Fluctuation Diffusion in Quark matter

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Abstract

A unique transition state is a precursor of quark matter formation. We point out the hallmarks of this state and evaluate the quark diffusion coefficient in the vicinity of the QCD critical line.

During the last decade the investigation of the quark matter at finite temperature and density became a compelling topic in QCD. Drawn in the (T, μ) plane with μ being the quark chemical potential the QCD phase diagram [1, 2] embodies several domains with quite different and sometimes poorly understood properties. The critical line starts at the point ($T = T_c \simeq 170$ MeV, $\mu = 0$) and terminates at ($T = 0, \mu \simeq 300 - 500$ MeV). From T_c with μ increasing the critical line presumably corresponds to the analytic crossover which ends at the critical point of the second order from which the first order transition line originates ending in turn at ($T = 0, \mu \simeq 300 - 500$ MeV). The whole critical line spans over the region of strong coupling QCD regime which fails us for the first principle calculations. Lattice simulations have been performed along the T axis at $\mu = 0$ while models like NJL have been used to investigate the transition in the vicinity of the other end point of the critical line.

Lattice simulations have been performed along the T axis at $\mu = 0$ and recently have been extended to nonzero but small values of $\mu, \mu/T \lesssim 1$ [4]. At the other end of the critical line at small T and moderate μ model calculations suggest that the system is unstable with respect to the formation of quark-quark Cooper pair condensate [5, 6]. It took quite some time to realize that the nonzero value of the gap obtained within the NJL type calculations for $\mu \simeq 300 - 500$ MeV does not mean the onset of color superconducting regime similar to the BCS one [7, 8].

Nonzero value of the gap is only a signal of the presence of fermion pairs. Depending on the strength of the interaction, on the fermion density, and on the temperature such pairs may be either stable, or fluctuating in time, may form a BCS condensate, or a dilute Bose gas, or undergo a Bose-Einstein condensation. It was shown [8, 9] that the critical line at small T and moderate μ corresponds to the crossover from strong coupling regime of composite nonoverlapping bosons (diquarks) to the weak coupling regime of macroscopic overlapping Cooper pair condensate (with possible LOFF phase [10] in between the two regimes). The dimensionless crossover parameter is $n^{1/3}\xi$, where n is the quark density, and ξ is the characteristic length of pair correlation when the system is in the BCS regime and the root of the mean-square radius of the bound state when the system is in the strong coupling regime. The crossover (called BEC-BCS crossover) occurs at $n^{1/3}\xi \sim 1$. It can be shown [11] that the same dimensionless parameter defines the Ginzburg-Levanyuk number Gi which characterize the fluctuation contribution to the physical quantities and the width of the fluctuation region

$$Gi = \frac{27\pi^4}{28\zeta(3)} \left(\frac{T_c}{E_F}\right)^4 = \frac{21\zeta(3)}{64} (k_F\xi)^{-4} = \frac{5 \cdot 10^{-2}}{(n^{1/3}\xi)^4}, \tag{1}$$

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where $\zeta(3) = 1.2, T_c \simeq (0.04 - 0.05)$ GeV is the critical temperature. From(1) we obtain $Gi \gtrsim 10^{-2}$, while for the normal superconductors $Gi \simeq 10^{-12} - 10^{-14}$. We see that the fluctuation effects in a quark system at moderate density are very strong. The transport coefficients in strong fluctuation regime can be expressed in terms of the time-dependent propagator [12]. In particular, we can derive an analytic expression for the quark diffusion coefficient. The only relevant parameter entering into the diffusion coefficient will be the relaxation time τ. In writing the expression for the fluctuation quark propagator (FQP) we assume that at moderate values of μ under consideration the quark Fermi surface is already formed and hence momentum integration can be performed around it in the same way as in the BCS theory of superconductivity. The FQP is defined as

$$L^{-1}(\mathbf{p}, \omega) = -\frac{1}{g} + F(\mathbf{p}, \omega), \tag{2}$$

$$F(\mathbf{p}, \omega) = \sum_k G^R(\mathbf{k}, k_4) G^A(\mathbf{p} - \mathbf{k}, \omega - k_4), \tag{3}$$

where g is the coupling constant with the dimension m^{-2}, the sum over k implies the momentum integration and Matsubara summation, $k_4 = -\pi(2n+1)T$, G^R is the thermal retarded Green's function which reads

$$G^R(\mathbf{k}, k_4) = i(\boldsymbol{\gamma}\mathbf{k} + \gamma_4 k_4 - im + i\mu\gamma_4 - \frac{1}{2\tau})^{-1}, \tag{4}$$

where τ is the relaxation time and $G^A = (G^R)^*$. We shall compute $F(\mathbf{p}, \omega)$ in the long-wave fluctuation approximation

$$F(\mathbf{p}, \omega) \simeq A(\omega) + B\mathbf{p}^2, \tag{5}$$

where the frequency dependence of B is neglected.

First we compute

$$tr\left\{G^R(\mathbf{k}, k_4) G^A(-\mathbf{k}, \omega - k_4)\right\} = 2\left\{\frac{1}{\tilde{k}_4^2 + (E - \tilde{\mu})^2} + \frac{1}{\tilde{k}_4^2 + (E + \tilde{\mu})^2}\right\}, \tag{6}$$

where trace is over the Lorentz indices, $E^2 = \mathbf{k}^2 + m^2$, $\tilde{k}_4 = k_4 - \omega/2$, $\tilde{\mu} = \mu - i\omega/2$.The second term in (6) corresponds to antiquarks and integration around the Fermi surface suppresses its contribution though as shown in [11] the interplay of the quark and antiquark modes may result in instability in the chiral limit. We shall omit the antiquark contribution and return to this question elsewhere. Performing momentum integration around the Fermi surface we obtain

$$A(\omega) = \nu \sum_{n \geq 0} \frac{1}{(n + \frac{1}{2} + \frac{\omega}{4\pi T})}, \tag{7}$$

where $\mu = 2\mu k_F/\pi^2$ is the density of states at the Fermi surface for two quark flavors. To evaluate B we act by the operator $(\mathbf{p}\frac{\partial}{\partial \mathbf{k}})^2$ on the second Green's function in (2). Then following the BCS recipe we express the constant g in (2) in terms of the critical temperature T_c. Collecting all the pieces together we arrive at the FQP containing the diffusion mode

$$L(\mathbf{p}, \omega) = -\frac{1}{\nu} \frac{1}{\varepsilon + \frac{\pi}{2T}(-i\omega + \hat{D}\mathbf{p}^2)}, \tag{8}$$

where $\varepsilon = \frac{T_c - T}{T_c}$. The diffusion coefficient $\hat{D}$ given by

$$\hat{D} = -\frac{8T\tau^2 v_F^2}{3\pi}\left\{\psi\left(\frac{1}{2} + \frac{1}{4\pi T\tau}\right) - \psi\left(\frac{1}{2}\right) - \frac{1}{4\pi T\tau}\psi'\left(\frac{1}{2}\right)\right\}, \tag{9}$$

where $v_F = \frac{\partial E}{\partial k}(k = k_F)$, $\psi(z)$ is the logarithmic derivative of the Γ-function. The expression is valid both below and above T_c. From (9) we obtain the two limiting regimes

$$\hat{D} \simeq \begin{cases} \frac{1}{3} v_F^2 \tau, & T\tau \ll 1. \quad (10) \\ v_F^2/6T, & T\tau \gg 1. \quad (11) \end{cases}$$

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