# Conformal invariance and the expressions for $C_F^4\alpha_s^4$ contributions to the Bjorken polarized and Gross-Llewellyn Smith sum rules

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**Abstract**

Considering massless axial-vector-vector triangle diagram in the conformal invariant limit and the results of the analytical distinguished calculations of the 5-loop single-fermion circle corrections to the QED $\beta$-function, we derive the analytical expressions for the $C_F^4\alpha_s^4$-contributions to the Bjorken polarized and Gross-Llewellyn Smith sum rule. This visible in future evaluation can shed extra light on the reliability of the appearance of $\zeta_3$-term in the explicitly known part of 5-loop corrections to the QED $\beta$-function which are proportional to the conformal invariant set of $C_F^4\alpha_s^4$ -contributions into to the $e^+e^-$- annihilation Adler function.

In this article, which is more detailed version of the talk at Quarks-2008 International Seminar, presented prior the presentation at the same Seminar of the results of calculations of the complicated analytical expression for the non-singlet order $\alpha_s^4$ contribution to the $e^+e^-$ annihilation Adler function function

$$D^{NS}(Q^2) = Q^2\int_0^\infty \frac{R(s)}{(s+Q^2)^2}ds = 3\sum_F Q_F^2 C_D^{NS}(a_s(Q^2)) = 3\sum_F Q_F^2\left[1+\sum_{n=1}^{n=4} d_n^{NS}a_s^n\right] \qquad (1)$$

[1] we will try to clarify why to our point of view it is interesting and important to perform independent calculations of the $C_F^4\alpha_s^4$ contributions to the Bjorken polarized or Gross-Llewellyn Smith sum rules of deep-inelastic lepton-nucleon scattering. In brief, these arguments were published in Ref.[2].

In Eq.(1) $R(s)$ is the well-known $e^+e^-$ ratio, $Q_F$ are the quarks charges, $a_s = \alpha_s(Q^2)/\pi$ and $\alpha_s(Q^2)$ is the $\overline{\rm MS}$-scheme QCD coupling constant, which obeys the property of asymptotic freedom at large $Q^2$. Note, that in the theoretical part of Eq.(1) the contributions of singlet-type diagrams were omitted, in view of the fact that at the 5-loop level they are not yet calculated, but presumably and hopefully will be small.

The calculation of $d_4^{NS}$ [1] is the third step after **complete** analytical calculations of the $\alpha_s^2$ [3] and $\alpha_s^3$ corrections [4], [5] to the Adler function of vector currents. The analytical result for the $\alpha_s^3$ coefficient, obtained in Refs.[4], [5] was confirmed later on in Ref. [6]. Its $SU(N)$-group structure was analyzed in detail in Ref. [7]. Some special features found in this work served the first theoretical argument in favor of the correctness of the results of Ref.[4].

Unfortunately, due to technical reasons (related to the desire to minimize the huge time of the complicated computer-based calculations), the expression for the $\alpha_s^4$ contribution to Eq.(1) was presented in the case of $SU(3)$ group only, without singling out the corresponding Casimir operators $C_F$ and $C_A$ [1]. This does not allow one to study possible special theoretical features of both $\alpha_s^4$-coefficient to $D^{NS}(Q^2)$ and to the photon vacuum polarization constant $Z_{ph}$ in particular. The latter consideration was done at the $\alpha_s^3$-level in Ref. [4], where the observation was made that in the case of $C_A = C_F = Tf/2 = N$, which corresponds to the case of $SU(4)$ Supersymmetric Yang-Mills (SYM) theory, first studied at three-loop order in Ref. [8], $\zeta_3$-term, which appear in the 4-loop approximation for $Z_{ph}$ in QCD, is canceling out. This observation gave the authors of Ref. [4] some additional confidence in favor of the validity of the results obtained. It will be highly desirable to understand whether the similar features are manifesting themselves at the 5-loop level.

Another interesting part of analytical result of Ref.[1], namely the the five-loop single-fermion contribution to the QED $\beta$-function, which is proportional to the single-fermion QED contribution to Eq.(1), was already presented in the literature some time ago [9]. It has the following form

$$\begin{aligned} \beta^{[1]}_{QED} &= \frac{4}{3}\mathrm{A} + 4\mathrm{A}^2 - 2\mathrm{A}^3 - 46\mathrm{A}^4 + \left(\frac{4157}{6} + 128\zeta_3\right)\mathrm{A}^5 \\ &= \frac{4}{3}\mathrm{A} \times C^{NS}_D(\mathrm{A}) \end{aligned} \quad (2)$$

where $\mathrm{A} = \alpha/(4\pi)$ and $\alpha$ is the QED coupling constant, which does not depend on any scale. Indeed, the coefficients of Eq.(2) are scheme-independent, at least in the schemes, not related to the lattice regularization [10]. The performed by different methods calculations of the order $O(\mathrm{A}^4)$-approximation to Eq.(2) [11], [12] are supporting this feature.

It is interesting, that the analytical structure of the 5-loop result of Ref. [9] **differs** from the structure of the previously known terms: it contains $\zeta_3$-term in the presented 5-loop coefficient.

Indeed, at the intermediate stages of calculations of the 3-loop correction to Eq.(2), which were done in Refs. [13], [14] for the purpose of more detailed study of the "finite QED program" [15], $\zeta_3$-terms were appearing, but they canceled out in the ultimate result. Moreover, in Ref. [14] this feature was related to the property of the conformal invariance of this part of QED $\beta$-function, though no proofs or references were given. Note, that the calculations of Ref.[13] were done within the context of regularization with upper cutt-off in the momentum space. The results of these calculations were confirmed in the dimensional regularization in the unpublished preprint of Ref.[16]. In this work the manifestation of the feature of cancellation of $\zeta_3$-term in the 3-loop calculations was clearly demonstrated at the diagrammatic language.

Next, in the process of evaluation of the 4-loop term in Eq.(2) [11] the contributions with two transcendentalities $\zeta_3$ and $\zeta_5$ appeared at the intermediate stages of calculations, but these contributions canceled in the final result. This feature was also expected [17]. A knot-theory explanation of the cancellation of transcendentalities in the discussed above 3-loop and 4-loop QED results was given in Ref. [18]. Moreover, the strong statement that the complete cancellation of transcendentalities from this part of QED $\beta$-function may be expected at every order, was made by the authors of Ref.[18]. Thus, at the 5-loop level one may expect, that $\zeta_3$, $\zeta_5$ and $\zeta_7$ should appear at the intermediate stages of concrete calculations, but cancel down in the final result. However, Eq.(2) demonstrate that for $\zeta_5$ and $\zeta_7$ this property is valid, while for $\zeta_3$ this is not the case!

I do not know whether this observation may be related to the unproved property of "maximal transcendentality", which at present is widely discussed while considering perturbative series for different quantities in the conformal invariant $N = 4$ SYM theory (see e.g. [19], [20]). Thus we do not know whether the appearance of the transcendental term may be considered *pro or contra* the validity of the results of Refs. [9], [1]. In any case, to clarify the status of this new feature of perturbative series in QED it is highly desirable to perform *independent calculational studies* of the results of Ref. [9], [1].

Let me discuss in more detail one of the possibilities for performing some clarifications of the status of the results of Ref.[9] (for brief presentation see Ref.[2]). It is based on the property of the conformal symmetry of the sets of the diagrams considered and the relation, derived by Crewther in Ref. [21] using the concept of conformal symmetry, typical to quark-parton model of strong interactions. The consequences of this relation were studied in Ref. [22] and Ref. [7] in the conformal-invariant limit of QCD and were generalized by different ways to the case of full QCD with non-zero QCD $\beta$-function in the works of Ref.[7], [23],[24],[25], [26],[27] (for a review see [28]).

We will follow here the work of Ref.[23]. Let us translate the investigations of Refs. [21], [22], performed in the $x$-space, to the language of the momentum space following the presentations, given in Ref.[23] (some additional details can be found and in Ref. [25]).

Consider first the axial-vector-vector (AVV) 3-point function

$$T^{abc}_{\mu\alpha\beta}(p,q) = i\int <0|TA^a_\mu(y)V^b_\alpha(x)V^c_\beta(0)|0> e^{ipx+iqy}dxdy = d^{abc}T_{\mu\alpha\beta}(p,q) \tag{3}$$

where $A^a_\mu(x) = \overline{\psi}\gamma_\mu\gamma_5(\lambda^a/2)\psi$, $V^a_\mu(x) = \overline{\psi}\gamma_\mu(\lambda^a/2)\psi$ are the axial and vector non-singlet quark currents. The r.h.s. of Eq.(3) can be expanded in a basis of 3 independent tensor structures under the condition $(pq) = 0$ as

$$\begin{aligned} T_{\mu\alpha\beta}(p,q) &= \xi_1(q^2,p^2)\epsilon_{\mu\alpha\beta\tau}p^\tau \\ &+ \xi_2(p^2,q^2)(q_\alpha\epsilon_{\mu\beta\rho\tau}p^\rho q^\tau - q_\beta\epsilon_{\mu\alpha\rho\tau}p^\rho q^\tau) \\ &+ \xi_3(p^2,q^2)(p_\alpha\epsilon_{\mu\beta\rho\tau}p^\rho q^\tau + p_\beta\epsilon_{\mu\alpha\rho\tau}p^\rho q^\tau) \quad . \end{aligned} \tag{4}$$

Taking now the divergency of axial current one can get the following relation for the invariant amplitude $\xi_1(q^2,p^2)$:

$$q_\beta T_{\mu\alpha\beta}(p,q) = \epsilon_{\mu\alpha\rho\tau}q^\rho p^\tau \xi_1(q^2,p^2) \tag{5}$$

while the property of the conservation of the vector currents implies that

$$lim|_{p^2\to\infty}p^2\xi_3(q^2,p^2) = -\xi_1(q^2,p^2) \tag{6}$$

(see Ref.[29] for the discussions of the details of the derivation of Eqs.(4)-(6)).

In order to clarify the meaning of the second invariant amplitude $\xi_2(q^2,p^2)$, let us first define the characteristics of the deep-inelastic processes, namely the polarized Bjorken sum rule

$$Bjp(Q^2) = \int_0^1 [g_1^{ep}(x,Q^2) - g_1^{en}(x,Q^2)]dx = \frac{1}{6}g_A C_{Bjp}(a_s) = \frac{1}{6}g_A\left[1+\sum_{n=1}^{n=4}c_n a_s^n\right] \tag{7}$$

and the Gross-Llewellyn Smith sum rule

$$GLS(Q^2) = \frac{1}{2}\int_0^1 \left[F_3^{\nu p}(x,Q^2) + F_3^{\overline{\nu}p}(x,Q^2)\right]dx = 3C_{GLS}(a_s) \quad . \tag{8}$$

where $a_s = \alpha_s/\pi$. The coefficient function $C_{Bjp}(a_s)$ can be found from the operator-product expansion of two non-singlet vector currents [30]

$$i\int TV^a_\alpha(x)V^b_\beta(0)e^{ipx}dx|_{p^2\to\infty} \approx C^{P,abc}_{\alpha\beta\rho}A^c_\rho(0) + other\ structures \tag{9}$$

with

$$C^{P,abc}_{\alpha\beta\rho} \sim id^{abc}\epsilon_{\alpha\beta\rho\sigma}\frac{p^\sigma}{P^2}C_{Bjp}(a_s) \quad . \tag{10}$$

and $P^2 = -p^2$. In the case of the definition of the coefficient function of the Gross-Llewellyn Smith sum rule one should consider the operator-product expansion of the axial and vector non-singlet currents

$$i\int TA^a_\mu(x)V^b_\nu(0)e^{iqx}dx|_{q^2\to\infty} \approx C^{V,ab}_{\mu\nu\alpha}V_\alpha(0) + other\ structures \tag{11}$$

where

$$C^{V,ab}_{\mu\nu\alpha} \sim i\delta^{ab}\epsilon_{\mu\nu\alpha\beta}\frac{q^\beta}{Q^2}C_{GLS}(a_s) \tag{12}$$

and $Q^2 = -q^2$. The third important quantity, which will enter into our analysis, is the QCD coefficient function $C_D^{NS}(a_s)$ of the Adler $D$-function of the non-singlet axial currents

$$D^{NS}(a_s) = -12\pi^2 q^2 \frac{d}{dq^2}\Pi_{NS}(q^2) = 3\sum_F Q_F^2 C_D^{NS}(a_s) \tag{13}$$

with $\Pi_{NS}(q^2)$ defined as

$$i\int < 0|TA_\mu^a(x)A_\nu^b(0)|0 > e^{iqx}dx = \delta^{ab}(g_{\mu\nu}q^2 - q_\mu q_\nu)\Pi_{NS}(q^2) \quad . \tag{14}$$

At this point we will stop with definitions of the basic quantities and return to the consideration of the 3-point function of Eq.(3). Following original work [21] one can apply to this correlation function an operator-product expansion in the limit $|p^2| >> |q^2|$, $p^2 \to \infty$, namely expand first the $T$-product of two non-singlet vector currents via Eq.(9) and then take the vacuum expectation value of the $T$-product of two remaining non-singlet axial currents defined through Eq.(14). These studies imply that [23]

$$\xi_2(q^2, p^2)|_{|p^2|\to\infty} \to \frac{1}{p^2}C_{Bjp}(a_s)\Pi_{NS}(a_s) \tag{15}$$

and thus

$$q^2\frac{d}{dq^2}\xi_2(q^2, p^2)|_{|p^2|\to\infty} \to \frac{1}{p^2}C_{Bjp}(a_s)C_D^{NS}(a_s) \quad . \tag{16}$$

Equations (15),(16) reflect the physical meaning of the invariant amplitude $\xi_2(q^2, p^2)$ and should be considered together with the relations for the invariant amplitudes $\xi_1(q^2, p^2)$ of Eq.(5) and $\xi_3(q^2, p^2)$ from Eq.(6)).

On the other hand, it was shown in Ref.[31] that in a conformal invariant (c-i) limit the three-index tensor of Eq.(3) is proportional to the fermion triangle one-loop graph, constructed from the massless fermions:

$$T_{\mu\alpha\beta}^{abc}(p, q)|_{c-i} = d^{abc}K(a_s)\Delta_{\mu\alpha\beta}^{1-loop}(p, q) \quad . \tag{17}$$

In other words, in a conformal invariant limit one has

$$\xi_1^{c-i}(q^2, p^2) = K(a_s)\xi_1^{1-loop}(q^2, p^2) \quad , \tag{18}$$
$$\xi_2^{c-i}(q^2, p^2) = K(a_s)\xi_2^{1-loop}(q^2, p^2) \quad , \tag{19}$$
$$\xi_3^{c-i}(q^2, p^2) = K(a_s)\xi_3^{1-loop}(q^2, p^2) \quad . \tag{20}$$

Moreover, in view of the Adler-Bardeen theorem [32] the invariant amplitude $\xi_1(q^2, p^2)$, related to the divergency of axial current (see Eq.(5)), has no radiative corrections. Therefore one has $K(a_s) = 1$. The 3-loop light-by-light-type scattering graphs, which were calculated in Ref. [33] and analyzed in Ref. [34] , do not affect this conclusion. Indeed, in the case of the 3-point function of the non-singlet axial-vector-vector currents they are contributing to the higher order QED corrections, while the QCD corrections of the similar origin are appearing only in the 3-point function with the singlet axial current in one of the vertexes, which will be not discussed here.

Taking into account the property $K(a_s) = 1$ for the 3-point function of the non-singlet axial-vector-vector currents allows us to derive the fundamental Crewther relation

$$C_{Bjp}(a_s(Q^2))C_D^{NS}(a_s(Q^2))|_{c-i} = 1 \quad , \tag{21}$$

which is valid in the conformal invariant limit in all orders of perturbation theory. The similar relation is also true for the coefficient function $C_{GLS}(a_s)$, defined by Eqs.(8), (11), (12) [22]

. Indeed, considering first the operator-product expansion of the axial and vector non-singlet currents in the 3-point function of Eq.(3) (see Eq.(11) and Eq.(12)), taking the $T$-product of the remaining vector currents and repeating the above discussed analysis, one can find that in the conformal invariant limit the second identity takes place:

$$C_{GLS}(a_s(Q^2))C_D^V(a_s(Q^2))|_{c-i} = 1 \tag{22}$$

where $C_D^V(a_s)$ is the coefficient function of the Adler $D$-function of two vector currents, which coincide with $C_D^{NS}(a_s(Q^2))$ when the high-order singlet light-by-light type contributions to $C_D^V$, which appear first at the $\alpha_s^3$-level [4], are not yet taken into account (see Ref.[1]).

The results of calculations of Ref.[9] are equivalent to the following expression for the $C_D^{NS}(a_s)$ function

$$C_D^{NS}(a_s) = \left[1 + \frac{3}{4}\mathrm{C_F}a_s - \frac{3}{32}\mathrm{C_F^2}a_s^2 - \frac{69}{128}\mathrm{C_F^3}a_s^3 + \left(\frac{4157}{2048} + \frac{3}{8}\zeta_3\right)\mathrm{C_F^4}a_s^4\right] \tag{23}$$

where $\mathrm{C_F} = (N^2-1)/(2N)$. Using now Eq.(21) and Eq.(22) we get the new analogous scheme-independent contributions to the Bjorken polarized sum rule and Gross-Llewellyn Smith sum rule

$$C_{Bjp}(a_s) = C_{GLS}(a_s) = 1 - \frac{3}{4}\mathrm{C_F}a_s + \frac{21}{32}\mathrm{C_F^2}a_s^2 - \frac{3}{128}\mathrm{C_F^3}a_s^3 - \left(\frac{4823}{2048} + \frac{3}{8}\zeta_3\right)\mathrm{C_F^4}a_s^4 \tag{24}$$

Note, that the order $a_s$, $a_s^2$ and $a_s^3$ coefficients are in agreement with the result of explicit calculations, performed in Refs.[35], [30] and [36] respectively. Thus, the direct calculation of the predicted $a_s^4$ coefficient may be rather useful for the independent cross-checks of the results of Ref.[9] and the verification of the appearance of $\zeta_3$-term at the 5-loop level.

These calculations may give us the hint whether the appearance of of $\zeta_3$-term in the result of Eq.(23) *is the new mathematical feature*, which appear in higher orders of perturbation theory.

If results of possible directcalculations of the contributions to the Bjorken sum rule will agree with the presented expression, then the appearance of $\zeta_3$-term in the 5-loop correction to the QED $\beta$-function and in the $C_F^4\alpha_s^4$ contribution into the $e^+e^-$ annihilation Adler function may get independent support and may be analyzed within the framework of the recently introduced concept of "maximal transcendentality".

After the possible cross-check of the results of 5-loop direct calculations it may be of interest to study why the the Lipatov-type estimates originally proposed in Ref. [37] for the sign-alternating series in the $g\Phi^4$-theory (for a review see Ref. [38]) are not working properly in QED. Indeed, theoretical works of Refs.[39], [40] indicate, that the QED series for $\beta_{QED}^{[1]}$-function should also have sign-alternating behavior, in contradiction to the concrete existing results of Ref.[11] and Ref. [9] (see Eq.(2)).

**Acknowledgments**

I wish to thank P.A. Baikov, K.G. Chetyrkin and J.H. Kuhn for discussions and private communications of the problems, related to their intriguing 5-loop project. I am grateful to D. J. Broadhurst and G. T. Gabadadze for pleasant collaboration and useful discussions of various scientific problems, related to single-scale generalizations of the Crewther studies. It is the pleasure to thank G.P. Korchemsky for useful comments concerning the current situation in $N = 4$ SYM theory and J.L. Rosner for his interest in the results, published in Ref.[2], useful questions and pointing out the existence of the knot-theory study of Ref.[18]. The important contributions to to the long-standed calculational research of all those, closely related to Physics Department of Moscow States University, which is celebrating 70-th anniversary in October 2008, is acknowledged in the concrete references below. This work is done within the framework of RFBR Grants Grants 08-01-00686-a and 06-02-16659-a.

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