The coupled-channel analysis of D_s and B_s mesons

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Abstract

In the framework of the coupled channel model the mass shifts of the P–wave excitations of D_s and B_s mesons have been calculated. The corresponding coupling to DK and BK channels is provided by the effective chiral Lagrangian which is deduced from QCD and does not contain fitting parameters. The strong mass shifts down for 0^+ and $1^{+'}$ states have been obtained, while $1^{+''}$ and 2^+ states remain almost at rest. Two factors are essential for large mass shifts: strong coupling of the 0^+ and $1^{+'}$ states to the S-wave decay channel, containing a Nambu-Goldstone meson, and the chiral flip transitions due to the bispinor structure of both heavy-light mesons. The masses $M(B_s^*(0^+)) = 5710(15)$ MeV and $M(B_s(1^{+'})) = 5730(15)$ MeV are predicted. Experimental limit on the width $\Gamma(D_{s1}(2536)) < 2.3$ MeV puts strong restrictions on admittable mixing angle between the 1^+ and $1^{+'}$ states.

1 Introduction

The heavy-light (HL) mesons play a special role in hadron spectroscopy. First of all, a HL meson is the simplest system, containing one light quark in the field of almost static heavy antiquark, and that allows to study quark (meson) chiral properties. The discovery of the $D_s(2317)$ and $D_s(2460)$ mesons [1, 2] with surprisingly small widths and low masses has given an important impetus to study chiral dynamics and raised the question why their masses are considerably lower than expected values in single channel potential models. The question was studied in different approaches: in relativistic quark model calculations [3]–[6], on the lattice [7], in QCD Sum Rules [8, 9], in chiral models [10]–[12] (for reviews see also [13, 14]). The masses of $D_s(0^+)$ and $D_s(1^{+'})$ in closed-channel approximation typically exceed by ~ 140 and 90 MeV their experimental numbers.

Thus main theoretical goal is to understand dynamical mechanism responsible for such large mass shifts of the 0^+ and $1^{+'}$ levels (both states have the light quark orbital angular momentum $l = 0$ and $j = 1/2$) and explain why the position of other two levels (with $j = 3/2$) remains practically unchanged. The importance of second fact has been underlined by S.Godfrey in [5].

The mass shifts of the $D_s(0^+, 1^{+'})$ mesons have already been considered in a number of papers with the use of unitarized coupled-channel model [15], in nonrelativistic Cornell model [16], and in different chiral models [17]–[19]. Here we address again this problem with the aim to calculate also the mass shifts of the $D_s(1^{+''})$ and $B_s(0^+, 1^{+'})$ states and the widths of the 2^+ and 1^+ states, following the approach developed in [18], for which strong coupling to the S-wave decay channel, containing a pseudoscalar (P) Nambu-Goldstone (NG) meson, is crucially important. Therefore in this approach principal difference exists between vector-vector (VV) and VP (or PP) channels. This analysis of two-channel system is performed with the use of the chiral quark-pion Lagrangian which has been derived directly from the QCD Lagrangian

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[20] and does not contain fitting parameters, so that the shift of the $D_s^*(0^+)$ state ~ 140 MeV is only determined by the conventional decay constant f_K.

Here the term "chiral dynamics" implies the mechanism by which in the transition from one HL meson to another the octet of the NG mesons ϕ is emitted. The corresponding Lagrangian ΔL_{FCM},

$$\Delta L_{FCM} = \bar{q}(\sigma r)\exp(i\gamma_5\phi/f_\pi)q, \tag{1}$$

contains the light-quark part, $\exp(i\gamma_5\phi/f_\pi)$, where ϕ is the $SU(3)$ octet of NG mesons and the important factor γ_5 is present. In the lowest order in ϕ this Lagrangian coincides with well-known effective Lagrangian $\Delta L_{\rm eff}$ suggested in [21],[22], where, however, an arbitrary constant g_A is introduced . At large N_c, as argued in [21], this constant has to be equal unity, $g_A = 1$. In [10, 17, 22] this effective Lagrangian was applied to describe decays of HL mesons taking $g_A < 0.80$.

More general Lagrangian ΔL_{FCM} (1) was derived in the framework of the field correlator method (FCM) [20, 23], in which the constant $g_A = 1$ in all cases, and which contains NG mesons to all orders, as seen from its explicit expression (1).

In [24] with the use of the Dirac equation it was shown that in the lowest order in ϕ $\Delta L_{FCM} = \Delta L_{\rm eff}$, if indeed $g_A = 1$. In our calculations the ΔL_{FCM} was used to derive the nonlinear equation for the energy shift and width, $\Delta E = \Delta\bar{E} - \frac{i\Gamma}{2}$, as in [18]. We do not assume any chiral dynamics for the unperturbed levels, which are calculated here with the use of the QCD string Hamiltonian [25, 26], because the mass shift ΔE appears to be weakly dependent on the position of unperturbed level. Nevertheless, the uncertainty in the final mass values is due to a poor knowledge of the fine structure (FS) interaction in the initial (unperturbed) P-wave masses.

It is essential that resulting shifts of the $J^P(0^+, 1^{+'})$ levels are large only for the D_s, B_s mesons, which lie close to the DK, D^*K, BK, B^*K thresholds, but not for the $D(1P), B(1P)$ mesons, in this way violating symmetry between them (this symmetry is possible in close-channel approximation). In our calculations shifted masses of the $D_s(0^+)$ and $B_s(0^+)$ practically coincide with those for the $D(0^+)$ and $B(0^+)$, in agreement with the experimental fact that $M_{\rm exp}(D(0^+)) = 2350 \pm 50$ MeV [27] is equal or even larger than $M_{\rm exp}(D_s(0^+)) = 2317$ MeV. The states with $j = 3/2$ $D_s(1^+, 2^+)$ and $D(1^+, 2^+)$ have no mass shifts and for them the mass difference is ~ 100 MeV, that just corresponds to the mass difference between the s and light quark dynamical masses.

For the $D_s(1^{+'})$ and $B_s(1^{+'})$ mesons calculated masses are also close to those of the D and B mesons. Therefore for given chiral dynamics the $J^P(0^+, 1^{+'})$ states cannot be considered as the chiral partners of the ground-state multiplet $J^P(0^-, 1^-)$, as suggested in [11].

We also analyse why two other members of the 1P multiplet, with $J^P = 2^+$ and 1^+, do not acquire the mass shifts due to decay channel coupling (DCC) and have small widths. Such situation occurs if the states 1^+ and $1^{+'}$ appear to be almost pure $j = \frac{3}{2}$ and $j = \frac{1}{2}$ states. Still small mixing angle between them, $|\phi| < 6°$, is shown to be compatible with experimental restriction on the width of $D_{s1}(2536)$, admitting possible admixture of other component in the wave function (w.f.) $\lesssim 10\%$.

In our analysis the 4-component (Dirac) structure of the light quark w.f. is crucially important. Specifically, the emission of a NG meson is accompanied with the γ_5 factor which permutes higher and lower components of the Dirac bispinors. For the $j = 1/2, P$ -wave and the $j = 1/2, S$ -wave states it is exactly the case that this "permuted overlap" of the w.f. is maximal because the lower component of the first state is similar to the higher component of the second state and vice versa. We do not know other examples of such a "fine tuning". On the other hand in the first approximation we neglect an interaction between two mesons in the continuum, like DK,etc.

In present paper we concentrate on the P-wave B, B_s mesons and the effects of the channel coupling. While the 1P levels of the D, D_s mesons are now established with good accuracy

[1],[2],[27], for the B, B_s mesons only relatively narrow $2^+, 1^+$ states have been recently observed [28],[29]. According to these data the splitting between the 2^+ and 1^+ levels is small, $\sim 20-10$ MeV, while the mass difference between $B_s(2^+)$ and $B(2^+)$ states is large ~ 100 MeV, as for the $D_s(2^+)$ and $D(2^+)$ mesons.

The actual position of the $B(1P), B_s(1P)$ levels is important for several reasons. Firstly, since dynamics of $(q\bar{b})$ mesons is very similar to that of $q\bar{c}$, the observation of predicted large mass shifts of the $B_s(0^+, 1^{+'})$ levels would give a strong argument in favour of the decay channel mechanism suggested here and in [18]. It has been shown in [30] that the mass of $B_s(0^+)$ can change by 150 MeV in different chiral models. Secondly, experimental observation of all P-wave states for the B,B_s mesons could clarify many unclear features of spin-orbit and tensor interactions in mesons. Understanding of the decay channel coupling (DCC) mass shifts could become an important step in constructing chiral theory of strong decays with emission of one or several NG particles.

2 Mixing of the 1^+ and $1^{+'}$ states

It is well known that in single-channel approximation, due to spin-orbit and tensor interactions the P-wave multiplet of a HL meson is split into four levels with $J^P = 0^+, 1^+_L, 1^+_H, 2^+$ [31]. Here we use the notation H(L) for the higher (lower) 1^+ eigenstate of the mixing matrix because a priori one cannot say which of them mostly consists of the light quark $j = 1/2$ contribution. For a HL meson, strongly coupled to a nearby decay channel (DC), some member(s) of the P-wave multiplet can be shifted down while another not. Just such situation takes place for the $D_s(1P)$ multiplet. The position of the levels with $j = \frac{3}{2}$, which remains unshifted, will be important in our analysis.

The scheme of classification, adapted to a HL meson, in the first approximation treats the heavy quark as a static one and therefore the Dirac equation can be used to define the light quark levels and wave functions [10]. Starting with the Dirac's P-wave levels, one has the states with $j = 1/2$ and $j = 3/2$. Since the light quark momentum j and the quantum number $\varkappa$ are conserved[1], they run along the following possible values:

even l

J^P	j	l	$\varkappa$
0^-	1/2	0	-1
1^-	1/2	0	-1
1^-	3/2	2	+2

odd l

J^P	j	l	$\varkappa$
0^+	1/2	1	+1
1^+	1/2	1	+1
1^+	3/2	1	-2
2^+	3/2	1	-2
2^+	5/2	3	+3

(2)

The HL meson w.f. can be expressed in terms of the light quark w.f. – the Dirac bispinors $\psi^{jlM}_{q,s}$:

$$\Psi_D\left(J^-_{1/2}, M_f\right) = C^{J,M_f}_{\frac{1}{2},M_f-\frac{1}{2};\frac{1}{2},+\frac{1}{2}} \psi_q^{\frac{1}{2},0,M_f-\frac{1}{2}} \otimes |\bar{c}\uparrow\rangle + C^{J,M_f}_{\frac{1}{2},M_f+\frac{1}{2};\frac{1}{2},-\frac{1}{2}} \psi_q^{\frac{1}{2},0,M_f+\frac{1}{2}} \otimes |\bar{c}\downarrow\rangle, \tag{3}$$

$$\Psi_{D_s}\left(J^+_j, M_i\right) = C^{J,M_i}_{j,M_i-\frac{1}{2};\frac{1}{2},+\frac{1}{2}} \psi_s^{j,1,M_i-\frac{1}{2}} \otimes |\bar{c}\uparrow\rangle + C^{J,M_i}_{j,M_i+\frac{1}{2};\frac{1}{2},-\frac{1}{2}} \psi_s^{j,1,M_i+\frac{1}{2}} \otimes |\bar{c}\downarrow\rangle, \tag{4}$$

where $C^{JM}_{j_1M_1;j_2M_2}$ are the corresponding Clebsch–Gordan coefficients.

Later in the w.f. we neglect possible (very small) mixing between the $D(1^-_{1/2})$, $D(1^-_{3/2})$ states and also between $D_s(2^+_{3/2})$, $D_s(2^+_{5/2})$ states; however, physical $D_s(1^+)$ states can be mixed via

[1]we use here the standard notation $\varkappa = \mp|j + \frac{1}{2}|$ for $j = \left\{ \begin{array}{l} l + \frac{1}{2} \\ l - \frac{1}{2} \end{array} \right.$

open channels and tensor interaction. The eigenstates, defining the higher 1^+_H and lower 1^+_L levels, can be parameterized by introducing the mixing angle ϕ:

$$|1^+_H\rangle = \cos\phi|j=\frac{1}{2}\rangle + \sin\phi|j=\frac{3}{2}\rangle, \tag{5}$$

and

$$|1^+_L\rangle = -\sin\phi|j=\frac{1}{2}\rangle + \cos\phi|j=\frac{3}{2}\rangle. \tag{6}$$

Later we will show that just the 1^+_L level with small $|\phi| \lesssim 6°$, being almost pure $j=\frac{3}{2}$ state, has no DC (hadronic) mass shift. In opposite case when 1^+_H is mostly $j=\frac{3}{2}$ state it is convenient to redefine in the equations (5), (6) the mixing angle as $\phi \to 90° - \phi$, performing similar analysis.

In general, the structure of the mixing is important because it defines the order of levels, the mass shift down of the $1^{+'}$ state, as well as the mass shift and the width of another 1^+ level. One of our goals here is to understand why if the coupling to nearby continuum channel is taken into account, the position of the 2^+ and 1^+ levels does not change (within 1-3 MeV) while $0^+, 1^{+'}$ levels acquire large DC shifts.

3 Chiral Transitions

To obtain the mass shift due to DCC effect we use here the chiral Lagrangian (1), which includes both effects of confinement (embodied in the string tension) and Chiral Symmetry Breaking (CSB) (in Euclidean notations):

$$L_{FCM} = i\int d^4x\psi^+(\hat{\partial} + m + \hat{M})\psi \tag{7}$$

with the mass operator $\hat{M}$ given as a product of the scalar function $W(r)$ and the SU(3) flavor octet,

$$\hat{M} = W(r)\exp(i\gamma_5\frac{\varphi_a\lambda_a}{f_\pi}), \tag{8}$$

where

$$\varphi_a\lambda_a = \sqrt{2}\begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta^0}{\sqrt{6}}, & \pi^+, & K^+ \\ \pi^-, & \frac{\eta^0}{\sqrt{6}} - \frac{\pi^0}{\sqrt{2}}, & K^0 \\ K^-, & \bar{K}^0, & -\frac{2\eta^0}{\sqrt{6}} \end{pmatrix}. \tag{9}$$

Taking the meson emission to the lowest order, one obtains the quark-pion Lagrangian in the form

$$\Delta L_{FCM} = -\int \psi^+_i(x)\sigma|\mathbf{x}|\gamma_5\frac{\varphi_a\lambda_a}{f\pi}\psi_k(x)d^4x. \tag{10}$$

Writing the equation (10) as $\Delta L_{FCM} = -\int V_{if}dt$, one obtains the operator matrix element for the transition from the light quark state i (i.e. the initial state i of a HL meson) to the continuum state f with the emission of a NG meson ($\varphi_a\lambda_a$). Thus we are now able to write the coupled channel equations, connecting any state of a HL meson to a decay channel which contains another HL meson plus a NG meson.

In the case, when interaction in each channel and also in the transition operator is time-independent, one can write following system of equations (see [32] for a review)

$$[(H_i - E)\delta_{il} + V_{il}]G_{lf} = 1. \tag{11}$$

Such two-channel system of the equations can be reduced to one equation with additional DCC potential, or the Feshbach potential [33],

$$(H_1 - E)G_{11} - V_{12}\frac{1}{H_2 - E}V_{21}G_{11} = 1. \tag{12}$$

Considering a complete set of the states $|f\rangle$ in the decay channel 2 and the set of unperturbed states $|i\rangle$ in channel 1, one arrives at the nonlinear equation for the shifted mass E,

$$E = E_1^{(i)} - \sum_f \langle i|V_{12}|f\rangle \frac{1}{E_2^{(f)} - E} \langle f|V_{21}|i\rangle. \tag{13}$$

Here the unperturbed values of $E_1^{(i)}$ are assumed to be known beforehand, while the interaction U_{if} is defined in (10). A solution of the nonlinear equation (13) yields (in general a complex number $E = \bar{E} - \frac{i\Gamma}{2}$) one or more roots on all Riemann sheets of the complex mass plane.

4 Calculation of the DCC shifts

To calculate explicitly the mass shifts, we will use the Eq. (13) in the following form:

$$m[i] = m^{(0)}[i] - \sum_f \frac{|<i|\hat{V}|f>|^2}{E_f - m[i]}, \tag{14}$$

where $m^{(0)}[i]$ is the initial mass, $m[i]$ – is the final one, $E_f = \omega_D + \omega_K$ is the energy of the final state, and the operator $\hat{V}$ provides the transitions between the channels (see the comment after Eq. (10)).

In our approximation we do not take into account the final state interaction in the DK system and neglect the D-meson motion, so the w.f. of the i, f-states are:

$$|f> = \Psi_K(\mathbf{p}) \otimes \Psi_D(M_f), \quad |i> = \Psi_{D_s}(M_i), \tag{15}$$

where

$$\Psi_K(\mathbf{p}) = \frac{e^{i\mathbf{pr}}}{\sqrt{2\omega_K(\mathbf{p})}} \tag{16}$$

is the plane wave describing the K-meson and $\Psi_D(M_f)$, $\Psi_{D_s}(M_i)$ are the HL meson w.f. at rest with the spin projections M_f, M_i, respectively.

We introduce the following notations:

$$\omega_K(\mathbf{p}) = \sqrt{\mathbf{p}^2 + m_K^2}, \quad \omega_D(\mathbf{p}) = \sqrt{\mathbf{p}^2 + m_D^2}, \tag{17}$$

so that in the final state the total energy is $E_f = \omega_D + \omega_K$, while

$$T_f = E_f - m_D - m_K \tag{18}$$

is the kinetic energy. Also it is convenient to define other masses with respect to nearby threshold: $m_{thr} = m_K + m_D$,

$$E_0 = m^{(0)}[D_s] - m_D - m_K, \quad \delta m = m[D_s] - m^{(0)}[D_s], \quad \Delta = E_0 + \delta m = m[D_s] - m_D - m_K, \tag{19}$$

where Δ determines the deviation of the D_s meson mass from the threshold. In what follows we consider unperturbed masses $m_0(J^P)$ of the $(Q\bar{q})$ levels as given (our results do not change if we slightly vary their position, in this way the analysis is actually model-independent).

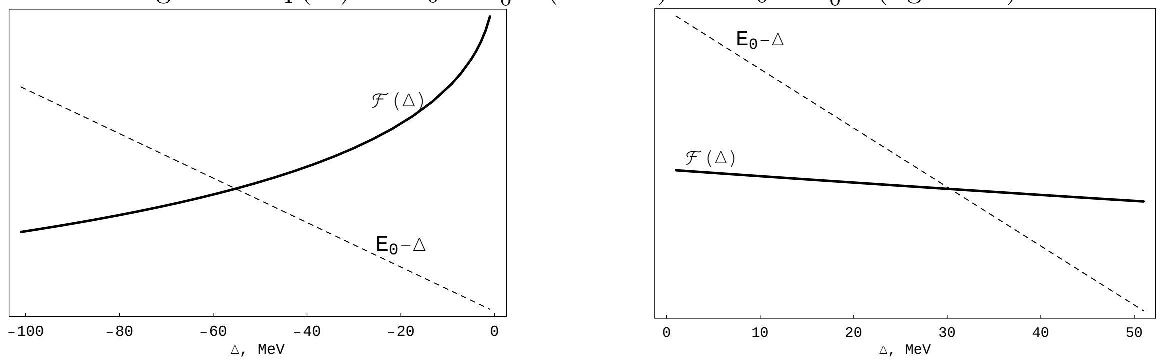
Using these notations, the Eq.(13) can be rewritten as

$$E_0 - \Delta = \mathcal{F}(\Delta), \tag{20}$$

where

$$\mathcal{F}(\Delta) \stackrel{\text{def}}{=} \int \frac{d^3\mathbf{p}}{(2\pi)^3} \sum_{M_f} \frac{\left|\left\langle M_i \middle| \hat{V} \middle| \mathbf{p}, M_f \right\rangle\right|^2}{T_f(\mathbf{p}) - \Delta} \tag{21}$$

Figure 1: Eq.(20) for $E_0 < E_0^{\rm crit}$ (left side) and $E_0 > E_0^{\rm crit}$ (right side)



and

$$\left\langle M_i \left| \hat{V} \right| \mathbf{p}, M_f \right\rangle = -\int \Psi^{\dagger}_{D_s}(M_i)\, \sigma|\mathbf{r}|\gamma_5 \frac{\sqrt{2}}{f_K}\, \Psi_D(M_f)\, \frac{e^{i\mathbf{pr}}}{\sqrt{2\omega_K(\mathbf{p})}}\, d^3\mathbf{r}, \tag{22}$$

The function $\mathcal{F}(\Delta)$ for negative Δ diminishes monotonously so there exists a final (critical) point,

$$E_0^{\rm crit} = \mathcal{F}(-0). \tag{23}$$

Thus, while solving the Eq.(20), one has two possible situations: $E_0 < E_0^{\rm crit}$ and $E_0 > E_0^{\rm crit}$ (see Fig. 1).

In the first case Eq.(20) has a negative real root $\Delta < 0$ and the resulting mass of the D_s meson appears to be under the threshold. In the second case Eq.(20) has a complex root $\Delta = \Delta' + i\Delta''$ with positive real part $\Delta' > 0$ and negative imaginary part $\Delta'' < 0$. To find latter solutions one should make analytic continuation of the solution(s) from the upper halfplane of Δ under the cut, which starts at the threshold, to the lower halfplane (second sheet). This solution can be also obtained by deforming the integration contour in $T_f(p)$. In actual calculations we take infinitesimal imaginary part Δ'', proving that Δ does not change much for finite Δ'' (the similar procedure has been used in [18]). Finally, the resulting mass of the D_s meson proves to be in the complex plane at the position $\Delta' - i|\Delta''|$, i.e. the meson has the finite width $\Gamma = 2\Delta''$.

For further calculations we should insert the explicit meson w.f. to the matrix element (22). As discussed above, in a HL meson we consider a light quark q moving in the static field of a heavy antiquark $\bar{Q}$, and therefore its w.f. can be taken as the Dirac bispinor:

$$\psi_q^{jlM} = \begin{pmatrix} g(r)\Omega_{jlM} \\ (-1)^{\frac{1+l-l'}{2}} f(r)\Omega_{jl'M} \end{pmatrix}, \quad \int\limits_0^\infty \left(f^2+g^2\right) r^2 dr = 1, \tag{24}$$

where the functions $g(r)$ and $f(r)$ are the solutions of the Dirac equation:

$$\begin{aligned} g' + \frac{1+\varkappa}{r} g - \left(E_q + m_q + U - V_C\right) f = 0, \\ f' + \frac{1-\varkappa}{r} f + \left(E_q - m_q - U - V_C\right) g = 0. \end{aligned} \tag{25}$$

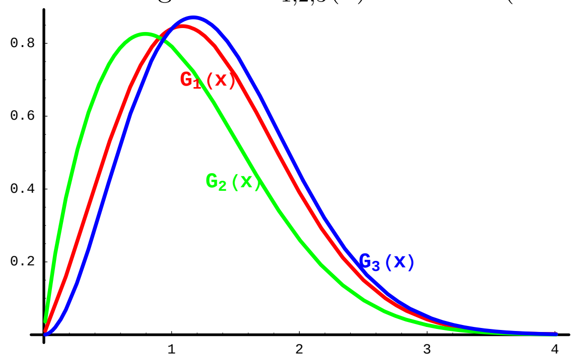
Here the interaction between the quark and the antiquark is described by a sum of linear scalar potential and the vector Coulomb potential with $\alpha_s = \text{const}$:

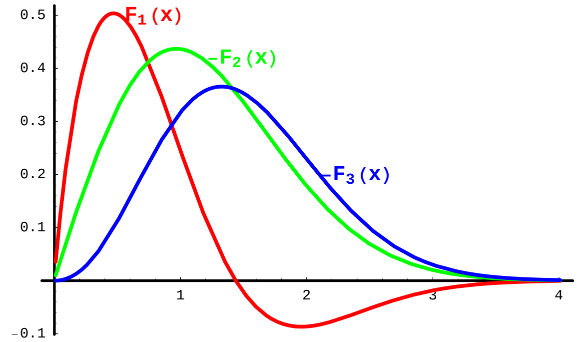
$$U = \sigma r, \quad V_C = -\frac{\beta}{r}, \quad \beta = \frac{4}{3}\alpha_s. \tag{26}$$

Introducing new dimensionless variables

$$x = r\sqrt{\sigma}, \quad \varepsilon_q = E_q/\sqrt{\sigma}, \quad \mu_q = m_q/\sqrt{\sigma}, \tag{27}$$

Figure 2: $G_{1,2,3}(x)$ functions (left side) and $F_{1,2,3}(x)$ functions (right side)





and new dimensionless functions

$$g = \sigma^{3/4} \frac{G(x)}{x}, \quad f = \sigma^{3/4} \frac{F(x)}{x}, \quad \int\limits_0^\infty \left(F^2 + G^2\right) dx = 1, \tag{28}$$

we come to the following system of equations:

$$\begin{aligned} G' + \frac{\varkappa}{x} G - \left(\varepsilon_q + \mu_q + x + \frac{\beta}{x}\right) F = 0, \\ F' - \frac{\varkappa}{x} F + \left(\varepsilon_q - \mu_q - x + \frac{\beta}{x}\right) G = 0. \end{aligned} \tag{29}$$

This system has been solved numerically.

Using the parameters from the papers [34]:

$$\begin{aligned} \sigma = 0.18 \text{ GeV}^2, \quad \alpha_s = 0.39, \\ m_s = 210 \text{ MeV}, \quad m_q = 4 \text{ MeV}, \end{aligned} \tag{30}$$

we obtain the following Dirac eigenvalues ε:

$\varkappa$	$\bar{Q}q$, $\mu_q = 0.01$	$\bar{Q}s$, $\mu_s = 0.5$
-1	1.0026	1.28944
+1	1.7829	2.08607
-2	1.7545	2.08475

(31)

and corresponding eigenfunctions G, F are given in Fig. 2.

Our choice of σ and α_s is a common one in the frame of the FCM approach, and the value of the light quark mass really does not influence here on any physical results because of its smallness in comparison with the natural mass scale $\sqrt{\sigma}$. The strange quark mass is taken from [35], where it was found from the ratio of experimentally measured decay constants $f(D_s)/f(D)$; the same value can be obtained by a renormalization group evolution starting from the conventional value $m_s(2 \text{ GeV}) = 90 \pm 15$ GeV.

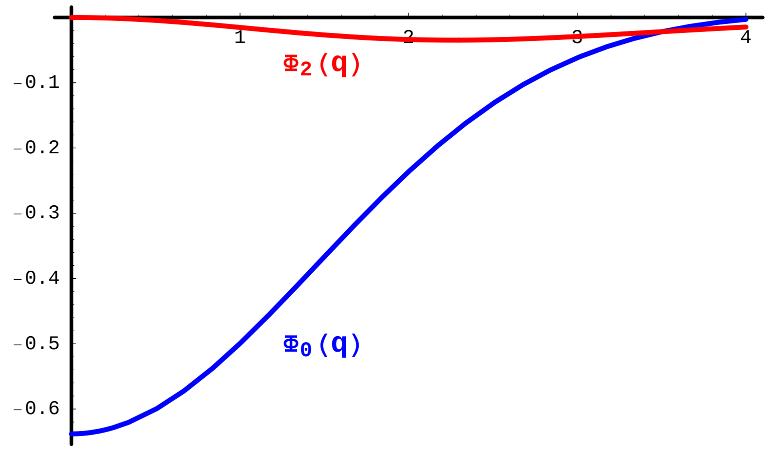
Later we use the simplified notations for the quark bispinors:

$$\psi_1(M_1) \stackrel{\text{def}}{=} \psi_s^{\frac{1}{2},1,M_1}, \quad \psi_2(M_2) \stackrel{\text{def}}{=} \psi_q^{\frac{1}{2},0,M_2}, \quad \psi_3(M_3) \stackrel{\text{def}}{=} \psi_s^{\frac{3}{2},1,M_3}. \tag{32}$$

Now, using explicit expressions for the spherical spinors,

$$\Omega_{l+1/2,l,M} = \begin{bmatrix} \sqrt{\frac{j+M}{2j}}\, Y_{l,M-1/2} \\ \sqrt{\frac{j-M}{2j}}\, Y_{l,M+1/2} \end{bmatrix}, \qquad \Omega_{l-1/2,l,M} = \begin{bmatrix} -\sqrt{\frac{j-M+1}{2j+2}}\, Y_{l,M-1/2} \\ \sqrt{\frac{j+M+1}{2j+2}}\, Y_{l,M+1/2} \end{bmatrix}, \tag{33}$$

Figure 3: $\Phi_{0,2}(q)$ functions



and the expansion :

$$e^{i\mathbf{pr}} = 4\pi \sum_{l,M} i^l j_l(pr) Y^*_{l,M}\left(\frac{\mathbf{p}}{p}\right) Y_{l,M}\left(\frac{\mathbf{r}}{r}\right) , \tag{34}$$

after cumbersome transformations (which are omitted in the text) we obtain the transition matrix elements:

$$\left\|\mathcal{V}_{12}\right\|_{M_1,M_2} = -\int \psi_1^\dagger(M_1)\,\sigma|\mathbf{r}|\gamma_5 \frac{\sqrt{2}}{f_K}\,\psi_2(M_2)\,\frac{e^{i\mathbf{pr}}}{\sqrt{2\omega_K(\mathbf{p})}}\,d^3\mathbf{r} = \frac{\sqrt{\sigma}}{f_K\sqrt{\omega_K(p)}}\Phi_0\left(\frac{p}{\sqrt{\sigma}}\right)\sqrt{4\pi}Y^*_{0,M_1-M_2}\left(\frac{\mathbf{p}}{p}\right), \tag{35}$$

$$\begin{aligned}\left\|\mathcal{V}_{32}\right\|_{M_3,M_2} &= -\int \psi_3^\dagger(M_3)\,\sigma|\mathbf{r}|\gamma_5 \frac{\sqrt{2}}{f_K}\,\psi_2(M_2)\,\frac{e^{i\mathbf{pr}}}{\sqrt{2\omega_K(\mathbf{p})}}\,d^3\mathbf{r}\\ &= -\frac{\sqrt{\sigma}}{f_K\sqrt{\omega_K(p)}}\Phi_2\left(\frac{p}{\sqrt{\sigma}}\right)\sqrt{\frac{4\pi}{5}}Y^*_{2,M_3-M_2}\left(\frac{\mathbf{p}}{p}\right)\times\begin{bmatrix}-1 & +2\\ -\sqrt{2} & +\sqrt{3}\\ -\sqrt{3} & +\sqrt{2}\\ -2 & +1\end{bmatrix}.\end{aligned} \tag{36}$$

where

$$\begin{aligned}\Phi_0(q) &= \int_0^\infty j_0(qx)xdx\Big[G_1(x)F_2(x) - F_1(x)G_2(x)\Big],\\ \Phi_2(q) &= \int_0^\infty j_2(qx)xdx\Big[G_3(x)F_2(x) - F_3(x)G_2(x)\Big].\end{aligned} \tag{37}$$

Notice that because of different signs of the $F_1(x)$ and $F_{2,3}(x)$ functions (while the $G_{1,2,3}$ functions are all positive) on almost all real axis, the integral Φ_2 appears to be strongly suppressed in comparison with the integral Φ_0. This fact is confirmed by numerical simulations (see Fig. 3).

Finally, introducing universal functions

$$\begin{aligned}\tilde{\mathcal{F}}_{0,2}(\Delta) &= \frac{\sigma}{2\pi^2 f_K^2}\int_0^\infty \frac{p(T_f)\omega_D(T_f)dT_f}{T_f + m_D + m_K}\cdot\frac{\Phi^2_{0,2}\left(\frac{p(T_f)}{\sqrt{\sigma}}\right)}{T_f - \Delta},\\ \tilde{\Gamma}_{0,2}(T_f) &= \frac{\sigma}{\pi f_K^2}\cdot\frac{p(T_f)\omega_D(T_f)}{T_f + m_D + m_K}\cdot\Phi^2_{0,2}\left(\frac{p(T_f)}{\sqrt{\sigma}}\right),\end{aligned} \tag{38}$$

Table 1: $D_s(0^+)$-meson mass shift due to the DK decay channel and $B_s(0^+)$-meson mass shift due to the BK decay channel (all in MeV)

state	$m^{(0)}$	$m^{(\text{theor})}$	$m^{(\text{exp})}$	δm
$D_s(0^+)$	2475 (30)	2330(20)	2317	-145
$B_s(0^+)$	5814(15)	5709 (15)	not seen	-105

we come to the following equations to determine meson masses and widths:

$$
\begin{array}{r||l}
D_s(0^+) & E_0[0^+] - \Delta = \tilde{\mathcal{F}}_0(\Delta), \\
D_s(1^+_L) & E_0[1^+_L] - \Delta = \cos^2\phi \cdot \tilde{\mathcal{F}}_0(\Delta) + \sin^2\phi \cdot \tilde{\mathcal{F}}_2(\Delta), \\
D_s(1^+_H) & E_0[1^-_H] - \Delta' = \sin^2\phi \cdot \tilde{\mathcal{F}}_0(\Delta') + \cos^2\phi \cdot \tilde{\mathcal{F}}_2(\Delta'), \\
 & \Gamma[1^+_H] = \sin^2\phi \cdot \tilde{\Gamma}_0(\Delta') + \cos^2\phi \cdot \tilde{\Gamma}_2(\Delta'), \\
D_s(2^+_{3/2}) & E_0[2^+_{3/2}] - \Delta' = \frac{3}{5} \cdot \tilde{\mathcal{F}}_2(\Delta'), \\
 & \Gamma[2^+_{3/2}] = \frac{3}{5} \cdot \tilde{\Gamma}_2(\Delta').
\end{array}
\tag{39}
$$

5 Results and discussion

In this chapter, using the expressions (39) to define the D_s and B_s meson mass shifts, we present and discuss our results. We will take into account the following pairs of mesons in coupled channels (i refers to first (initial) channel, while f refers to second (decay) one):

$$
\begin{array}{c||c|c|c}
i & D_s(0^+) & D_s(1^+) & D_s(2^+) \\
\hline
f & D(0^-) + K(0^-) & D^*(1^-) + K(0^-) & D^*(1^-) + K(0^-)
\end{array}
$$

$$
\begin{array}{c||c|c|c}
i & B_s(0^+) & B_s(1^+) & B_s(2^+) \\
\hline
f & B(0^-) + K(0^-) & B^*(1^-) + K(0^-) & B^*(1^-) + K(0^-)
\end{array}
\tag{40}
$$

In our calculations we use the following meson masses and thresholds (in MeV):

$$
\begin{aligned}
m_{D^+} &= 1869, \quad m_{D^+} + m_{K^-} = 2363, \\
m_{D^{*+}} &= 2010, \quad m_{D^{*+}} + m_{K^-} = 2504, \\
m_{B^+} &= 5279, \quad m_{B^+} + m_{K^-} = 5772, \\
m_{B^*} &= 5325, \quad m_{B^*} + m_{K^-} = 5819.
\end{aligned}
\tag{41}
$$

The results of our calculations are presented in Tables 1–3. *A priori* one cannot say whether the $|j = \frac{1}{2}\rangle$ and $|j = \frac{3}{2}\rangle$ states are mixed or not. If there is no mixing at all, in this case the width $\Gamma(D_{s1}(2536)) = 0.3$ MeV is obtained in [36], while the experimental limit is $\Gamma < 2.3$ MeV [27] and recently in [37] the width $\Gamma = 1.0 \pm 0.17$ MeV has been measured. Therefore small mixing is not excluded and here we take the mixing angle ϕ slightly deviated from $\phi = 0^\circ$ (no mixing case). Then we define those angles ϕ which are compatible with experimental data for the masses and widths of both 1^+ states.

The small value $\phi = 5.7^\circ$ provides large mass shift (~ 100 MeV) of the for the $1^+_H(j = 1/2)$ level and at the same time does not produce the mass shift of the 1^+_L level, which is almost pure

Table 2: The $D_s(1^+)$, $D_s(2^+)$ meson mass shifts and widths due to the D^*K decay channel for the mixing angle 4° (all in MeV)

state	$m^{(0)}$	$m^{(\mathrm{theor})}$	$m^{(\mathrm{exp})}$	$\Gamma^{(\mathrm{theor})}_{(D^*K)}$	$\Gamma^{(\mathrm{exp})}_{(D^*K)}$	δm
$D_s(1^+_H)$	2568(15)	2458(15)	2460	$\times$	$\times$	-110
$D_s(1^+_L)$	2537	2535(15)	2535(1)	1.1	< 1.3	-2
$D_s(2^+_{3/2})$	2575	2573	2573(2)	0.03	not seen	-2

Table 3: The $B_s(1^+)$, $B_s(2^+)$ meson mass shifts and widths due to the B^*K decay channel for the mixing angle 4° (all in MeV)

state	$m^{(0)}$	$m^{(\mathrm{theor})}$	$m^{(\mathrm{exp})}$	$\Gamma^{(\mathrm{theor})}_{(B^*K)}$	$\Gamma^{(\mathrm{exp})}_{(B^*K)}$	δm
$B_s(1^+_H)$	5835(15)	5727	not seen	$\times$	$\times$	-108
$B_s(1^+_L)$	5830(fit)	5828	5829 (1)	0.8	< 2.3	-2
$B_s(2^+_{3/2})$	5840(fit)	5838	5839(1)	$< 10^{-3}$	not seen	-2

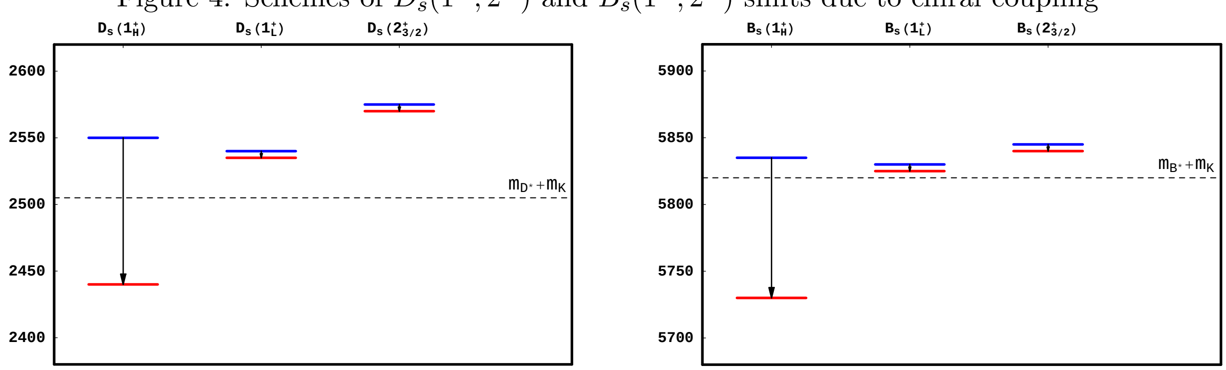
$j = \frac{3}{2}$ state. For illustration we show the scheme of the 1^+, 2^+ shifts on Fig. 4. We would like to stress here that the mass shifts weakly differ for D_s and B_s, or weakly depend on the heavy quark mass: this can be directly illustrated using in the Eq.(39) the expansion via the inverse heavy quark mass.

6 Conclusions

We have studied the mass shifts of the $D_s(0^+, 1^{+'})$ and $B_s(0^+, 1^{+'})$ mesons due to strong coupling to the decay channels DK, D^*K and BK, B^*K. To this end the chiral quark-pion Lagrangian without fitting parameters has been used.

We have shown that the emission of a NG meson, accompanied with the γ_5 factor, gives rise to maximal overlapping between the higher component with $j = \frac{1}{2}$ of the P-wave meson (D_s, B_s) bispinor w.f. and the lower component (also with $j = \frac{1}{2}$) of the S-wave HL meson w.f. in considered S-wave decay channel. Due to this effect, while taking the w.f. of the $1P$ and $1S$ states with the use of the Dirac equation, large mass shifts of the $0^+, 1^{+'}$ states are obtained. In particular, the shifted masses $M(B_s, 0^+) = 5710(15)$ MeV and $M(B_s, 1^{+'}) = 5730(15)$ MeV were calculated in agreement with the predictions in [14] and of S.Narison [9] and by ~ 100

Figure 4: Schemes of $D_s(1^+, 2^+)$ and $B_s(1^+, 2^+)$ shifts due to chiral coupling



MeV lower than in [3],[4],[10].

The widths of $D_{s1}(2536)$ and $B_{s1}(5830)$ are also calculated. To satisfy the experimental condition $\Gamma(D_{s1}(2536)) < 2.3$ MeV the following limit on the mixing angle ϕ (between the $|j = \frac{3}{2} >$ and $|j = \frac{1}{2} >$ states) is obtained: $|\phi| \lesssim 6°$.

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