

Scalar field potential in the Standard Model

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Abstract

The mechanism of particle mass generation in the Standard Model is discussed. It is shown that non-zero vacuum expectation value of a scalar field together with the proper symmetry of the Lagrangian allow a certain class of scalar field potentials providing generation of gauge boson and fermion masses and keeping the electroweak sector of the model unchanged. Applying the minimality principle and certain additional conditions one can reduce the number of free parameters in the model. Possible phenomenological consequences in the scalar sector for different choices of the potential are discussed.

1 Introduction

The Standard Model (SM) of elementary particles and fundamental interactions is a very successful physical theory providing theoretical predictions being in an excellent agreement with practically all present high energy physics experimental data. Nevertheless, for many reasons we suppose that the SM is not the ultimate theory of everything, but rather an effective model appropriate for the given energy range of modern experiments. Moreover, one of the most important ingredients of the model, the mechanism of electroweak (EW) symmetry breaking and particle mass generation, has not yet been directly verified. In the forthcoming experiments, particularly at the Large Hadron Collider (LHC) at CERN, we hope to access both the limit of the SM applicability and the mechanism of the EW symmetry breaking.

In the SM, the Higgs–Kibble mechanism [1, 2] together with the Yukawa interactions of the Higgs scalar with fermions are responsible for generation of masses for weak bosons and fermions, respectively. Note that in the SM the mass of the Higgs boson, m_h , appear as a free parameter and it is not *generated* contrary to all others. The direct experimental limit, $m_h > 114.4$ GeV at 95% C.L., and indirect upper bounds can be imposed on the mass of the SM Higgs particle (see *e.g.* Refs. [3, 4] and references therein). But still the nature of the very origin of the Higgs boson are not well understood and justified contrary to the ones of the gauge bosons, which are believed to be in a deep relation with the space-time symmetry properties. Moreover, there are several difficulties in the SM directly related to the Higgs potential: tachyon behavior of the Higgs field at large energies, monopole solutions in the classical Higgs potential, non-zero imaginary part of the effective potential, large Higgs self-coupling leading to non-perturbative effects and possibly to unitarity violation, and other problems. These difficulties and especially the naturalness (or the fine tuning) problem motivate us to look for models beyond SM. In this context the discussion of possible modifications of the scalar sector in the SM discussed in this paper might be of interest.

We suggest to consider a generalization of the scalar field potential. In fact, to generate the masses of electroweak bosons it is sufficient to have a non-zero vacuum expectation value of the scalar field

$$\sqrt{2}\langle 0|\Phi|0\rangle \equiv \eta = \left(\sqrt{2}G_{\text{Fermi}}\right)^{-1/2} \approx 246 \text{ GeV} \quad (1)$$

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together with a certain symmetry condition corresponding to the symmetry in the gauge sector of the model. On the other hand the standard Higgs potential with a tachyon mass parameter does not follow from any basic principle used to construct the SM. In particular the Higgs potential is not minimal as will be shown below. Moreover, it uses the correspondence to the Ginzburg-Landau superconductivity mechanism, where the appearance of such a potential is provided by external conditions, while in the SM this correspondence looks artificial since we to try to construct the model as a fundamental theory resulting from the *first principles*.

The paper is organized as follows. In the next Sect. we will consider in detail the mechanism of mass generation in the $U(1)$ abelian case. The conditions on the classical potential are discussed in Sect. 3. In Conclusions we discuss the $SU(2) \times U(1)$ case and peculiar properties of certain scalar potentials and their possible impact on the phenomenology.

2 Mass generation in the abelian case

Let us start with a model describing interactions of a scalar field Φ and a vector abelian gauge field A with the Lagrangian

$$\mathcal{L} = \partial_\mu \Phi^\dagger \partial^\mu \Phi - V(\Phi) - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + ig \left(\Phi^\dagger \partial_\mu \Phi - (\partial_\mu \Phi^\dagger) \Phi \right) A^\mu + g^2 \Phi^\dagger \Phi A_\mu A^\mu, \quad (2)$$

where g is the charge of the scalar field, and $V(\Phi)$ is the classical potential of the scalar field. We require the symmetry of the Lagrangian with respect to the transformations

$$\Phi \rightarrow e^{i\chi} \Phi, \quad \Phi^\dagger \rightarrow e^{-i\chi} \Phi^\dagger, \quad A_\mu \rightarrow A_\mu + \frac{1}{g} \partial_\mu \chi. \quad (3)$$

For the potential $V(\Phi)$ the above condition means that it should depend only on the product $\Phi^\dagger \Phi$ which is invariant under these transformations:

$$V(\Phi) = V(\Phi^\dagger \Phi). \quad (4)$$

The scalar field in the present case possesses two degrees of freedom. We can use the polar coordinates reflecting the symmetry of the theory and cast it in the form

$$\Phi(x) = \sigma(x) e^{i\theta(x)}, \quad \Phi^\dagger(x) = \sigma(x) e^{-i\theta(x)}. \quad (5)$$

The standard procedure of the gauge boson mass generation can be performed in these variables. Substituting (5) in the Lagrangian (2) we get

$$\mathcal{L} = \partial_\mu \sigma \partial^\mu \sigma - V(\Phi) + g^2 \sigma^2 \left(A_\mu + \frac{1}{g} \partial_\mu \theta \right) \left(A^\mu + \frac{1}{g} \partial^\mu \theta \right) - \frac{1}{4} F_{\mu\nu} F^{\mu\nu}. \quad (6)$$

To get a mass for the gauge boson it is sufficient to suppose that there is a non-zero vacuum expectation value of the radial degree of freedom of the scalar field:

$$\langle 0 | \sigma | 0 \rangle \equiv \sigma_0 = \frac{\eta}{\sqrt{2}}, \quad (7)$$

so that

$$\sigma(x) = \frac{\eta + h(x)}{\sqrt{2}}, \quad (8)$$

where h is a usual particle-like excitation which is called the Higgs boson, $\langle 0 | h(x) | 0 \rangle = 0$.

Note that the vacuum expectation value of the scalar field can be treated as its zeroth harmonic corresponding to an average over a large space volume V_0 :

$$\frac{1}{V_0} \int_{V_0} d^3x \sigma(x) = \sigma_0, \quad \frac{1}{V_0} \int_{V_0} d^3x h(x) = 0. \quad (9)$$

If we fix the gauge of our vector field as

$$A_\mu(x) \rightarrow B_\mu(x) = A_\mu(x) + \frac{1}{g}\partial_\mu\theta(x) \quad (10)$$

and apply the separation of the scalar field zeroth harmonic, we get the Lagrangian in the form

$$\begin{aligned} \mathcal{L} &= \frac{1}{2}\partial_\mu h\partial^\mu h - V\left(\frac{\eta + h(x)}{\sqrt{2}}\right) + \frac{1}{2}g^2\eta^2 B_\mu B^\mu + g^2\eta h B_\mu B^\mu + \frac{1}{2}g^2 h^2 B_\mu B^\mu \\ &- \frac{1}{4}\tilde{F}_{\mu\nu}\tilde{F}^{\mu\nu}, \quad \tilde{F}_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu. \end{aligned} \quad (11)$$

So the vector field B acquired a non-zero mass by absorbing the rotational degree of freedom of the scalar field. Note that this degree of freedom is massless just because it describes rotation without any reference to the Goldstone theorem. On the other hand, gauge fixing (10) can be considered as the $U(1)$ symmetry breaking. Nevertheless as discussed in Ref. [5] keeping the phenomenology unchanged, the whole procedure of mass generation performed in the polar variables can be interpreted in terms of supercurrents without any explicit gauge fixing.

Generation of the fermion masses can be performed in the usual way by introducing in the Lagrangian additional terms for free massless fermions and their interaction with our scalar field. Note that in the polar variables the Yukawa interaction term can be taken in the $U(1)$ symmetric form as

$$g_f|\Phi|\bar{f}f = g_f\sigma\bar{f}f = \frac{g_f\eta}{\sqrt{2}}\bar{f}f + \frac{g_f}{\sqrt{2}}h\bar{f}f. \quad (12)$$

3 The scalar field potential

Let us look now at the scalar field potential and define the class of possible choices of its form.

As concerns the symmetry condition for the potential, in these variables we explicitly see that it can be a bit extended with respect to the usual one defined by Eq. (4):

$$V(\Phi) = V(|\Phi|) = V(\sigma). \quad (13)$$

Cosmological observations show that the Higgs contribution to the Universe energy density vanishes (or is extremely small). This gives us an additional condition on the potential:

$$V(\sigma_0) + V_{\text{eff}}(\sigma_0) = 0, \quad (14)$$

where $V_{\text{eff}}(\sigma_0)$ is the effective Coleman-Weinberg additional part of the potential coming from loop corrections [6]. Note that this condition is a big puzzle for the SM, even so that it can be adjusted by *tuning* the divergence subtraction procedure in the V_{eff} calculations. In what follows we assume that we can choose such a classical potential that

$$V(\sigma_0) = 0, \quad V_{\text{eff}}(\sigma_0) = 0, \quad (15)$$

taking into account that a constant shift in the potential corresponds to adding a full derivative to the Lagrangian.

One more condition is coming from the stability condition: the point $|\Phi| = \sigma_0$ has to be a minimum of the potential, so that

$$\left.\frac{dV(\sigma)}{d\sigma}\right|_{\sigma=\sigma_0} = 0, \quad \left.\frac{d^2V(\sigma)}{d\sigma^2}\right|_{\sigma=\sigma_0} \geq 0. \quad (16)$$

This minimum has to be at least a local one. But from the general point of view it would be much better if that is the unique global minimum of the potential.

Let us limit the class of potentials by a polynomials of the 4th order or lower:

$$V_{\text{pol}}(\sigma) = c_0 + c_1\sigma + c_2\sigma^2 + c_3\sigma^3 + c_4\sigma^4. \quad (17)$$

For the minimality reason we do not consider now so-called quasi-potentials and higher order operators. Applying the considered conditions on the coefficients of the above potential and dropping terms linear in the Higgs field h because of Eq. (9), we get a class of potentials being different in the scalar sector but leading to the same effect of vector boson and fermion mass generation.

The standard Higgs potential

$$V_{\text{SM}}(\Phi) = \lambda \left(\Phi^\dagger \Phi - \frac{\eta^2}{2} \right)^2 = \lambda \eta^2 h^2 + \lambda \eta h^3 + \frac{\lambda}{4} h^4. \quad (18)$$

certainly satisfies the conditions listed above. In this case we have one free parameter λ , and the Higgs boson mass is defined from the first term on the right hand side, $m_h = \eta\sqrt{2\lambda}$. This form of the potential is known to be the source of many difficulties in the SM. In particular, the presence of two minima gives rise to an imaginary part in the one-loop effective potential leading to instability of quantum states in it as discussed in Refs. [7, 8].

Let us look at a sub-class of the general potential (17) with a single global minimum and even in powers of $(\sigma - \sigma_0)$:

$$V_I(\sigma) = m_h^2(\sigma - \sigma_0)^2 + \lambda(\sigma - \sigma_0)^4 = \frac{m_h^2}{2}h^2 + \frac{\lambda}{4}h^4, \quad (19)$$

where we have two free parameters: the Higgs mass m_h and the self-coupling λ . Note that the triple Higgs self-interaction is absent in this case. Potential V_I has one more free parameter with respect to the standard case. But since there is no any relation between the parameters, one can consider three special cases.

First we can drop the Higgs self-interaction term:

$$V_{II}(\sigma) = m_h^2(\sigma - \sigma_0)^2 = \frac{m_h^2}{2}h^2. \quad (20)$$

In this case the potential is reduced just to a mass term for the physical Higgs field. The number of free parameters is the same as in the standard case. Note that Higgs self-interactions will still appear due to vector boson and fermion loop corrections.

There is another special case, when the Higgs mass term does not come from the classical potential:

$$V_{III}(\sigma) = \lambda(\sigma - \sigma_0)^4 = \frac{\lambda}{4}h^4. \quad (21)$$

This type of potential looks unrealistic from the phenomenological point of view.

Moreover, introduction of the mass into the model *by hands* can be avoided just by setting all the free parameters to zero:

$$V_{IV}(\sigma) \equiv 0. \quad (22)$$

In the last two cases, the observable Higgs boson mass should be generated by a certain additional mechanism and taking into account radiative corrections.

Certainly the last choice of the Higgs potential is the minimal one. For all the four choices (19—22) with a single minimum we have differences from the standard case only in the Higgs self-interaction sector of the Lagrangian including the Higgs mass term. Phenomenological consequences and theoretical aspects of the different choices have to be studied and discussed.

4 Conclusions

If we take the full electroweak sector of the Standard Model with the $SU(2)_L \times U(1)_Y$ symmetry, we can generate the masses of the EW vector bosons and of all fermions in the standard way again using the polar coordinates (see *e.g.* Ref. [5]). Again the key conditions are the proper symmetry of the potential and the non-zero vacuum expectation value. In the same way as for the abelian case we can generalize the possible class of the scalar potential to the form (17) and consider the possibility to have a concave potential with a single minimum. The unitarity in electroweak boson interaction is preserved for all choices of classical potentials discussed above, since the EW sector of the SM including interactions of leptons and EW bosons with the Higgs scalar are not affected. For the same reason, one can achieve renormalizability of the full theory as a sum of the renormalizable EW sector with a simple Φ^4 scalar one. Even so that the resulting theory is not linear in the *initial* (unphysical) fields Φ and Φ^\dagger . Of course to prove renormalizability one has to go beyond the classical level, which is beyond the scope of the present discussion.

There is a statement [9, 10] that for the scalar sector of the SM, being a Φ^4 theory, to remain perturbative at all scales one needs to have the trivial case without self-interactions of the scalar bosons, *i.e.* $\lambda = 0$. This possibility can be accessed for certain choices of the parameters in the generalized potential as discussed above. Note that in this case the theory would be free at the classical level from the Higgs self-interactions and contain only the gauge and Yukawa ones.

In this way we suggest to generalize the class of classical potentials of a scalar field, which can be used to generate masses of the SM particles. Certainly, different classical potential lead to different quantum theories. Several problems such as radiative corrections to the Higgs boson mass and the Coleman-Weinberg effective potential can be approached for the suggested potentials by compilation of the existing SM calculations. Moreover, the choice of the classical potential in a general case should be motivated by a certain basic principle of the theory under construction, *e.g.* the correspondence principle or a conformal symmetry. So we have to look for such a motivation. Discussion of these questions will be presented elsewhere.

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