically viable vacuum. Notice that these vev's also appear in the standard case where messenger masses are $SU(5)$ symmetric.

3.2.2 Generation of the μ and $B\mu$ terms

As stressed in the introduction, due to the hierarchy of scales $m_{3/2} \ll M_{GM}$, the Giudice-Masiero mechanism fails to generate a μ term of the appropriate magnitude for radiative electroweak symmetry breaking. Fortunately, there are other sources for μ and $B\mu$ in our scenario.

A crucial (but standard) hypothesis is the absence of a direct coupling between the hidden SUSY breaking sector and the observable sector (i.e. the MSSM). In particular, the coupling XH_uH_d should be absent from the superpotential. The fields of the ISS sector therefore couple to the MSSM fields only via non-renormalizable interactions and via the messengers. It is easy to check that non-renormalizable interactions involving the ISS and MSSM fields have a significant effect only on the μ and $B\mu$ terms, whereas they induce negligible corrections to the MSSM soft terms and Yukawa couplings. The most natural operators mixing the two sectors, which are local both in the electric and in the magnetic phases of the ISS model, are the ones built from the mesons X. It turns out, however, that such operators do not generate μ and $B\mu$ parameters of the appropriate magnitude.

Fortunately, a more interesting possibility arises in our scenario, thanks to the loop-induced vev of the meson fields discussed in the previous subsection. Indeed, the Planck-suppressed operator

$$\lambda_1 \frac{q\tilde{q}}{M_P} H_u H_d \ , \tag{28}$$

in spite of being of gravitational origin, yields a μ term that can be parametrically larger than $m_{3/2}$. This allows us to assume $m_{3/2} \ll M_{GM}$, as needed to suppress the most dangerous FCNC transitions, consistently with electroweak symmetry breaking (which typically requires a μ term of the order of the squark and gluino masses). More precisely, the operator (28) generates

$$\mu \;=\; \frac{\lambda_1}{h} \frac{N}{\sqrt{N_c}} \sqrt{3}\, m_{3/2} \ , \tag{29}$$

$$B \;=\; -2h\langle Y_0^\star \rangle \;=\; -\frac{5N_m\, |\mathrm{Tr}'\lambda|^2\, (\mathrm{Tr}''\lambda)}{96\pi^2 N \sqrt{N_c}} \frac{M_P}{\lambda_\Sigma v} \sqrt{3}\, m_{3/2} \ , \tag{30}$$

where we used $m_{3/2} = \sqrt{\sum_{i=N+1}^{N_f} |F^i_{X,i}|^2} / \sqrt{3} M_P = \sqrt{N_c}\, h f^2 / \sqrt{3} M_P$. Using Eqs. (26) and (27), it is easy to convince oneself that one can obtain $\mu \sim 1$ TeV for e.g. $m_{3/2} \sim (10 - 100)$ GeV, by taking a small enough ISS coupling h. As a numerical example, one can consider for instance $m_{3/2} = 50$ GeV, $N_c = 5$, $N_f = 7$ and $\lambda_1/h = 10$, in which case $\mu = 775$ GeV. As for the B parameter, it turns out to be somewhat smaller than $m_{3/2}$. For instance, taking as above $N_c = 5$, $N_f = 7$ and assuming further $N_m = 1$, $|\mathrm{Tr}'\lambda|^2 = 10^{-3}$ and $\lambda_\Sigma v = 10^{13}$ GeV, one obtains $B = -0.49\, (\mathrm{Tr}''\lambda)\, m_{3/2}$. This will in general be too small for a proper electroweak symmetry breaking, even if $\mathrm{Tr}''\lambda \sim 1$ is possible in principle (contrary to $\mathrm{Tr}'\lambda$, $\mathrm{Tr}''\lambda$ is not constrained by the lifetime of the ISS vacuum). However, $B\mu$ also receives a contribution from the non-renormalizable operator

$$\lambda_2 \frac{XX}{M_P} H_u H_d \ , \tag{31}$$

which gives a negligible contribution to μ, but yields $B\mu = -\lambda_2 \sqrt{3N_c}\, \langle X_0 \rangle\, m_{3/2}$. Using Eq. (29), one then obtains

$$B \;=\; -\,\lambda_2 \frac{h}{\lambda_1} \frac{N_c}{N} \langle X_0 \rangle \;=\; -\frac{\lambda_2}{\lambda_1} \frac{5N_m\, |\mathrm{Tr}'\lambda|^2\, (\mathrm{Tr}'\lambda)^\star}{12(\ln 4 - 1) h^2 N^2 \sqrt{N_c}} \frac{M_P}{\lambda_\Sigma v} \sqrt{3}\, m_{3/2} \ , \tag{32}$$

which is enhanced with respect to Eq. (30) by the absence of the loop factor and by the presence of h^2 in the denominator. It is then easy to obtain the desired value of the B parameter. As an illustration, choosing the same parameter values as in the above numerical examples and taking $h = 0.1$, one obtains $B/\lambda_2 = 7.9$ TeV, while choosing $\mathrm{Tr}'\lambda = 10^{-2}$ (instead of $10^{-3/2}$) gives $B/\lambda_2 = 250$ GeV.

We conclude that Planck-suppressed operators can generate μ and $B\mu$ parameters of the appropriate size in our scenario, thanks to the vev's of the meson fields induced by messenger loops, which are crucial for the generation of $B\mu$. As mentioned in the previous subsection, these vev's appear independently of whether the messenger masses are split or not. Therefore, the μ and $B\mu$ terms can be generated in the same way in more standard gauge mediation models with $SU(5)$ symmetric messenger masses.

Notice that there is a price to pay for the above solution to the $\mu/B\mu$ problem: the interaction term (28), which is local in the magnetic ISS description, becomes non-local in the electric description, analogously to the $qX\tilde{q}$ coupling of the magnetic Seiberg duals [22].

4 Low-energy phenomenology

The phenomenology of minimal gauge mediation has been investigated in detail in the past (see e.g. Ref. [18]). The main distinctive feature of our scenario with respect to standard gauge mediation is the presence of a light neutralino, with a mass of a few tens of GeV in the picture where $M_1 \sim m_{3/2} \sim (10-100)$ GeV. As is well known, such a light neutralino is not ruled out by LEP data: the usually quoted lower bound $M_{\tilde{\chi}^0_1} \gtrsim 50$ GeV assumes high-scale gaugino mass unification, and can easily be evaded once this assumption is relaxed[4]. The other features of the superpartner spectrum depend on the messenger representation. Particularly striking is the lightness of the $\tilde{l}_R$ with respect to other sfermions (including the $\tilde{l}_L$) in the case of $(10, \overline{10})$ messengers. The values of the soft terms at the reference messenger scale[5] $M_{mess} = 10^{13}$ GeV are given by Eqs. (16) to (19). One can derive approximate formulae for the gaugino and the first two generation sfermion masses at low energy by neglecting the Yukawa contributions in the one-loop renormalization group equations, as expressed by Eq. (A.4). At the scale $\mu = 1$ TeV, one thus obtains

$$M_2 \simeq 0.25 N_m M_{GM} \,, \quad M_3 \simeq N_m M_{GM} \,, \tag{33}$$

$$m^2_{Q_{1,2}} \simeq (0.79 + 0.69 N_m) N_m M^2_{GM} \,, \quad m^2_{U^c_{1,2}} \simeq (0.70 + 0.66 N_m) N_m M^2_{GM} \,, \tag{34}$$

$$m^2_{D^c_{1,2}} \simeq (0.68 + 0.66 N_m) N_m M^2_{GM} \,, \quad m^2_{L_{1,2}} \simeq (0.14 + 0.03 N_m) N_m M^2_{GM} \,, \tag{35}$$

$$m^2_{E^c_{1,2}} \simeq 0.08 N_m M^2_{GM} + 0.12 M_1^2 \,, \tag{36}$$

for $(5, \bar{5})$ messengers, and

$$M_2 \simeq 2.2 N_m M_{GM} \,, \quad M_3 \simeq 3.5 N_m M_{GM} \,, \tag{37}$$

$$m^2_{Q_{1,2}} \simeq (8.8 + 10.4 N_m) N_m M^2_{GM} \,, \quad m^2_{U^c_{1,2}} \simeq (5.6 + 8.1 N_m) N_m M^2_{GM} \,, \tag{38}$$

$$m^2_{D^c_{1,2}} \simeq (5.5 + 8.1 N_m) N_m M^2_{GM} \,, \quad m^2_{L_{1,2}} \simeq (3.3 + 2.3 N_m) N_m M^2_{GM} \,, \tag{39}$$

$$m^2_{E^c_{1,2}} \simeq 0.17 N_m M^2_{GM} + 0.12 M_1^2 \,, \tag{40}$$

for $(10,\overline{10})$ messengers, where $M_{GM} = (\alpha_3(M_{mess})/4\pi)(\lambda_X F_X/\lambda_\Sigma v)$. Furthermore, one has in both cases:

$$M_{\tilde{\chi}^0_1} \approx 0.5 M_1 \,. \tag{41}$$

[4]More precisely, for a mostly-bino neutralino (as in our scenario, where $M_1 \ll M_2, |\mu|$), there is no mass bound from LEP if either $M_{\tilde{\chi}^0_1} + M_{\tilde{\chi}^0_2} > 200$ GeV or selectrons are very heavy [23]. The former constraint is satisfied by all superpartner mass spectra considered in this section. Furthermore, a mostly-bino neutralino has a suppressed coupling to the Z boson and thus only gives a small contribution to its invisible decay width.

[5]As explained in Appendix B.2, the requirement that our metastable vacuum is sufficiently long lived constrains the messenger scale $M_{mess} \equiv \lambda_\Sigma v$ to lie below 10^{14} GeV or so. Demanding $M_{GM}/m_{3/2} \sim 10$ further pushes it down to 10^{13} GeV.

In Eqs. (33) to (40), the unknown gravitational contribution to the soft terms is not taken into account, apart from M_1 which is taken as an input (we neglected subdominant terms proportional to M_1^2 in all sfermion masses but $m^2_{E^c_{1,2}}$). These formulae fit reasonably well the results obtained by evolving the soft terms from $M_{mess} = 10^{13}$ GeV down to $\mu = 1$ TeV with the code SUSPECT [24]. For the third generation sfermion masses, most notably for $m^2_{Q_3}$ and $m^2_{U^c_3}$, the Yukawa couplings contribute sizeably to the running and the above formulae do not apply. The Higgs and neutralino/chargino spectrum also depend on $\tan\beta$ and on the values of the μ and $B\mu$ parameters, which are determined from the requirement of proper radiative electroweak symmetry breaking. As for the lightest neutralino, Eq. (41) implies that $M_{\tilde{\chi}^0_1} < m_{3/2}$ as long as $M_1 \lesssim 2m_{3/2}$, a condition which is unlikely to be violated if M_1 is of gravitational origin, and we can therefore safely assume that the lightest neutralino is the LSP. The gravitino is then the NLSP, and its late decays into $\tilde{\chi}^0_1\gamma$ tend to spoil the successful predictions of Big Bang nucleosynthesis (BBN) if it is abundantly produced after inflation. This is the well-known gravitino problem [25], and it is especially severe for a gravitino mass in the few 10 GeV range, as in our scenario. We are therefore led to assume a low reheating temperature in order to reduce the gravitino abundance, typically $T_R \lesssim (10^5 - 10^6)$ GeV, which strongly disfavor baryogenesis mechanisms occuring at very high temperatures, such as (non-resonant) thermal leptogenesis.

While the lightness of $\tilde{\chi}^0_1$ is a welcome feature from the point of view of distinguishing the present scenario from other supersymmetric models (for recent studies of the collider signatures of a light neutralino, see e.g. Refs. [13, 14]), it might be a problem for cosmology. Indeed, a neutralino with a mass below, say, 50 GeV will generally overclose the universe, unless some annihilation processes are very efficient [11, 12, 13]: (i) the annihilation into $\tau^+\tau^-$ and $b\bar{b}$ via s-channel exchange of the CP-odd Higgs boson A, or (ii) the annihilation into a fermion-antifermion pair via t- and u-channel exchange of a light sfermion. The process (i) can bring the relic neutralino abundance down to the observed dark matter level (namely, $\Omega_{DM}h^2 = 0.1099 \pm 0.0062$ [26]) if A is light, $\tan\beta$ is large and $\tilde{\chi}^0_1$ contains a sizeable higgsino component (which requires $|\mu| \sim 100$ GeV). More precisely, $\tilde{\chi}^0_1$ can be as light as 6 GeV for $M_A \sim 90$ GeV and $\tan\beta > 30$ [12, 13], in the anti-decoupling regime for the lightest Higgs boson h. The process (ii) is more efficient for light sleptons ($\tilde{l}_R$) and large values of $\tan\beta$. In particular, in the large m_A region where the process (i) is not relevant, $\tilde{\chi}^0_1$ can be as light as 18 GeV without exceeding the observed dark matter density if $m_{\tilde{\tau}_1}$ is close to its experimental bound of 86 GeV and $\tan\beta \sim 50$ [11, 13]. Note that experimental limits on superpartner masses and rare processes have been imposed in deriving these bounds.

We were not able to find values of M_{GM}, N_m and $\tan\beta$ leading to a light A boson (say, $M_A \leq 120$ GeV); hence we must considerer $M_{\tilde{\chi}^0_1} > 18$ GeV in order to comply with the dark matter constraint. In Table 1, we display 6 representative spectra with $20\,\text{GeV} \leq M_{\tilde{\chi}^0_1} \leq 45\,\text{GeV}$ and light $\tilde{l}_R$ masses (apart from model 1), corresponding to different numbers and types of messengers, and different values of M_{GM} and $\tan\beta$. The superpartner masses were obtained by running the soft terms from $M_{mess} = 10^{13}$ GeV down to low energy with the code SUSPECT. Apart from M_1, which is taken as an input, the unknown subdominant gravitational contributions to the soft terms have not been included (we shall comment on this later). As is customary, $\tilde{f}_1$ and $\tilde{f}_2$ refer to the lighter and heavier $\tilde{f}$ mass eigenstates; for the first two generations of sfermions, they practically coincide with $\tilde{f}_R$ and $\tilde{f}_L$. We also indicated in Table 1 the bino and down higgsino components of the lightest neutralino, in the notation $\tilde{\chi}^0_1 = Z_{11}\tilde{B} + Z_{12}\tilde{W}^3 + Z_{13}\tilde{H}^0_d + Z_{14}\tilde{H}^0_u$.

Let us now comment on these spectra. In the case of messengers in $(5, \bar{5})$ representations, taking into account the LEP lower bound on the lightest Higgs boson mass ($m_h \geq 114.4$ GeV) and the experimental limits on the superpartner masses generally leads to relatively heavy $\tilde{l}_R$ (see model 1), although larger values of $\tan\beta$ yield a lighter $\tilde{\tau}_1$ (for instance, shifting $\tan\beta$ from 30 to 50 in model 1 gives $m_{\tilde{\tau}_1} = 150$ GeV). However, one can accommodate a lighter $\tilde{\tau}_1$ if one assumes a large number of messengers, as exemplified by model 2. Light sleptons are more

model	1	2	3	3 bis	4	5	6
$N_{(5,\bar{5})}$	1	6	0	0	0	1	3
$N_{(10,\overline{10})}$	0	0	1	1	4	1	1
M_{GM}	1000	200	300	300	110	220	160
M_1	50	50	50	85	80	85	85
$\tan\beta$	30	24	15	15	9	15	15
sign(μ)	+	+	+	+	+	+	+
h	114.7	115.0	115.2	115.2	116.5	114.6	114.8
A	779.2	645.4	892.2	892.4	1015	735.8	662.7
H^0	779.2	645.5	892.4	892.6	1015	735.9	662.8
$H^\pm$	783.3	650.3	895.7	895.9	1018	740.1	667.5
$\tilde{\chi}_1^\pm$	259.4	305.0	560.2	560.3	676.7	408.0	223.9
$\tilde{\chi}_2^\pm$	747.8	636.8	693.9	694.0	970.4	590.4	597.5
$\tilde{\chi}_1^0$	24.5	23.5	23.2	42.9	38.1	43.0	42.9
$\tilde{\chi}_2^0$	259.4	305.0	560.1	560.3	677.1	408.0	223.9
$\tilde{\chi}_3^0$	743.3	629.8	596.9	597.1	691.0	570.8	589.2
$\tilde{\chi}_4^0$	745.7	634.7	693.8	693.9	970.4	590.4	596.3
$\|Z_{11}\|$	0.9982	0.9975	0.9971	0.9971	0.9978	0.9968	0.9969
$\|Z_{13}\|$	0.0599	0.0708	0.0750	0.0755	0.0648	0.0792	0.0772
$\tilde{g}$	1064	1207	1097	1097	1527	1028	1063
$\tilde{t}_1$	984.6	927.3	861.7	861.6	1080	795.7	809.5
$\tilde{t}_2$	1156	1074	1240	1240	1468	1058	1002
$\tilde{u}_1, \tilde{c}_1$	1195	1087	1135	1135	1361	1006	987.9
$\tilde{u}_2, \tilde{c}_2$	1240	1115	1327	1327	1555	1118	1043
$\tilde{b}_1$	1128	1040	1123	1123	1356	995.4	966.2
$\tilde{b}_2$	1169	1079	1224	1224	1451	1038	987.1
$\tilde{d}_1, \tilde{s}_1$	1184	1085	1134	1134	1360	1005	987.1
$\tilde{d}_2, \tilde{s}_2$	1243	1117	1329	1329	1557	1121	1046
$\tilde{\tau}_1$	242.2	99.0	86.3	89.3	87.0	96.7	95.2
$\tilde{\tau}_2$	420.3	289.4	696.2	696.3	753.1	498.6	349.8
$\tilde{e}_1, \tilde{\mu}_1$	294.4	150.6	131.5	133.6	105.4	123.6	117.4
$\tilde{e}_2, \tilde{\mu}_2$	413.4	275.1	699.1	699.2	754.1	500.1	348.5
$\tilde{\nu}_\tau$	396.6	260.5	691.4	691.5	749.0	491.4	337.6
$\tilde{\nu}_e, \tilde{\nu}_\mu$	405.8	263.6	694.8	694.9	750.1	493.9	339.5
$\Omega_{\tilde{\chi}_1^0} h^2$	6.40	0.428	0.279	0.122	0.124	0.118	0.116

Table 1: Supersymmetric mass spectra obtained by running the soft terms from $M_{mess} = 10^{13}$ GeV down to low energy with the code SUSPECT (all masses in GeV).

easily obtained with messengers in $(10,\overline{10})$ representations (models 3/3bis and 4), or in both $(5,\bar{5})$ and $(10,\overline{10})$ representations (models 5 and 6). Note that both $m_{\tilde{\tau}_1}$ and $m_{\tilde{\mu}_1,\tilde{e}_1}$ are close to their experimental limits in model 4. Apart from the mass of the lightest neutralino (and to a lesser extent of $\tilde{l}_R$), the low-energy spectrum very weakly depends on the actual value of M_1 (compare models 3 and 3bis, which only differ by the value of M_1). In the last column of Table 1, we give the relic density of $\tilde{\chi}^0_1$ computed by the code micrOMEGAs [27, 28]. One can see that, for $M_{\tilde{\chi}^0_1} \sim (20-25)$ GeV, $\Omega_{\tilde{\chi}^0_1} h^2$ lies above the observed dark matter density, even though $\tilde{l}_R$ are light (models 1 to 3); this can be traced back to the small higgsino admixture of $\tilde{\chi}^0_1$, which suppresses the Z boson exchange contribution [13]. Larger values of $M_{\tilde{\chi}^0_1}$ enable the relic density to fall in the 2σ WMAP range (models 3bis to 6).

We conclude that the scenario of supersymmetry breaking considered in this paper can provide supersymmetric models with a light neutralino ($M_{\tilde{\chi}^0_1} \sim 40$ GeV) accounting for the dark matter of the universe. We have checked that the models of Table 1 are consistent with the negative results from direct dark matter detection experiments such as CDMS [29] and XENON [30]. Since the spin-independent (spin-dependent) neutralino-nucleon cross section is dominated by Higgs boson and squark exchange diagrams (Z boson and squark exchange diagrams), it is expected to be rather small in our scenario, in which squarks are heavy and the neutralino is mostly a bino. This is confirmed by a numerical computation with MicrOMEGAs, which gives typical values of $(10^{-46} - 10^{-45})$ cm^2 for the spin-independent cross-section, and of $(10^{-46} - 10^{-45})$ cm^2 for the spin-dependent cross-section.

Let us add for completeness that models 1 to 3 can be made consistent with the observed dark matter density by assuming a small amount of R-parity violation [10]. In fact, in the presence of R-parity violation, nothing prevents us from considering even smaller neutralino masses by lowering[6] $m_{3/2}$.

Some comments are in order regarding the subdominant supergravity contributions to the soft terms and their effects in flavor physics. First of all, these contributions will shift the values of the soft terms at M_{mess} by a small amount and correspondingly affect the spectra presented in Table 1. Since supergravity contributions are parametrically suppressed with respect to gauge contributions by a factor $m_{3/2}/(N_m M_{GM})$ for gaugino masses, and by a factor $m_{3/2}/(\sqrt{N_m} M_{GM})$ for scalar masses, we do not expect them to change the qualitative features of the spectra[7]. Also, the gravity-mediated A-terms are suppressed by the small gravitino mass, and they should not affect the sfermion masses in a significant way. The most noticeable consequence of the supergravity contributions is actually to introduce flavor violation in the sfermion sector at the messenger scale:

$$(M^2_\chi)_{ij} \;=\; m^2_\chi\, \delta_{ij} \,+\, (\lambda_\chi)_{ij}\, m^2_{3/2} \qquad (\chi = Q, U^c, D^c, L, E^c)\ , \tag{42}$$

where $m^2_\chi\, \delta_{ij}$ is the flavor-universal gauge-mediated contribution, and the coefficients $(\lambda_\chi)_{ij}$ are at most of order one. As is well known, flavor-violating processes are controlled by the mass insertion parameters (here for the down squark sector):

$$(\delta^d_{LL})_{ij} \;\equiv\; \frac{(M^2_Q)_{ij}}{\bar{m}^2_{\tilde{d}}}\ , \quad (\delta^d_{RR})_{ij} \;\equiv\; \frac{(M^2_{D^c})_{ij}}{\bar{m}^2_{\tilde{d}}}\ , \quad (\delta^d_{LR})_{ij} \;\equiv\; \frac{(A_d)_{ij} v_d}{\bar{m}^2_{\tilde{d}}} \qquad (i \neq j)\ , \tag{43}$$

where $(M^2_Q)_{ij}$, $(M^2_{D^c})_{ij}$ and $(A_d)_{ij} v_d$ are the off-diagonal entries of the soft scalar mass matrices renormalized at low energy and expressed in the basis of down quark mass eigenstates, and $\bar{m}_{\tilde{d}}$ is an average down squark mass.

[6] Assuming $M_1 \sim m_{3/2}$, one can reach $M_{\tilde{\chi}^0_1} \sim 5$ GeV by choosing $m_{3/2} \sim 10$ GeV. We refrain from considering much lower values of $m_{3/2}$, which would render the generation of $\mu \sim M_{GM}$ less natural. However, we note that in recent models of moduli stabilization [31, 32], gravity (moduli) contributions to gaugino masses are typically smaller than $m_{3/2}$ by one order of magnitude.

[7] For values of $m_{3/2}$ as large as $80-85$ GeV, however, the supergravity contribution to the $\tilde{l}_R$ masses is expected to be comparable to the GMSB one. In this case the parameters of the models in Table 1 must be adjusted in order to keep the sleptons sufficiently light.

Neglecting the RG-induced flavor non-universalities, which are suppressed by a loop factor and by small CKM angles, the mass insertion parameters $(\delta^d_{MM})_{ij}$ $(M = L, R)$ arising from the non-universal supergravity contributions are suppressed by a factor $m^2_{3/2}/\bar{m}^2_{\tilde{d}}$, and possibly also by small coefficients $(\lambda_{Q,D^c})_{ij}$. For $m_{3/2} = 85$ GeV and $\bar{m}_{\tilde{d}} \sim 1$ TeV as in the spectra displayed in Table 1, we find $(\delta^d_{LL})_{ij} \sim 7 \times 10^{-3}\,(\lambda_Q)_{ij}$ and $(\delta^d_{RR})_{ij} \sim 7 \times 10^{-3}\,(\lambda_{D^c})_{ij}$, which is sufficient to cope with all experimental constraints (in the presence of large CP-violating phases, however, ϵ_K further requires $\sqrt{(\lambda_Q)_{12}(\lambda_{D^c})_{12}} \lesssim 0.04$, see e.g. Ref. [33]). As for the $(\delta^d_{LR})_{ij}$, they are typically suppressed by $m_{3/2} m_{d_i}/\bar{m}^2_{\tilde{d}}$ and are therefore harmless.

The situation is much more problematic in the slepton sector, where processes such as $\mu \to e\gamma$ and $\tau \to \mu\gamma$ put strong constraints on the $(\delta^e_{MN})_{ij}$, $M, N = L, R$ (see e.g. Ref. [34]). Indeed, the leptonic δ's are less suppressed than the hadronic ones, due to the smallness of the slepton masses: for $m_{3/2} = 50$ GeV and $m_{L_i} \sim 500$ GeV, $m_{E^c_i} \sim 100$ GeV, one e.g. finds $(\delta^e_{LL})_{ij} \sim 10^{-2}(\lambda_L)_{ij}$ and $(\delta^e_{RR})_{ij} \sim 0.3\,(\lambda_{E^c})_{ij}$. To cope with the experimental constraints, which are particularly severe in the presence of a light neutralino and of light sleptons, we need to assume close to universal supergravity contributions to slepton soft masses, perhaps due to some flavor symmetry responsible for the Yukawa hierarchies. Possible other sources of lepton flavor violation, e.g. radiative corrections induced by heavy states, should also be suppressed. Let us stress that the same problem is likely to be present in any light neutralino scenario in which the neutralino annihilation dominantly proceeds through slepton exchange. Alternatively, in models where the relic density of $\tilde{\chi}^0_1$ is controlled by a small amount of R-parity violation, all sleptons can be relatively heavy as in model 1, thus weakening the constraints from the non-observation of lepton flavor violating processes.

Throughout this paper, we assumed that the non-renormalizable operator $\Phi\Sigma^2\tilde{\Phi}/M_P$ is absent from the superpotential and that M_1 is purely of gravitational origin. Let us mention for completeness the alternative possibility that this operator is present and gives the dominant contribution to M_1. In this case, the lightest neutralino mass is no longer tied up with the mass of the gravitino, which can be the LSP as in standard gauge mediation. This makes it possible to solve the lepton flavor problem by taking $m_{3/2} \lesssim 10$ GeV and considering a model with relatively heavy $\tilde{l}_R$. Such a scenario is still characterized by a light neutralino, but it is no longer the LSP, and the dark matter abundance is no longer predicted in terms of parameters accessible at high-energy colliders. Furthermore, since the NLSP is the lightest neutralino, some amount of R-parity violation is needed to avoid the strong BBN constraints [25].

5 Conclusions

In this paper, we have shown that models in which supersymmetry breaking is predominantly transmitted by gauge interactions lead to a light neutralino if the messenger mass matrix is oriented with the hypercharge generator, $M \sim vY$. This arises naturally if the main contribution to messenger masses comes from a coupling to the adjoint Higgs field of an underlying $SU(5)$ theory. In this case, the bino receives its mass from gravity mediation, leading to a light neutralino which is then the LSP. While from a model building perspective the gravitino, hence the neutralino, could be much lighter, we considered a typical neutralino mass in the $(20 - 45)$ GeV range and worked out the corresponding low-energy superpartner spectrum. We noticed that, in the case of $(10, \overline{10})$ messengers or of a large number of $(5, \bar{5})$ messengers, the scalar partners of the right-handed leptons are much lighter than the other sfermions, making it possible for a neutralino with a mass around 40 GeV to be a viable dark matter candidate. However, such a SUSY spectrum also creates potential FCNC problems in the lepton sector, which asks for a high degree of universality or alignment in slepton masses.

In the hybrid models of supersymmetry breaking considered in this paper, the gravity-mediated contributions, although subdominant, are essential in generating the μ and $B\mu$ terms through Planck-suppressed operators. We studied the case where the SUSY breaking sector is provided by the ISS model and found that, as expected, messenger loops induce a breaking of the R-symmetry in the ISS vacuum. The associated meson vev's happen to be of the appropriate size for generating the $B\mu$ term needed for electroweak symmetry breaking. We stress that this mechanism also works for more general messenger mass matrices than the one studied in this paper, in particular in the simpler case of $SU(5)$ symmetric messenger masses.

While the vanishing of the GMSB contribution to the bino mass is a simple consequence of the messenger mass matrix (6) and of the embedding of the hypercharge into a simple gauge group, the other features of the superpartner spectrum depend on the representation of the messengers, in contrast to minimal gauge mediation. For example, the gluino to wino mass ratio is $|M_3/M_2| = 3\alpha_3/2\alpha_2$ for $(5, \bar{5})$ messengers and $|M_3/M_2| = 7\alpha_3/12\alpha_2$ for $(10, \overline{10})$ messengers. The experimental evidence for one of these mass ratios at the LHC, together with the discovery of a light neutralino LSP, would be a clear signature of the hybrid models of supersymmetry breaking studied in this paper. In most high-energy scenarios, gaugino masses are assumed to be universal, leading to the hierarchy $M_1 : M_2 : M_3 = \alpha_1 : \alpha_2 : \alpha_3$ at low energy. The possibility that non-universal gaugino masses be related to the lightness of the neutralino LSP by an underlying GUT structure appears to be appealing and deserves further investigation.

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A Gauge contributions to the MSSM gaugino and scalar masses

In this appendix, we compute the gauge-mediated contributions to the MSSM soft terms in the scenario with a GUT-induced messenger mass splitting considered in this paper. We use the method of Ref. [35], appropriately generalized to the case of several types of messengers with different masses.

A.1 General formulae

The gauge-mediated contributions to gaugino masses are encoded in the running of the gauge couplings [35]:

$$\frac{1}{g_a^2(\mu)} = \frac{1}{g_a^2(\Lambda_{UV})} - \frac{b_a}{8\pi^2} \ln\left(\frac{\Lambda_{UV}}{\mu}\right) + \sum_i \frac{2T_a(R_i)}{8\pi^2} \ln\left(\frac{\Lambda_{UV}}{\mathrm{M}_i}\right) , \tag{A.1}$$

where $b_a = 3C_2(G_a) - \sum_R T_a(R)$ is the beta function coefficient of the gauge group factor G_a, and the sum runs over several types of messengers $(\phi_i, \tilde{\phi}_i)$ with masses M_i $(\mu < M_i < \Lambda_{UV})$ belonging to the SM gauge representations R_i. $T_a(R_i)$ is the Dynkin index of the representation R_i, normalized to 1/2 for fundamental representations of $SU(N)$. For $U(1)$, we use the $SU(5)$ normalization $\alpha_1 = \frac{5}{3}\alpha_Y$; correspondingly, $T_1(R_i)$ should be understood as $3Y_i^2/5$, where the hypercharge Y is defined by $Y = Q - T_3$ (so that $Y_Q = 1/6$, $Y_{U^c} = -2/3$, $Y_{D^c} = 1/3$, $Y_L = -1/2$ and $Y_{E^c} = 1$). The one-loop gaugino masses are then given by [35]

$$M_a(\mu) = \frac{\alpha_a(\mu)}{4\pi} \sum_i 2T_a(R_i) \frac{\partial \ln(\det M_i)}{\partial \ln X} \frac{F_X}{X}\bigg|_{X=X_0} . \tag{A.2}$$

The gauge-mediated contributions to scalar masses are encoded in the wave-function renormalization of the MSSM chiral superfields χ [35]:

$$Z_\chi(\mu) \;=\; Z_\chi(\Lambda_{UV}) \prod_a \left(\frac{\alpha_a(\Lambda_{UV})}{\alpha_a(M_2)}\right)^{\frac{2C^a_\chi}{b_{a,2}}} \left(\frac{\alpha_a(M_2)}{\alpha_a(M_1)}\right)^{\frac{2C^a_\chi}{b_{a,1}}} \left(\frac{\alpha_a(M_1)}{\alpha_a(\mu)}\right)^{\frac{2C^a_\chi}{b_a}} , \tag{A.3}$$

where $\mu < M_1 < M_2 < \Lambda_{UV}$, $b_{a,1} \equiv b_a - 2N_1 T_a(R_1)$, $b_{a,2} \equiv b_{a,1} - 2N_2 T_a(R_2)$, and C^a_χ are the quadratic Casimir coefficients for the superfield χ, normalized to $C(N) = (N^2-1)/2N$ for the fundamental representation of $SU(N)$ and to $C^1_\chi = 3Y^2_\chi/5$ for $U(1)$. In Eq. (A.3), we considered for simplicity only 2 types of messengers, characterized by their masses $M_{1,2}$ (which should not be confused with the bino and wino masses), SM gauge representations $R_{1,2}$ and multiplicities $N_{1,2}$. Following Ref. [35], we obtain for the soft mass parameter m^2_χ:

$$\begin{aligned} m^2_\chi \;=\;& 2\sum_a C^a_\chi \left(\frac{\alpha_a(\mu)}{4\pi}\right)^2 \Bigg\{ \left[2N_2 T_a(R_2)\xi^2_{a,2} + \frac{(2N_2 T_a(R_2))^2}{b_{a,1}} (\xi^2_{a,1} - \xi^2_{a,2}) \right] \left|\frac{\partial \ln M_2}{\partial \ln X}\right|^2 \\ &+ 2N_1 T_a(R_1)\xi^2_{a,1} \left|\frac{\partial \ln M_1}{\partial \ln X}\right|^2 + \frac{1-\xi^2_{a,1}}{b_a} \left|\frac{\partial \ln(\det M)}{\partial \ln X}\right|^2 \Bigg\} \left|\frac{F_X}{X}\right|^2 \Bigg|_{X=X_0} , \end{aligned} \tag{A.4}$$

where $\xi_{a,i} \equiv \frac{\alpha_a(M_i)}{\alpha_a(\mu)}$ $(i=1,2)$ and $\det M = M_1^{N_1} M_2^{N_2}$. In Eq. (A.4), the first term in square brackets contains the contribution of the messengers of mass M_2 renormalized at the scale M_1, the second term represents the contribution of the messengers of mass M_1, and the third term the running from the messenger scale M_1 down to the low-energy scale μ.

A.2 $(5,\,\bar{5})$ and $(10,\,\overline{10})$ messengers with GUT-induced mass splitting

We are now in a position to evaluate the MSSM gaugino and scalar masses induced by N_m $(5,\bar{5})$ messenger pairs with a mass matrix $M(X)$ given by Eq. (6). Inside each pair, the $SU(3)_C$ triplets have a mass $2\lambda_\Sigma v$, while the $SU(2)_L$ doublets have a mass $-3\lambda_\Sigma v$ (we omit the contribution of $X_0 \neq 0$, which as discussed in Section 3.2 turns out to be negligible). Applying Eq. (A.2), we obtain for the one-loop gaugino masses:

$$M_3 \;=\; \frac{1}{2} N_m \frac{\alpha_3}{4\pi} \frac{\lambda_X F_X}{\lambda_\Sigma v} , \qquad M_2 \;=\; -\frac{1}{3} N_m \frac{\alpha_2}{4\pi} \frac{\lambda_X F_X}{\lambda_\Sigma v} , \qquad M_1 \;=\; 0 . \tag{A.5}$$

In computing scalar masses, we neglect for simplicity the running of the gauge couplings between different messenger scales, which amounts to set $\alpha_a(M_1) = \alpha_a(M_2) \equiv \alpha_a(M_{mess})$ in Eq. (A.4), where M_{mess} is an average messenger mass. Summing up all gauge contributions, we can cast the scalar masses in the form

$$m^2_\chi(M_{mess}) \;=\; N_m \sum_a d^a_\chi \left(\frac{\alpha_a}{4\pi}\right)^2 \left|\frac{\lambda_X F_X}{\lambda_\Sigma v}\right|^2 , \tag{A.6}$$

where $\alpha_a = \alpha_a(M_{mess})$ and the coefficients d^a_χ are given in the following table:

d^a_χ	$SU(3)_C$	$SU(2)_L$	$U(1)$
Q	2/3	1/6	1/180
U^c	2/3	0	4/45
D^c	2/3	0	1/45
L	0	1/6	1/20
E^c	0	0	1/5
H_u, H_d	0	1/6	1/20

Consider now N_m $(10,\overline{10})$ messenger pairs. Inside each pair, the $(\phi_{3,2,+1/6},\ \tilde{\phi}_{\bar{3},2,-1/6})$ fields have a mass $\lambda_\Sigma v$, $(\phi_{\bar{3},1,-2/3},\ \tilde{\phi}_{\bar{3},1,+2/3})$ have a mass $-4\lambda_\Sigma v$, and $(\phi_{1,1,+1},\ \tilde{\phi}_{1,1,-1})$ have a mass $6\lambda_\Sigma v$. Then the gaugino masses are given by

$$M_3 \;=\; \frac{7}{4}\, N_m\, \frac{\alpha_3}{4\pi}\, \frac{\lambda_X F_X}{\lambda_\Sigma v}\;, \qquad M_2 \;=\; 3\, N_m\, \frac{\alpha_2}{4\pi}\, \frac{\lambda_X F_X}{\lambda_\Sigma v}\;, \qquad M_1 \;=\; 0\;, \tag{A.7}$$

and the scalar masses by Eq. (A.6), with coefficients d^a_χ given by:

d^a_χ	$SU(3)_C$	$SU(2)_L$	$U(1)$
Q	11/2	9/2	1/90
U^c	11/2	0	8/45
D^c	11/2	0	2/45
L	0	9/2	1/10
E^c	0	0	2/5
H_u, H_d	0	9/2	1/10

B Quantum corrections and metastability of the vacuum

B.1 Tree-level vacuum structure

We are searching for the minima of the scalar potential

$$V \;=\; |F^a_X|^2 + |F^b_X|^2 + |F_q|^2 + |F_{\tilde{q}}|^2 + |F_\phi|^2 + |F_{\tilde{\phi}}|^2 + |F_\Sigma|^2\;, \tag{B.1}$$

where

$$\begin{aligned}
|F^a_X|^2 &= \sum_{i=1}^{N} \left| h q^i_a \tilde{q}^a_i - h f^2 + \lambda^i_{X,i} \phi\tilde{\phi} \right|^2\;, \\
|F^b_X|^2 &= \sum_{(i,j)\notin\{i=j=1\ldots N\}} \left| -h f^2 \delta^i_j + \lambda^i_{X,j} \phi\tilde{\phi} \right|^2\;, \\
|F_q|^2 &= \sum_{a,i=1\ldots N} \left| h X^j_i \tilde{q}^a_j \right|^2\;, \\
|F_{\tilde{q}}|^2 &= \sum_{a,j=1\ldots N} \left| h q^i_a X^j_i \right|^2\;, \\
|F_\phi|^2 &= \left| (\lambda_X X + \lambda_\Sigma \Sigma)\tilde{\phi} \right|^2\;, \\
|F_{\tilde{\phi}}|^2 &= |\phi(\lambda_X X + \lambda_\Sigma \Sigma)|^2\;, \\
|F_\Sigma|^2 &= \left| \lambda_\Sigma \phi\tilde{\phi} + \frac{\partial W_{GUT}}{\partial \Sigma} \right|^2\;.
\end{aligned} \tag{B.2}$$

We choose a basis in which $q^i_a \tilde{q}^a_j$ is a rank N diagonal matrix:

$$\left(\begin{array}{ccc|cc}
q_1\tilde{q}_1 & 0 & 0 & 0 & 0 \\
0 & \cdots & 0 & 0 & 0 \\
0 & 0 & q_N\tilde{q}_N & 0 & 0 \\
\hline
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0
\end{array}\right)\;. \tag{B.3}$$

The potential (B.1) does not contain the supergravity contributions nor the corresponding soft terms, which are expected to have a negligible impact in the present discussion. $W_{GUT}(\Sigma)$

is the superpotential for the $SU(5)$ adjoint Higgs field Σ, whose vev is responsible for the spontaneous breaking of $SU(5)$.

We find that all the F-terms, except F_X^b, can be set to zero. However, $F_\phi = F_{\tilde{\phi}} = 0$ has two types of solutions. More precisely, for values of X such that the matrix (acting on $SU(5)$ gauge indices) $\lambda_X X + \lambda_\Sigma \Sigma$ is

- invertible, then $\phi = \tilde{\phi} = 0$;
- non invertible, then both ϕ and $\tilde{\phi}$ can have a non zero vev.

Indeed, if $\lambda_X X + \lambda_\Sigma \sigma_i = 0$, where σ_i is an eigenvalue of Σ, the values of ϕ and $\tilde{\phi}$ are not fixed by the constraint $F_\phi = F_{\tilde{\phi}} = 0$. The equation $F_\Sigma = 0$ implies that they must be of the form $\phi = (0, \dots, 0, \phi^{\alpha_0}, 0, \dots, 0)$ and $\tilde{\phi}^T = (0, \dots, 0, \tilde{\phi}_{\alpha_0}, 0, \dots, 0)$. Indeed, one has $F^\alpha_{\Sigma,\beta} = f'(\Sigma)^\alpha_\beta - \rho\, \delta^\alpha_\beta + \lambda_\Sigma \phi^\alpha \tilde{\phi}_\beta = 0$, where $\alpha, \beta = 1 \dots 5$ are $SU(5)$ indices and $f(\Sigma)$ is defined by $W_{GUT}(\Sigma) = f(\Sigma) - \rho\, \mathrm{Tr}\Sigma$ (the specific form of the function f is irrelevant here). Working in a $SU(5)$ basis in which Σ^β_α is diagonal, one concludes that at most one component in ϕ and $\tilde{\phi}$ can be nonzero, and it must be the same component. As for F_q and $F_{\tilde{q}}$, they can always be fixed to zero by choosing the matrix X^j_i to be symmetric, with the vectors $q^i_a = \tilde{q}^a_i$ $(a = 1 \dots N)$, solutions of $h q^i_a \tilde{q}^a_i - h f^2 + \lambda^i_{X,i} \phi \tilde{\phi} = 0$ (so as to satisfy the constraint $|F^a_X|^2 = 0$), belonging to its kernel. Note that the value of X is not completely determined at this level.

We have succeeded to set all F-terms but F^b_X to zero without completely fixing the value of X. For generic couplings $\lambda^i_{X,j}$, it is still possible to arrange for the matrix $\lambda_X X + \lambda_\Sigma \Sigma$ to have a zero eigenvalue, in which case ϕ and $\tilde{\phi}$ can be nonzero. We can minimize $|F^b_X|^2$ in both cases ($\phi\tilde{\phi} = 0$ versus $\phi\tilde{\phi} \neq 0$), which yields two types of local supersymmetry-breaking minima:

- $\phi\tilde{\phi} = 0$, with the ISS energy $V_0 = (N_f - N) h^2 f^4$;
- $\phi\tilde{\phi} = \dfrac{-\sum_{i=N+1}^{N_f} \lambda^i_{X,i} h f^2}{\sum_{(i,j)\notin\{i=j=1\dots N\}} \left|\lambda^i_{X,j}\right|^2}$, with $V_0 = h^2 f^4 \left(N_f - N - \dfrac{\left|\sum_{i=N+1}^{N_f} \lambda^i_{X,i}\right|^2}{\sum_{(i,j)\notin\{i=j=1\dots N\}} \left|\lambda^i_{X,j}\right|^2} \right)$.

B.2 Lifetime of the metastable vacuum

Following Ref. [9], we evaluate the lifetime of the metastable ISS vacuum in the triangle approximation. The decay rate is proportional to

$$\exp\left(-\frac{(\Delta\phi)^4}{\Delta V}\right), \quad \text{with} \quad \frac{\Delta V}{(\Delta\phi)^4} = \sum_{(i,j)\notin\{i=j=1\dots N\}} |\lambda^i_{X,j}|^2 \equiv \overline{\lambda^2}\,. \tag{B.4}$$

In order for the metastable vacuum to be sufficiently long lived, we require $\overline{\lambda^2} \lesssim 10^{-3}$. The individual couplings $\lambda^i_{X,j}$ must then typically be of order 10^{-2}, except the ones corresponding to $i = j = 1 \dots N$, which can in principle be larger. From Eq. (4) we see that, for $\mathrm{Tr}'\lambda \equiv \sum_{i=N+1}^{N_f} \lambda^i_{X,i} = 10^{-2}$, $M_{GM}/m_{3/2} \sim 10$ corresponds to a messenger scale $\lambda_\Sigma v \sim 10^{13}$ GeV, which in turn requires $\lambda_\Sigma \sim 10^{-3}$.

B.3 Quantum corrections to the scalar potential

As explained in Ref. [9], the ISS model has a tree-level flat direction along the $i,j = (N+1) \dots N_f$ components of X^j_i. In the absence of messengers, quantum corrections enforce $\langle X \rangle = 0$. In this section, we add the contribution of the messengers to the one-loop effective potential for X and study its behaviour around $\phi = \tilde{\phi} = 0$. Our aim is to determine whether the ISS vacuum remains metastable and long lived in our scenario after quantum corrections have been included.

We parametrize the quantum fluctuations in the following way:

$$X = \begin{pmatrix} \tilde{Y} & \delta Z^\dagger \\ \delta\tilde{Z} & \tilde{X} \end{pmatrix}, \qquad q = \begin{pmatrix} fe^{\theta} + \delta\chi, \delta\rho \end{pmatrix}, \qquad \tilde{q} = \begin{pmatrix} fe^{-\theta} + \delta\tilde{\chi}^\dagger \\ \delta\tilde{\rho}^\dagger \end{pmatrix}, \tag{B.5}$$

with $\tilde{X} = X_0 + \delta\hat{X}$ and $\tilde{Y} = Y_0 + \delta\hat{Y}$. The only F-term from the ISS sector that is relevant for the computation of the messenger contribution to the one-loop effective potential is the one of $\tilde{X}$:

$$-F^\star_{\tilde{X}_{ff'}} = h\,\mathrm{Tr}_{N_c}(\delta\rho\,\delta\tilde{\rho}^\dagger)_{ff'} - hf^2\,\delta_{ff'} + \lambda_{X,ff'}\,\delta\phi\,\delta\tilde{\phi}\ , \tag{B.6}$$

where $f, f' = (N+1)\dots N_f$. The terms of the scalar potential that contribute to the scalar messenger mass matrix are:

$$|h\mathrm{Tr}_{N_c}(\delta\rho\delta\tilde{\rho}^\dagger)_{ff'} - hf^2\delta_{ff'} + \lambda_{X,ff'}\delta\phi\delta\tilde{\phi}|^2 + |(\lambda_X X + m)\delta\tilde{\phi}|^2 + |\delta\phi(\lambda_X X + m)|^2. \tag{B.7}$$

Around the vacuum with zero messenger vev's, $\phi = \tilde{\phi} = 0$, there is no quadratic mixing between the ISS and messenger fields. We can therefore compute separately the contributions of the ISS and messenger sectors to the effective potential.

Let us first consider the messenger sector. With the notations $\tilde{M}_I \equiv \lambda_X X + m_I$ (where the index I refers to different components of the messenger fields in definite SM gauge representations, and $m_I = 6\lambda_\Sigma Y_I v$), $\mathrm{Tr}'\lambda \equiv \sum_{i=N+1}^{N_f} \lambda^i_{X,i}$ and $t \equiv hf^2\,\mathrm{Tr}'\lambda$, the scalar mass matrix reads:

$$\begin{pmatrix} \phi_I^\dagger & \tilde{\phi}_I^\dagger & \phi_I & \tilde{\phi}_I \end{pmatrix} \begin{pmatrix} |\tilde{M}_I|^2 & & & -t^* \\ & |\tilde{M}_I|^2 & -t^* & \\ & -t & |\tilde{M}_I|^2 & \\ -t & & & |\tilde{M}_I|^2\ . \end{pmatrix} \begin{pmatrix} \phi_I \\ \tilde{\phi}_I \\ \phi_I^\dagger \\ \tilde{\phi}_I^\dagger \end{pmatrix} . \tag{B.8}$$

We then find the mass spectrum (which is non-tachyonic since $|t| = |\lambda_X F_X| \ll \lambda_\Sigma^2 v^2 \sim m_I^2$):

$$m^2_{0,I} = |\tilde{M}_I|^2 \pm |t| = |\lambda_X X + m_I|^2 \pm hf^2|\mathrm{Tr}'\lambda|\ . \tag{B.9}$$

The contribution of the messenger sector to the effective potential is then:

$$V^{(1)}_{\phi,\tilde{\phi}} = \frac{1}{64\pi^2}\,\mathrm{Str}M^4\,\ln\left(\frac{M^2}{\Lambda^2}\right) = \frac{2N_m}{64\pi^2}\left(20|t|^2 + 2|t|^2\ln\left(\frac{\det \tilde{M}^\dagger\tilde{M}}{\Lambda^2}\right)\right) . \tag{B.10}$$

As for the contribution of the ISS sector, it is given by [9]:

$$V^{(1)}_{ISS} = \frac{1}{64\pi^2}\,8\,h^4 f^2(\ln 4 - 1)N(N_f - N)|X_0|^2\ , \tag{B.11}$$

where we have set $\tilde{X} = X_0\,I_{N_f - N}$, $\tilde{Y} = Y_0\,I_N$, and we have omitted a term proportional to $|Y_0|^2$, which is subleading with respect to the tree-level potential for Y_0, $V^{(0)}_{ISS}(Y_0) = 2Nh^2f^2|Y_0|^2$ (by contrast, the term proportional to $|X_0|^2$ in $V^{(1)}_{ISS}$ is fully relevant, since there is no tree-level potential for X_0). To $V^{(0)}_{ISS} + V^{(1)}_{ISS}$, we add the linearized field-dependent one-loop contribution of the messenger sector, using the fact that $|t| \ll \lambda_\Sigma^2 v^2$:

$$V^{(1)}_{\phi,\tilde{\phi}} = \frac{N_m|\mathrm{Tr}'\lambda|^2h^2f^4}{64\pi^2}\left[-\frac{35}{18\lambda_\Sigma^2 v^2}\,(\lambda_X X)^2 + \frac{10}{3\lambda_\Sigma v}\,\lambda_X X + \mathrm{h.c.}\right] . \tag{B.12}$$

As will become clear after minimization of the full one-loop effective potential, the quadratic term in $V^{(1)}_{\phi,\tilde{\phi}}$ is suppressed with respect to the quadratic terms in V_{ISS} by $\langle X\rangle \ll \lambda_\Sigma v$, and can therefore be dropped. Minimizing $V^{(0)}_{ISS} + V^{(1)}_{ISS} + V^{(1)}_{\phi,\tilde{\phi}}$, one finds that the contribution of the messenger fields to the effective potential destabilizes the tree-level ISS vacuum and creates small tadpoles for the meson fields:

$$\langle X_0 \rangle \;\simeq\; -\, \frac{5 N_m \, |\mathrm{Tr}'\lambda|^2 \, (\mathrm{Tr}'\lambda)^\star}{12(\ln 4 - 1) h^2 N (N_f - N)} \, \frac{f^2}{\lambda_\Sigma v} \;\ll\; \lambda_\Sigma v \; , \tag{B.13}$$

$$\langle Y_0 \rangle \;\simeq\; -\, \frac{5 N_m \, |\mathrm{Tr}'\lambda|^2 \, (\mathrm{Tr}''\lambda)^\star}{192 \pi^2 N} \, \frac{f^2}{\lambda_\Sigma v} \;\ll\; \lambda_\Sigma v \; , \tag{B.14}$$

where $\mathrm{Tr}''\lambda \equiv \sum_{i=1}^N \lambda^i_{X,i}$. The contribution of Eqs. (B.13) and (B.14) to the vacuum energy, being suppressed both by a loop factor and by $\langle X_0 \rangle, \langle Y_0 \rangle \ll \lambda_\Sigma v$, is negligible compared with the ISS energy. Hence, we still have a metastable vacuum around $\langle \phi \rangle = \langle \tilde{\phi} \rangle = 0$, with a small tadpole induced for X. This plays an important role in generating μ and $B\mu$ parameters of the appropriate size in the MSSM Higgs sector, as discussed in Section 3.

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