SELECTED PROBLEMS OF BARYONS SPECTROSCOPY: CHIRAL SOLITON VERSUS QUARK MODELS

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Abstract

Inconsistency between rigid rotator and bound state models at arbitrary number of colors, rigid rotator — soft rotator dilemma and some other problems of baryon spectroscopy are discussed in the framework of the chiral soliton approach (CSA). Consequences of the comparison of CSA results with simple quark models are considered. Strong dependence of the effective strange antiquark mass on the SU(3) multiplet is required to fit the CSA predictions. The difference of 'good' and 'bad' diquark masses, which is about 100 Mev, is in reasonable agreement with other estimates.

1 Introduction

In spite of (or due to?) recent dramatic events with (non)observation of pentaquark states, the studies of baryons spectrum — nonstrange, strange, and with heavy flavors — remain to be very actual for accelerator physics. Discovery of baryon states besides well established (e.g., octet, decuplet and certain resonances) could help to the progress in understanding hadrons structure.

In the absence of the complete theory of strong interactions there are different approaches and models of hadron structure; each has some advantages and certain drawbacks. Interpretation of hadrons spectra in terms of quark models (QM) is widely accepted, QM are "most successful tool for the classification and interpretation" (R.Jaffe) of hadrons spectrum. These models are so widely accepted because (probably) they correspond to our intuitive ideas how the bigger object — baryon, for example — can be made of smaller ones (quarks). However, our intuition, based on macroscopic experience, may be totally misleading in the world of elementary particles.

QM are to large extent phenomenological since there are no regular methods of solving relativistic many-body problem. The number of constituents (e.g. additional $q\bar{q}$-pairs) and their weight should not be fixed as starting conditions in a true relativistic theory, but should be obtained by means of solving adequite relativistic equations (and the quark confinement should be obtained in this way, as well!).

In view of this global unresolved problem alternative approaches are of interest. In particular, the chiral soliton approach (CSA) has certain advantages. It is based on few principles and ingrediemts incorporated in the model lagrangian. Baryons and baryonic systems are considered on equal footing (the look 'from outside'). CSA has many features of a true theory, but still it is a model: some elements of phenomenology are present necessarily in CSA as well.

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Results obtained within CSA mimic some features of baryons spectrum within quark models due to Gell-Mann — Okubo relations.

2 Features of the chiral soliton approach

The CSA is based on few principles and ingredients incorporated in the *truncated effective chiral lagrangian*:

$$L^{eff} = -\frac{F_\pi^2}{16} Tr l_\mu l_\mu + \frac{1}{32e^2} Tr[l_\mu l_\nu]^2 + \frac{F_\pi^2 m_\pi^2}{8} Tr\big(U + U^\dagger - 2\big) + ..., \tag{1}$$

the chiral derivative $l_\mu = \partial_\mu U U^\dagger$, $U \in SU(2)$ or $\in SU(3)$- unitary matrix depending on chiral fields, m_π is pion mass, F_π-pion decay constant taken from experiment, e - the only parameter of the model which defines the weight of the antisymmetric 4-th order in chiral derivatives term in the lagrangian (Skyrme term) [1]. The higher order in l_μ terms are not shown in (1). 6-th order term is taken into account in a number of calculations, and it does not change the properties of multiskyrmions considerably. The mass term $\sim F_\pi^2 m_\pi^2$, changes asymptotics of the profile f and the structure of multiskyrmions at large B. For the $SU(2)$ case

$$U = cos f + i\,(\vec{n}\vec{\tau}) sin\, f, \tag{2}$$

unit $\vec{n}$ depends on 2 functions, $\alpha,\ \beta$. Three profiles $\{f,\ \alpha,\ \beta\}(x, y, z)$ parametrize the unit vector on the 3-sphere S^3.

The soliton is configuration of chiral fields, possessing topological charge identified with the baryon number B (Skyrme, 1961):

$$B = \frac{-1}{2\pi^2} \int s_f^2 s_\alpha I\,[(f, \alpha, \beta)/(x, y, z)]\, d^3r \tag{3}$$

where I is the Jacobian of the coordinates transformation, $s_f = sin\, f$. So, the quantity B shows how many times S^3 is covered when integration over R^3 is made. Recall that surface of the unit sphere S^3 equals

$$\int s_f^2 s_\alpha df\, d\alpha d\beta = 2\pi^2. \tag{4}$$

Minimization of the mass functional M_{class} for each value of baryon number provides 3 profiles $f, =, \alpha,\ \beta$, the mass of static configuration and allows to calculate moments of inertia, etc. Binding energies of classical configurations, moments of inertia $\Theta_I,\ \Theta_J$ and some other characteristics of chiral solitons contain implicitly information about interaction between baryons.

3 Quantization and spectrum of baryons

The observed spectrum of states is obtained by means of quantization procedure and depends on quantum numbers and moments of inertia, Σ-term (Γ), etc. In $SU(2)$ case the rigid rotator model (RRM)[1] is most effective and successfull in describing the properties of nucleons, Δ, of light nuclei (talks at this meeting by N.Manton, S.Krusch and S.Wood, [2]) and also "symmetry energy" of nuclei with $A <\sim 20$ [3].

In the $SU(3)$ case different quantization models have been developed. Probably, mostly accepted way to get the spectrum of baryons is to place the established $SU(2)$ classical configuration (the so called 'hedgehog' for the $B = 1$ skyrmion) in the upper left corner of the $SU(3)$

[1] In some papers the constant F_π and even the mass m_π are considered as parameters, although they are fixed by existing data. Such approach is useful, however, for investigations of some global properties of chiral soliton models.

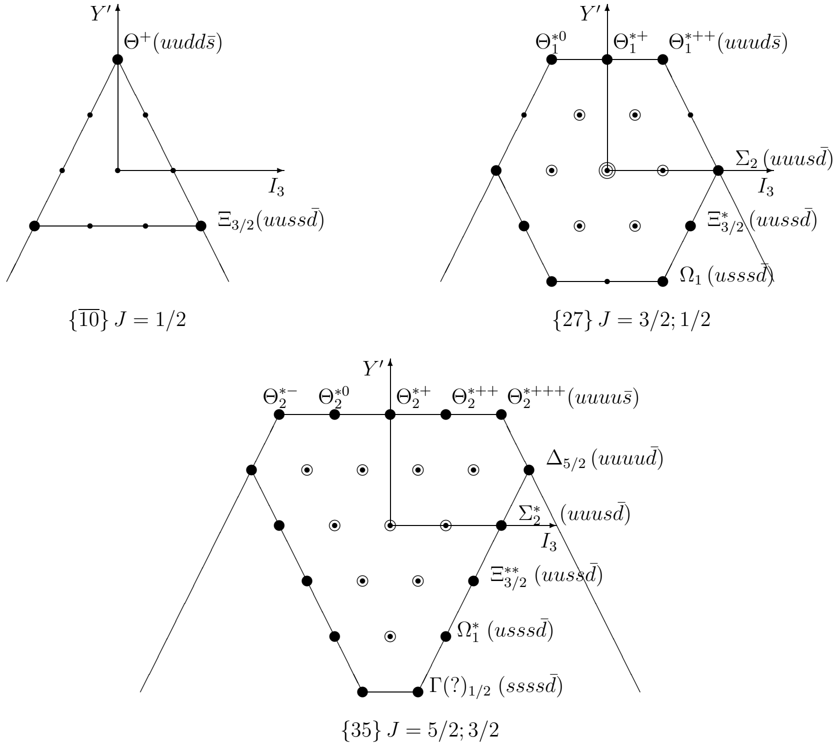


Figure 1: The $I_3\!-\!Y'$ diagrams ($Y' = S\!+\!1$) for multiplets of 'pentaquark' baryons, antidecuplet, $\{27\}$- and $\{35\}$-plets. For $N > 3$ these diagrams should be extended within long lines, as shown in the picture. Quark contents are given for manifestly exotic states, when $N_c = 3$.

matrix of chiral fields and to quantize the $SU(3)$ zero modes corresponding to rotations in the $SU(3)$ configuration space [4]. The following mass formula takes place, corresponding to this rigid rotator model:

$$M(p,q,Y,I,J) = M_{cl} + \frac{K(p,q,J)}{2\Theta_K} + \frac{J(J+1)}{2\Theta_\pi} + \delta M_{(p,q)}(Y,I), \tag{5}$$

$$\sim N_c \qquad \sim 1 \qquad \sim N_c^{-1} \qquad \sim 1,$$

it is in fact expansion in powers of $1/N_c$.

$$K(p,q,I_R) = C_2(SU3) - I_R(I_R+1) - N_c^2B^2/12, \quad C_2(SU3) = (p^2+q^2+pq)/3 + p + q,$$

p, q are the numbers of upper and lower indeces in the spinor describing the $SU(3)$ multiplet, I_R is the so called 'right' isospin, $I_R = J$ - the value of spin of the $B = 1$ state. Some paradox is in the fact that total splitting of the whole multiplet is $\sim N_c$. Mass splittings δM are due to the term in the lagrangian

$$\mathcal{L}_M \simeq -\tilde{m}_K^2\Gamma\frac{s_\nu^2}{2}, \tag{6}$$

ν is the angle of rotation into strange direction, $\tilde{m}_K^2 = F_K^2 m_K^2/F_\pi^2 - m_\pi^2$ includes $SU(3)$-symmetry violation in flavor decay constants. For accepted values of the model parameters numerical values of some important characteristics of the $B = 1$ skyrmion are: $\Gamma \sim 5\,Gev^{-1} \sim \Sigma$, moments of inertia $\Theta_\pi \sim (5-6)Gev^{-1}$, $\Theta_K \sim (2-3)Gev^{-1}$. All inertia are proportional to the number of colors, $\Theta \sim N_c$.

The multiplets of exotic baryons are shown in Fig. 1. The lower index in notations of states indicates the isospin of the state, e.g.

$$\Phi/\Xi_{3/2} = |\overline{10}, S = -2, I = 3/2 >,\ \Sigma_2 = |27, S = -1, I = 2 >,\ \Omega_1 = |27, S = -3, I = 1 > .$$

"Strangeness contents"

$$C_S =< s_\nu^2/2 >_B \tag{7}$$

can be calculated exactly with the help of wave functions in $SU(3)$ configuration space, for arbitrary number of colors N_c [5, 6].

Some examples of values of C_S at arbitrary number of colors N_c are:

$$C_S(\text{"}N\text{"}) = \frac{2(N_c+4)}{(N_c+3)(N_c+7)}, \quad C_S(\text{"}\Xi\text{"}) = \frac{4}{N_c+7}, \quad C_S(\text{"}\Delta\text{"}) = \frac{2(N_c+4)}{(N_c+1)(N_c+9)},$$

$$C_S(\text{"}\Theta\text{"}) = \frac{3}{N_c+9}, \quad C_S(\text{"}\Phi/Xi_{3/2}\text{"}) = \frac{6N_c+9}{(N_c+3)(N_c+9)}, \quad C_S(\text{"}\Theta_1\text{"}) = \frac{3N_c+23}{(N_c+5)(N_c+11)}. \tag{8}$$

Approximately at large N_c

$$C_S \simeq \frac{2+|S|}{N_c}. \tag{9}$$

The Gell-Mann - Okubo formula takes place in the form

$$C_S = -A(p,q)\,Y - B(p,q)\left[Y^2/4 - \vec{I}^2\right] + C(p,q), \tag{10}$$

$A_(p,q), B_(p,q), C_(p,q)$ depend on particular $SU(3)$ multiplet. For the 'octet', for example,

$$A(\text{"}8\text{"}) = \frac{N_c+2}{(N_c+3)(N_c+7)}, \quad B(\text{"}8\text{"}) = \frac{2}{(N_c+3)(N_c+7)}, \quad C(\text{"}8\text{"}) = \frac{3}{(N_c+7)}. \tag{11}$$

If we try to make expansion in $1/N_c$, then parameter is $\sim 7/N_c$. For 'decuplet' and 'antidecuplet' expansion parameter is $\sim 9/N_c$ and becomes worse for greater multiplets, "27"-plet, "35"-plet, etc. Apparently, for realistic world with $N_C = 3$ the $1/N_c$ expansion *does not work.*

Any chain of states connected by relation $I = C' \pm Y/2$ reveals linear dependence on hypercharge (strangeness), so, CSA *mimics the quark model* with effective strange quark mass

$$m_s^{eff} \sim \tilde{m}_K^2 \Gamma \left[A(p,q) \mp 3B(p,q)/2\right], \tag{12}$$

for decuplet (antidecuplet). This is valid if the flavor symmetry breaking (FSB) is included in the lowest order of perturbation theory. At large N_c

$$m_S^{eff} \sim \tilde{m}_K^2 \Gamma/N_c, \tag{13}$$

too much, about $\sim 0.6\,GeV$ if extrapolated to $N_c = 3$.

If we make expansion in RRM, we obtain for the 'octet' of baryons

$$\delta M_N = 2\tilde{m}_K^2 \frac{\Gamma}{N_c}\left(1 - \frac{6}{N_c}\right), \quad \delta M_\Lambda = \bar{m}_K^2 \frac{\Gamma}{N_c}\left(3 - \frac{21}{N_c}\right),$$

$$\delta M_\Sigma = \bar{m}_K^2 \frac{\Gamma}{N_c}\left(3 - \frac{17}{N_c}\right), \quad \delta M_\Xi = \tilde{m}_K^2 \frac{\Gamma}{N_c}\left(4 - \frac{28}{N_c}\right), \tag{14}$$

4 Bound state model

Within the **bound state model** (BSM) (Callan, Klebanov, Riska, Scoccola,.. '85, '86)[7] anti-kaon is bound by $SU(2)$ skyrmion. The mass formula takes place

$$M = M_{cl} + \omega_S + \omega_{\bar S} + |S|\omega_S + \Delta M_{HFS}, \tag{15}$$

where flavor and antiflavor excitation energies

$$\omega_S = N_c(\mu-1)/8\Theta_K, \;\; \omega_{\bar S} = N_c(\mu+1)/8\Theta_K, \tag{16}$$

$$\mu = \sqrt{1+\bar m_K^2/M_0^2} \simeq 1 + \frac{\bar m_K^2}{2M_0^2},$$

$$M_0^2 = N_c^2/(16\Gamma\Theta_K) \sim N_c^0, \quad \mu \sim N_c^0. \tag{17}$$

The expansion of μ made above really does not work well even for the case of strangeness, however, it is very useful for comparison of BSM and RRM.

The hyperfine splitting correction depending on hyperfine splitting constants c and $\bar c$, and "strange isospin" $I_S = |S|/2$ equals

$$\Delta M_{HFS} = \frac{J(J+1)}{2\Theta_\pi} + \frac{(c_S-1)[J(J+1)-I(I+1)] + (\bar c_S - c_S)I_S(I_S+1)}{2\Theta_\pi}. \tag{18}$$

$$c_S = 1 - \frac{\Theta_\pi}{2\mu\Theta_K}(\mu-1) \simeq 1 - 4\frac{\Theta_\pi \Gamma m_K^2}{N_c^2},$$

$$\bar c_S = \frac{\Theta_\pi}{\mu^2\Theta_K}(\mu-1) \simeq 1 - 8\frac{\Theta_\pi \Gamma m_K^2}{N_c^2}. \tag{19}$$

The approximate equality (right side) takes place when expansion in m_K^2 is possible. In this approximation $\bar c_S \simeq c^2$, as mentioned in the literature. It is crucially important that for the antiflavor (positive strangeness) the hyperfine splitting constants are different, they can be obtained by means of the change $\mu \to -\mu$ in above formulas:

$$c_{\bar S} = 1 - \frac{\Theta_\pi}{2\mu\Theta_K}(\mu+1) \simeq 1 - \frac{\Theta_\pi}{\Theta_K} + 4\frac{\Theta_\pi \Gamma m_K^2}{N_c^2} + O(m_K^4),$$

$$\bar c_{\bar S} = 1 + \frac{\Theta_\pi}{\mu^2\Theta_K}(\mu+1) \simeq 1 + 2\frac{\Theta_\pi}{\Theta_K} - 24\frac{\Theta_\pi \Gamma m_K^2}{N_c^2} + O(m_K^4), \tag{20}$$

and is small at large N_c, $\sim 1/N_c$, and for heavy flavors. For anti-flavor (positive strangeness) certain changes should be done: $\omega_S \to \omega_{\bar S}$ and $c_S \to c_{\bar S}$ in the last term.

In this way we obtain for the 'octet'

$$\delta M_N = 2\tilde m_K^2 \frac{\Gamma}{N_c}, \quad \delta M_\Lambda = \tilde m_K^2 \frac{\Gamma}{N_c}\left(3 - \frac{3}{N_c}\right),$$

$$\delta M_\Sigma = \tilde m_K^2 \frac{\Gamma}{N_c}\left(3 + \frac{1}{N_c}\right), \quad \delta M_\Xi = \tilde m_K^2 \frac{\Gamma}{N_c}\left(4 - \frac{4}{N_c}\right). \tag{21}$$

It is instructive to compare the total splitting of the 'octet' in the BSM and in RRM

$$\Delta_{tot}(`8', BSM) = \tilde m_K^2 \frac{\Gamma}{N_c}\left(2 - \frac{4}{N_c}\right), \quad \Delta_{tot}(`8', RRM) = \tilde m_K^2 \frac{\Gamma}{N_c}\left(2 - \frac{16}{N_c}\right). \tag{22}$$

In BSM mass splittings are bigger than in RRM.

The RRM used for prediction of pentaquarks [8] is *different* from the BSM model, used in N.Itzhaki et al (2004)[9] to disavow the Θ^+.

The case of exotic $S = +1$ Θ hyperons is especially interesting. In BSM we obtain using the above expressions for $c_{\bar S}$ and $\bar c_{\bar S}$:

$$M_{\Theta_0,J=1/2} = M_{cl} + \frac{2N_c+3}{4\Theta_K} + \frac{3}{8\Theta_\pi} + \bar m_K^2\Gamma\left(\frac{3}{N_c} - \frac{9}{N_c^2}\right),$$

$$M_{\Theta_1,J=3/2} = M_{cl} + \frac{2N_c+1}{4\Theta_K} + \frac{15}{8\Theta_\pi} + \bar m_K^2\Gamma\left(\frac{3}{N_c} - \frac{7}{N_c^2}\right),$$

$$M_{\Theta_2,J=5/2} = M_{cl} + \frac{2N_c-1}{4\Theta_K} + \frac{35}{8\Theta_\pi} + \bar m_K^2\Gamma\left(\frac{3}{N_c} - \frac{5}{N_c^2}\right). \qquad (23)$$

The terms $\sim 1/\Theta_K$ agree with those obtained in RRM for anti-decuplet, $\{27\}$- and $\{35\}$-plets (terms, proportional to $K(p,q,J)$ in the RRM mass formula). We should compare also the contributions $\sim \bar m_K^2\Gamma$ with the mass splitting correction from RRM:

$$\delta M^{RRM}_{\Theta_0,J=1/2} = \bar m_K^2\Gamma\left(\frac{3}{N_c} - \frac{27}{N_c^2}\right), \quad \delta M^{RRM}_{\Theta_1,J=3/2} = \bar m_K^2\Gamma\left(\frac{3}{N_c} - \frac{25}{N_c^2}\right),$$

$$\delta M^{RRM}_{\Theta_2,J=5/2} = \bar m_K^2\Gamma\left(\frac{3}{N_c} - \frac{23}{N_c^2}\right), \qquad (24)$$

and again — as in case of 'octet' and 'decuplet' — considerable difference takes place.

The addition of the term to the BSM result, *possible due to normal ordering ambiguity for the operators of (anti)strangeness production, present in BSM* (I.Klebanov, VBK, 2005, unpublished)

$$\Delta M_{BSM} = -6\bar m_K^2\frac{\Gamma}{N_c^2}(2+|S|) \qquad (25)$$

brings results of RRM and BSM in agreement — for nonexotic and exotic states. This procedure looks however not quite satisfactorily: if we believe to RRM, why we need BSM at all? Anyway, RRM and BSM in its accepted form are *different models*.

The rotation-vibration approach (RVA) by H.Weigel and H.Walliser [10] unifies RRM and BSM in some way, Θ^+ has been confirmed with somewhat higher energy and considerable width ($\Gamma_\Theta \sim 50\,MeV$)[2].

We conclude this section with the following discussion of the case of large value of the mass $\bar m_F$, which can be $\bar m_K$, $\bar m_D$ or $\bar m_B$. When this mass is large enough, the expansion of the quantity μ in (17) cannot be made, and instead of this expansion we have $\mu \simeq \bar m_F/M_0 = 4\bar m_F\sqrt{\Gamma\Theta_K}/N_c$. This linear dependence of μ and also flavor excitation energies ω_F, $\bar\omega_F$ given by (16) on the mass m_F is quite reasonable, but it is a challenging problem to get such behaviour of flavored states energies within rotator models, RRM or SRM. Probably, strong configuration mixing which should take place in this case, would be able to reduce the quadratic dependence on m_F and to convert it to linear dependence.

5 The role of configuration mixing

Configuration mixing due to the term $\sim m_K^2\Gamma s_\nu^2$ in the lagrangian [11] is important, e.g. the Δ state from decuplet of baryons is mixed with the Δ' state from $\{27\}$-plet, and as a result,

[2] The alternative RRM — BSM is not resolved consequently in the literature. In some cases when there is an ambiguity, the priority is given to the RR model (see, e.g. [10]).

the splitting between these states becomes larger: the mass of the Δ goes down, and the mass of Δ' increases. Similar mixing takes place for other baryon states which have equal values of strangeness and isospin but belong to different $SU(3)$ multiplets.

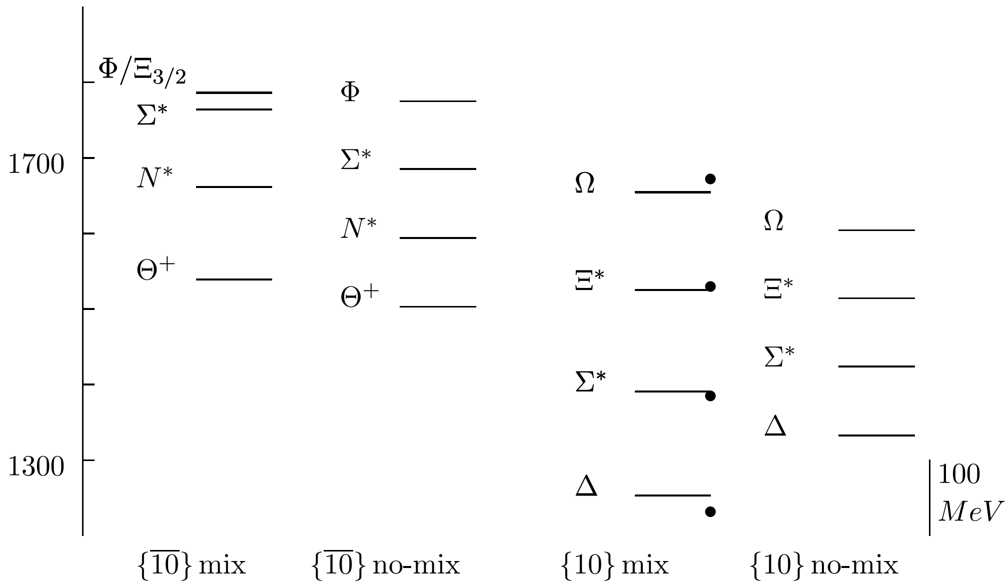


Figure 2: *Influence of the configuration mixing* (H.Yabu, K.Ando, 1988 [11]) on the mass splitting within antidecuplet and decuplet of baryons, RR model (variant by H.Walliser, VBK, 2003 [12])
. For decuplet the data are shown by black points.

For anti-decuplet mixing decreases slightly the total splitting, and pushes the N^* and Σ^* states toward higher energy. Mixing with components of the octet is important. Apparent contradiction with the simplest assumption of equality of masses of strange quarks and antiquarks $m(s) = m(\bar{s})$ takes place (see next section).

For decuplet mixing increases total splitting considerably, but approximate **equidistancy still remains**![3]. Mixing with the components of $\{27\}$-plet is important.

A note for the QM should be made: states with different numbers of $q\bar{q}$ pairs can mix, and such mixing *should be taken into account*. In the diquark-diquark-antiquark picture proposed in [13] the mixing of pentaquark states with the ground state baryon octet should be taken into account since strong interactions do not conserve the number of quark-antiquark pairs present in the hadron. This mixing pushes the pentaquark states towards higher energy and changes the whole picture of relative positions of baryon states.

6 Comparison of CSA results with simple quark model

It is possible to make comparison of CSA results with expectations from simple quark model in *pentaquark* approximation (projection of CSM on QM). The masses m_s, $m_{\bar{s}}$ and the mass of $s\bar{s}$ pair $m(s\bar{s})$ come into play, as presented in the Table for pure states (without mixing).

[3]Therefore, statement made in several papers that approximate equidistancy within the decuplet of baryons is an argument that configuration mixing is negligible, is not correct

$\|\overline{10},2,0>$	$\|\overline{10},1,\frac{1}{2}>$	$\|\overline{10},0,1>$	$\|\overline{10},-1,\frac{3}{2}>$		
$m_{\bar{s}}$	$2m_{s\bar{s}}/3$	$m_s+m_{s\bar{s}}/3$	$2m_s$		
564 600	655 722	745 825	836 847		
$\|27,2,1>$	$\|27,1,\frac{3}{2}>$	$\|27,0,2>$	$\|27,-1,\frac{3}{2}>$	$\|27,-2,1>$	
$m_{\bar{s}}$	$m_{s\bar{s}}/2$	m_s	$2m_s$	$3m_s$	
733 749	753 887	772 779	889 911	1005 1048	
$\|35,2,2>$	$\|35,1,\frac{5}{2}>$	$\|35,0,2>$	$\|35,-1,\frac{3}{2}>$	$\|35,-2,1>$	$\|35,-3,\frac{1}{2}>$
$m_{\bar{s}}$	0	m_s	$2m_s$	$3m_s$	$4m_s$
1152 1122	857 853	971 979	1084 1107	1197 1236	1311 1367

Table Strange quark (antiquark) masses contributions and calculation results within RRM, with and without configuration mixing (2-d and 1-st lines of numbers, correspondingly). For each value of strangeness the states with largest value of isospin are considered here.

Simple relations can be obtained from this Table for effective $s-$quark and antiquark masses m_s and $m_{\bar{s}}$ (see also Fig. 3): from the total splitting of antidecuplet

$$[2m_s - m_{\bar{s}}]_{\overline{10}} = 247\,MeV\ (272\,Mev). \tag{26}$$

Note, that if the mass of s-antiquark within antidecuplet were equal to that of s-quark (we call this variant *simplistic* model), then this splitting would be much smaller, about $130-150\,Mev$. It is remarkable that configuration mixing pushes the splitting towards simplistic quark model. If we assume that the s-quark mass in $\overline{10}$ is about $150\,Mev$, as in the decuplet, then strange antiquark within $\overline{10}$ should be very light, with the mass about $30-50\,Mev$.

From splittings within 27-plet we get

$$[m_s - m_{\bar{s}}]_{27} = 30\,Mev\ (39\,Mev),$$

$$[m_s]_{27} \simeq 135\,Mev\ (117\,Mev). \tag{27}$$

It is of interest that when configuration mixing is not included then the mass of strange quark-antiquark pair $m_{s\bar{s}} = (m_s+m_{\bar{s}})/2$ both for antidecuplet and $\{27\}$-plet. This relation is the consequence of Gell-Mann — Okubo relation, indeed.

From 35-plet we obtain

$$[m_s]_{35} = 130\,Mev\ (114\,Mev),$$

$$[m_{\bar{s}}]_{35} \simeq 270\,Mev\ (295\,Mev). \tag{28}$$

As can be seen from the Table, the mass of $s\bar{s}$ pair does not enter the masses of 35-plet components with largest values of isospin.

Strong dependence of the s-antiquark mass on the multiplet is required when we project the results of CSA on simple quark model: is it artefact of CSA, or is physically significant — is not clear now. The influence of the configuration mixing on the contribution of m_s, $m_{\bar{s}}$, $m_{s\bar{s}}$ to baryon states should be included in more detailed consideration.

7 Diquarks mass difference estimates

Estimates of the diquark mass differences can be made roughly from CSA. As it was suggested by F.Wilczek the singlet in spin diquark $[q_1q_2]$, which is antitriplet $\bar{3}_F$ in flavor, is called ‘good’ diquark d_0, The triplet in spin (q_1q_2), in flavor 6_F is called ‘bad’ diquark d_1.

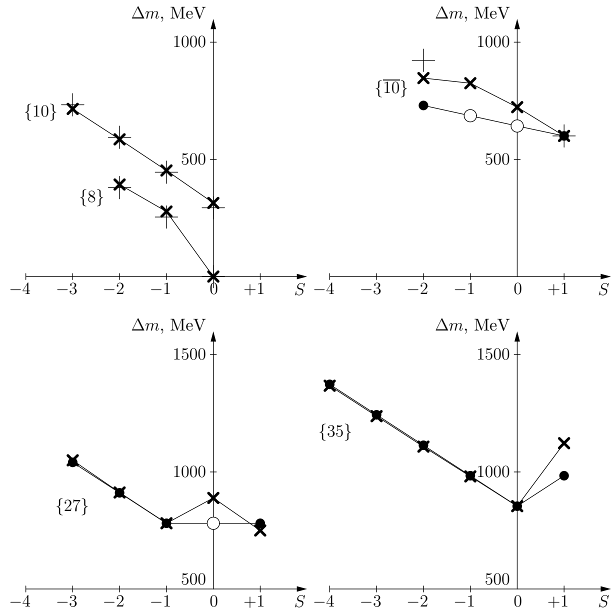


Figure 3: Location of baryon states - nonexotic and exotic - within multiplets of baryons. + show data, black circles - manifestly exotic states within simplistic quark model with the strange quark mass $m_s \simeq 130\ MeV$, empty circles - cryptoexotic states, ×- results of calculations within chiral soliton (rigid rotator) model (figure from [6]).

Examples of wave functions of pentaquarks (PQ-s) in the diquark-diquark-antiquark picture by R.Jaffe and F.Wilczek [13] are the following:

$\Theta_0 \in \{\overline{10}\} \sim [ud][ud]\bar{s}$,

$\Phi/\Xi^{--}_{3/2} \in \{\overline{10}\} \sim [sd][sd]\bar{u}$,

It is not possible to built {27} and {35}-plets from 'good' diquarks only, 'bad' diquarks are needed, as can be illustrated well by these examples of wave functions of positive strangeness baryons:

$\Theta_1^0 \in \{27\} \sim (dd)[ud]\bar{s}$, $\Theta_1^+ \in \{27\} \sim (ud)[ud]\bar{s}$, $\Theta_1^{++} \in \{27\} \sim (uu)[ud]\bar{s}$,

$\Theta_2^- \in \{35\} \sim (dd)(dd)\bar{s}$, $\Theta_2^0 \in \{35\} \sim (ud)(dd)\bar{s}$, ... $\Theta_2^{+++} \in \{35\} \sim (uu)(uu)\bar{s}$.

It seems to be natural to ascribe the difference of rotation energies for different multiplets to the difference of masses of 'bad' and 'good' diquarks. Since 'bad' diquark is heavier, this is obvious reason why Θ_1 is heavier than Θ_0, and Θ_2 is more heavy.

From the difference of {27}-plet and antidecuplet masses

$$M(d_1) - M(d_0) \sim \frac{3}{2\Theta_\pi} - \frac{1}{2\Theta_K} \sim 100\, MeV. \tag{29}$$

from {35}-plet and {27}-plet mass difference

$$M(d_1) - M(d_0) \sim \frac{5}{2\Theta_\pi} - \frac{1}{2\Theta_K} \sim 250\, MeV. \tag{30}$$

Qualitatively this result seems to be OK, in agreement with, e.g., lattice calculations [14], but this picture should be too naive. E.g., interaction between diquarks may be important, which makes the $\Theta_{5/2}$ even more heavier.

8 Rigid Rotator — Soft Rotator dilemma

The rigid rotator model is a limiting case of the rotator model when deformations of skyrmions during rotation in $SU(3)$ configuration space are totally neglected. In the soft rotator model, opposite to rigid rotator, it is supposed that soliton is deformed under influence of FSB forces: static energy minimization is made at fixed value of ν. Dependence on ν of static characterstics of skyrmions is taken into account in the quantization procedure.

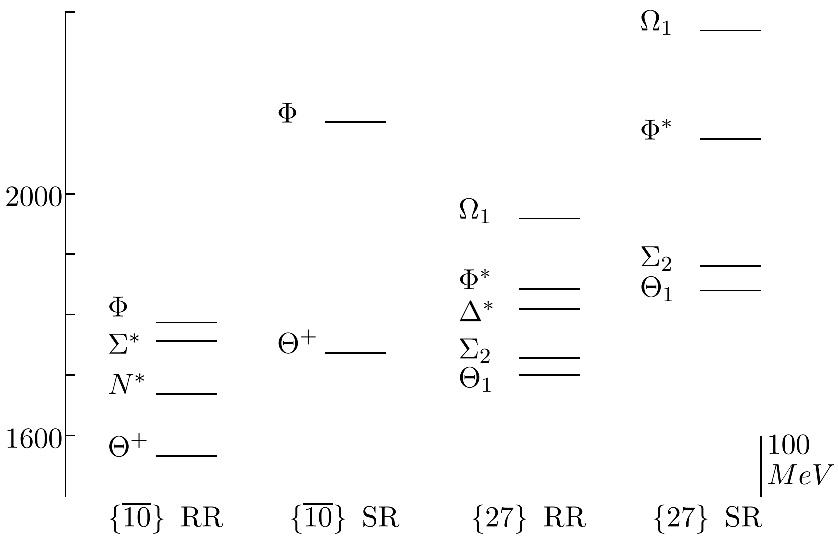


Figure 4: Comparison of the rigid rotator (RR) and soft rotator (SR) models predictions for the masses of exotic baryons, antidecuplet and {27}-plets. Not all states are shown for the SR model. The code for SR model used here was arranged by B.Schwesinger, H.Weigel (1992)[15].

Static characteristics of skyrmions depend on ν - the angle of rotation into 'strange' direction. For 'strange', or kaonic inertia it is most important:

$$\Theta_K = \frac{1}{8}\int (1-c_f)\left[F_K^2 - sin^2\nu(F_K^2 - F_\pi^2)(2-c_f)/2 + +\frac{1}{e^2}\left(f'^2 + \frac{2s_f^2}{r^2}\right)\right] d^3r. \qquad (31)$$

It is decreasing function of $sin^2\nu$. RRM corresponds to $\nu = 0$, the maximal value of kaonic inertia Θ_K and relatively low values of masses of exotic baryons (like Θ, $\Phi/\Xi_{3/2}$, etc.). Within SRM the masses of baryons from antidecuplet and {27} - plet are considerably greater than in RRM, mostly due to smaller value of Θ_K. The truth is somewhere between RR and SR models, but to make reasonable calculation seems to be unrealistic presently since the properties of baryonic matter are not known, in particular the response of matter relative to the FSB forces.

9 Multibaryons

Great advantage of CSA is that multibaryon states — nuclei, hypernuclei ... — can be considered on equal footing with the B=1 case. The rational map approximation proposed in [16] simplifies this work considerably. $\Theta_I \sim B$, $\Theta_J \sim B^2$ for $B \leq 20 - 30$. Some kind of the "bag model" can be obtained with the help of this ansatz, starting with effective lagrangian [19].

Ordinary nuclei and hypernuclei (ground states) should be ascribed to definite $SU(3)$ multiplets, as shown in Fig.5 for baryon numbers 3 and 4.

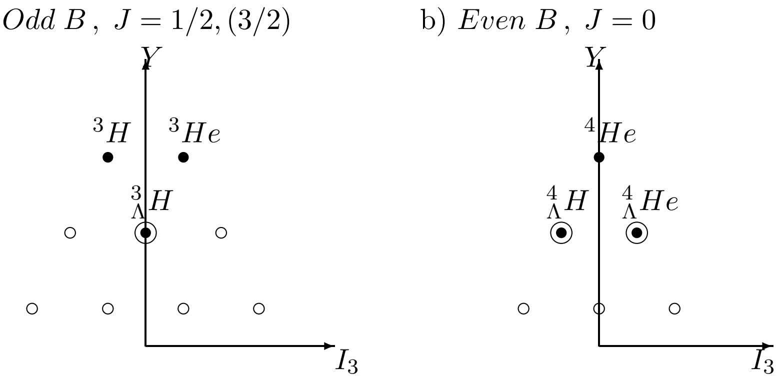


Figure 5: (a) The location of the isoscalar state (shown by double circle) with odd B-number and $|F| = 1$ in the upper part of the $(I_3 - Y)$ diagram. (b) The same for isodoublet states with even B. The case of light hypernuclei $_\Lambda H$ and $_\Lambda He$ is presented as an example. The lower parts of diagrams with $Y \leq B - 3$ are not shown here.

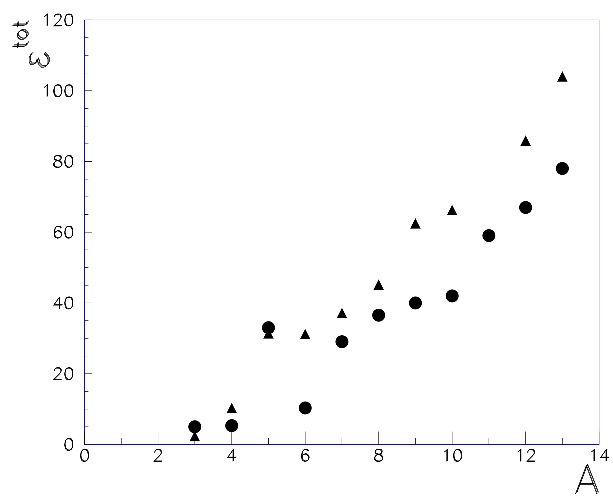
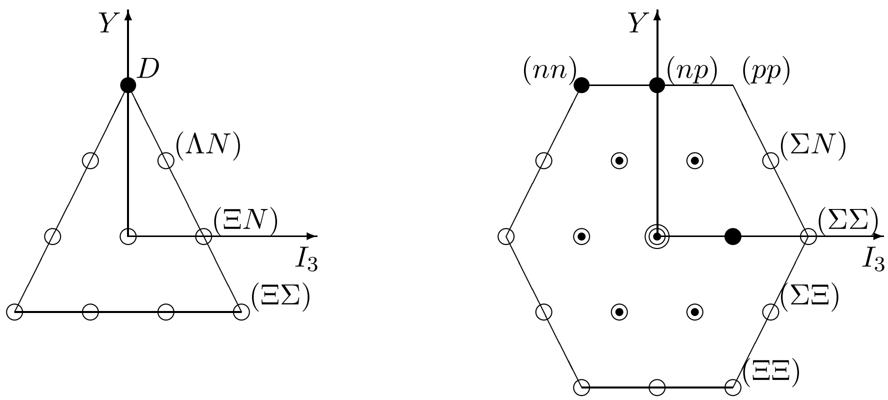


Figure 6: Total binding energies of light hypernuclei (in Mev). Full triangles — experimental data, full circles — theoretical results in a version of BSM.

In a version of BSM it is possible to describe total binding energies of light hypernuclei in qualitative, even semiquantitative agreement with data [20]. The collective motion of multi-skyrmion in the SU(3) collective coordinates space is taken into account. The results of such estimates within the rigid oscillator model (a variant of the bound state model) are presented in Fig.6, and quite satisfactory qualitative agreement with existing data on total binding energies takes place.



$B = 2,\ \{\overline{10}\},\ J = 1\,(2, ...)$ $\qquad$ $B = 2,\ \{27\},\ J = 0\,(1, ...)$

Figure 7: $I_3 - Y$ diagrams of multiplets of dibaryons, $B = 2$: the $J = 1$ antidecuplet (should not be mixed up with antidecuplet of pentaquarks, $B = 1$) and the $J = 0$ $\{27\}$-plet. Virtual levels (scattering states) are shown in brackets, e.g. (ΛN) scattering state which appears as near-threshold enhancement.

For the $B = 2$ case more detailed calculations have neen performed. The lowest multiplets of dibaryons are shown in Fig.7: left figure shows the antidecuplet of the $J = 1$ dibaryons, the $I = 0$ deuteron being the nonstrange state; right figure shows the $J = 0$ $\{27\}$-plet, the $I = 1$ nucleon-nucleon scattering state being the upper (nonstrange) component. There is also $\{35\}$-plet with the $N\Delta$ - like nonstrange upper component (isospin $I = 2$) and $\{28\}$ - plet with the $\Delta\Delta$ - like upper component (isospin $I = 3$). $\{28\}$ - plet contains the state with $S = -6$ (di-Omega). $\{35\}$-plet and $\{28\}$ - plet are not shown in Fig.7.

Calculations of spectrum of strange dibaryons have been performed in [17] in SRM which is more relevant for the $B = 2$ case than for the $B = 1$ case. When the NN-scattering state was fitted to be on the right place (the deuteron binding energy is then about 30 Mev), all strange and multistrange dibaryons are above threshold by few tens of Mev, so, they can appear as near-threshold enhancements in scattering cross sections of baryons with appropriate quantum numbers. These results are in qualitative agreement with quark models calculations [18].

Rotational excitations of any state have additional energy

$$\Delta E = \frac{J(J+1)}{2\Theta_J}. \tag{32}$$

$J = 2^+$ excited states have energy by $\sim 2/\Theta_J$ greater than $\overline{10}$. The state with $S = -1$, $I = 1$, $J^P = 2^+$ can be interpreted as $NN\bar{K}$ state with binding energy $\sim 100\ Mev$. For the $B = 2$ $\{27\}$-plet $J = 1$ states have energy by $1/\Theta_J \sim 60\,Mev$ greater than $J = 0$ ground states.

The orbital inertia grows fast with increasing baryon (atomic) number, $\Theta_J \sim B^p$, p is between 1 and 2. By this reason the number of rotational states becomes larger for large baryon numbers. Some of them can be interpreted as *deeply bound anti-kaon states* discussed intensively in [21] and other papers. More detailed investigations of this issue are necessary.

10 Summary and conclusions

We can summarize our discussion in the following way:

Expansion parameter in $1/N_c$ is large for the case of the baryon spectrum, extrapolation to real world is not possible in this way.

Rigid (soft as well) rotator and bound state models coincide in the first order of $1/N_c$ expansion, but *differ* in the next orders.

Configuration mixing is important, according to RRM, and makes substantial influence on the effective quark masses.

Transition to Soft Rotator Model from RRM may be crucial, leading to the increase of masses, especially for exotic states.

There is correspondence of chiral solitons (RRM) and quark model predictions for pentaquarks spectra in negative S sector of $\{27\}$ and $\{35\}$ plets: the effective mass of strange quark is about $135 - 130\, MeV$, slightly smaller for $\{35\}$.

For positive strangeness components the link between CSM and QM requires strong dependence of effective $\bar{s}$ mass on particular $SU(3)$ multiplet. Config. mixing pushes spectra towards *simplistic* model - nice property, but reasons are not clear. Diquarks mass difference estimates from CSA seems to be reasonable.

As conclusion, we state that chiral soliton models, based on few principles and ingredients incorporated in effective lagrangian, allow to describe qualitatively, in some cases even quantitatively, various chracteristics of baryons and nuclei - from ordinary ($S = 0$) nuclei to known hypernuclei.

This suggests that predictions of pentaquark states, as well as multibaryons with strangeness, should be considered seriously. Existence of PQ by itself is without any doubt, although very narrow PQ may not exist. Wide, even very wide PQ should exist. Searches for PQ-s remain to be an actual task.

There are, however, problems when one tries to project results of the CSA on the Quark Models: strong dependence of strange antiquark mass on the $SU(3)$ multiplet; difference of masses of 'bad' and 'good' diquarks is not unique in naive picture, at least.

In view of theoretical uncertainties, experimental investigations could play decisive role. LHC could help, of course, but studies at smaller energies may be of crucial importance. In particular, experiments at the J-PARC accelerator (50 GeV) can provide *great chance to shed more light on the puzzles of baryon spectroscopy.*

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