

New results on gauge-invariant TMD PDFs in QCD

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Abstract

The renormalization properties of unintegrated (transverse-momentum dependent) parton distribution functions (TMD PDF's) are used for analyzing their completely gauge-invariant definition. To this end, the UV anomalous dimension is calculated at the one-loop order in the light-cone gauge and a consistent treatment of the extra singularities, which produce undesirable contributions in the anomalous dimensions, is given. The generalized definition of a TMD PDF, based on the renormalization procedure for the Wilson exponentials with obstructions, is proposed. The reduction of the re-defined TMD PDF to the integrated PDF's, as well as their probabilistic interpretation, are discussed.

Introduction Parton distribution functions (PDF's) play a crucial role in QCD phenomenology [1, 2, 3]. In inclusive processes (e.g., DIS), the standard (integrated) PDF's, which originate from the parton model, are used. The integrated PDF's depend on the longitudinal fraction of the momentum, x , and on the scale of the hard subprocess Q^2 . The completely gauge invariant definition of integrated PDF's reads

$$\hat{f}_i(x) = \frac{1}{2} \int \frac{d\xi^-}{2\pi} e^{-ik^+\xi^-} \langle h(P) | \bar{\psi}_i(\xi^-, \mathbf{0}_\perp) [\xi^-, 0^-] \gamma^+ \psi_i(0^-, \mathbf{0}_\perp) | h(P) \rangle, \quad (1)$$

where the Wilson line (gauge link), ensuring gauge invariance, is defined as follows

$$[y, x|\Gamma] = \mathcal{P} \exp \left[-ig \int_{x[\Gamma]}^y dz_\mu A_a^\mu(z) t_a \right]. \quad (2)$$

The renormalization properties of these objects are described by the DGLAP evolution equation

$$\mu \frac{d}{d\mu} f_i(x, \mu) = \sum_j \int_x^1 \frac{dz}{z} P_{ij} \left(\frac{x}{z} \right) f_j(z, \mu), \quad (3)$$

where P_{ij} is the DGLAP integral kernel. The renormalization properties of the quantities under consideration (to be precise, their anomalous dimensions) are the cornerstone of our approach. The reason is that anomalous dimensions (within perturbative QCD) accumulate the main characteristics of Wilson lines in *local* form, while the gauge contours are *global* objects and, therefore, complicated to handle within a local-field theory framework.

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Unintegrated PDF's The study of semi-inclusive processes, such as SIDIS, or the Drell-Yan process, where the transverse momentum of the produced hadrons can be observed, requires the introduction of more complicated quantities, so-called unintegrated, or transverse-momentum dependent, PDF's. In this case, one does not integrate over the transverse component of the parton's momentum \mathbf{k}_\perp , and the corresponding distribution function looks like a generalization of the integrated PDF. The “naive” definition we start with reads

$$f_i(x, \mathbf{k}_\perp) = \frac{1}{2} \int \frac{d\xi^- d^2\xi_\perp}{2\pi(2\pi)^2} e^{-ik^+\xi^- + i\mathbf{k}_\perp \cdot \boldsymbol{\xi}_\perp} \langle p | \bar{\psi}_i(\xi^-, \boldsymbol{\xi}_\perp) [\xi^-, \boldsymbol{\xi}_\perp; \infty^-, \boldsymbol{\xi}_\perp;]^\dagger \times \gamma^+ [\infty^-, \mathbf{0}_\perp; 0^-, \mathbf{0}_\perp] \psi_i(0^-, \mathbf{0}_\perp) | p \rangle . \quad (4)$$

Formally, the integration over the transverse component of the parton's momentum is expected to yield the integrated distribution

$$\int d^2k_\perp f_i(x, \mathbf{k}_\perp) = \hat{f}_{i/h}(x) . \quad (5)$$

However, this definition, taken literally, suffers from several shortcomings (see, e.g, the recent works in Refs. [3, 8, 9, 10]):

- Gauge invariance, in fact, is not complete: in the light-cone gauge, the dependence on the pole prescription in the gluon propagator remains.
- Extra (rapidity) divergences arise, which are associated with the known features of the light-cone gauge, or the light-like Wilson lines, that cannot be removed by ordinary ultra-violet renormalization alone. Note, that in the integrated case, these divergences, though appearing at the intermediate steps of the calculation, they are absent in the final result due to the mutual cancellation between real and virtual gluon contributions.
- The reduction to the integrated case cannot be performed straightforwardly: the formal integration does not reproduce the correct result (i.e., the DGLAP kernel) because of additional uncanceled UV divergences.

The following methods to take care of the above-mentioned problems have been proposed in the literature:

- Gauge invariance is restored by means of an additional transverse Wilson line at light-cone infinity [4, 5, 6]. This gauge link contributes only in the light-cone gauge and cancels the pole-prescription dependence.
- Extra divergences can be avoided by using the non-light-like gauge connectors in covariant gauges, or an axial gauge off the light cone [7, 14]. This, however, entails the introduction of an additional rapidity parameter $\zeta = (p \cdot n)^2/n^2$ (with $n^2 \neq 0$) to encode the deviation from the light cone. To establish the independence from this arbitrary variable, an additional evolution equation to the standard one has to be fulfilled rendering the reduction to the integrated case questionable. Besides, factorization off the light cone also becomes problematic.
- Application of a generalized renormalization procedure for the light-like Wilson lines (or a subtractive method): as a result, extra divergences cancel by the additional “soft” factor, defined by the vacuum average of particular Wilson lines (demonstrated explicitly in a covariant gauge, in the 1-loop order) in [11, 13], see also [12].

In this work, we implement the analysis of anomalous dimensions within the latter approach. This allows us to figure out the necessary modifications of TMD PDF's in the most economic

way. Towards a “completely correct” definition, we calculate the anomalous dimension of the TMD PDF (in fact, we calculate the distribution of a “quark in a quark”) in the light-cone gauge and identify extra UV divergences in terms of the entailed defect of the anomalous dimension. Then, we perform a generalized renormalization procedure of the TMD PDF, similar to the renormalization of the Wilson contours with cusps or self-intersections [17, 18]. This renormalization cancels undesirable divergences and yields a completely gauge invariant definition of TMD PDF’s.

One-loop anomalous dimension In the tree approximation, the TMD PDF reads

$$f^{(0)}(x, \mathbf{k}_\perp) = \delta(1-x)\delta^{(2)}(\mathbf{k}_\perp) . \quad (6)$$

The one-gluon exchanges, contributing to the UV-divergences, are described by the diagrams Fig. 1(a, b).

The source of the uncertainties and extra divergences is the pole in the gluon propagator in the light-cone gauge:

$$D_{\text{LC}}^{\mu\nu}(q) = \frac{-i}{q^2} \left[g^{\mu\nu} - \frac{q^\mu n^{-\nu}}{[q^+]} - \frac{q^\nu n^{-\mu}}{[q^+]} \right] , \quad (7)$$

where $[q^+]$ stands for an undefined denominator. Consider now the following pole prescriptions:

$$\frac{1}{[q^+]_{\text{PV}}} = \frac{1}{2} \left(\frac{1}{q^+ + i\eta} + \frac{1}{q^+ - i\eta} \right) \quad \text{and} \quad \frac{1}{[q^+]_{\text{Adv/Ret}}} = \frac{1}{q^+ \mp i\eta} . \quad (8)$$

In what follows, we keep η small, but finite. To control UV singularities, dimensional regularization is used. Another widely used prescription is the Mandelstam-Leibbrand (ML) one:

$$\frac{1}{[q^+]_{\text{ML}}} = \frac{1}{q^+ + i\eta q^-} = \frac{q^-}{q^+ q^- + i\eta} \quad (9)$$

to be considered in a separate work.

The UV divergent part of the diagrams 1(a, b) (without their “mirror” contributions) reads

$$\Sigma_{\text{left}}^{UV}(p, \alpha_s; \epsilon) = -\frac{\alpha_s}{\pi} C_F \frac{1}{\epsilon} \left[-\frac{3}{4} - \ln \frac{\eta}{p^+} + \frac{i\pi}{2} + i\pi C_\infty \right] + \alpha_s C_F \frac{1}{\epsilon} [iC_\infty] , \quad (10)$$

where $C_F = (N_c^2 - 1)/2N_c = 4/3$ and the numerical factor C_∞ accumulates the pole-prescription uncertainty, being defined by

$$C_\infty = \begin{cases} 0 , & \text{Adv} : \frac{1}{[q^+]} = \frac{1}{q^+ - i\eta} \\ -1 , & \text{Ret} : \frac{1}{[q^+]} = \frac{1}{q^+ + i\eta} \\ -\frac{1}{2} , & \text{PV} : \frac{1}{[q^+]} = \frac{1}{2} \left(\frac{1}{q^+ - i\eta} + \frac{1}{q^+ + i\eta} \right) \end{cases} . \quad (11)$$

One immediately observes that the prescription dependence is canceled due to the contribution of the transverse gauge link at the light-cone infinity—diagram 1(b). Taking into account the “mirror” contributions (designated as “right” below), one gets the total real UV divergent part:

$$\Sigma_{\text{tot}}^{UV}(p, \alpha_s(\mu); \epsilon) = \Sigma_{\text{left}} + \Sigma_{\text{right}} = -\frac{\alpha_s}{4\pi} C_F \frac{2}{\epsilon} \left(-3 - 4 \ln \frac{\eta}{p^+} \right) . \quad (12)$$

The one-loop anomalous dimension is defined via the renormalization factor

$$\gamma = \frac{1}{2} \frac{1}{Z^{(1)}} \mu \frac{\partial \alpha_s(\mu)}{\partial \mu} \frac{\partial Z^{(1)}(\mu, \alpha_s(\mu); \epsilon)}{\partial \alpha_s} \quad (13)$$

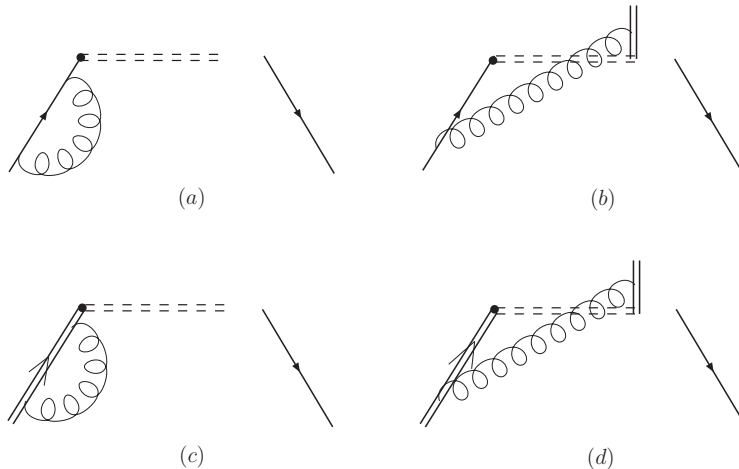


Figure 1: One-gluon exchanges in the TMD PDF in the light-cone gauge. Diagrams (a) and (b) produce UV divergences in the considered TMD PDF, while diagrams (c) and (d) correspond to the UV divergences of the soft factor. The conjugated (“mirror”) diagrams are not shown.

and, using Eq. (12), it reads

$$\gamma_{\text{LC}} = \gamma_{\text{smooth}} - \delta\gamma \quad , \quad \gamma_{\text{smooth}} = \frac{3}{4} \frac{\alpha_s}{\pi} C_F + O(\alpha_s^2) \quad . \quad (14)$$

Here we introduce the defect of the anomalous dimension

$$\delta\gamma = -\frac{\alpha_s}{\pi} C_F \ln \frac{\eta}{p^+} \quad , \quad (15)$$

which marks the deviation of the calculated quantity from the anomalous dimension of the two-quark operator with the smooth (i.e., direct) gauge connector. The latter equals the double anomalous dimension of the fermion field, while γ_{LC} contains an undesirable p^+ -dependent term that should be removed by an appropriate procedure. Note that $p^+ = (p \cdot n^-) \sim \cosh \chi$ defines, in fact, an angle χ between the direction of the quark momentum p_μ and the light-like vector n^- . In the large χ limit, $\ln p^+ \rightarrow \chi$, $\chi \rightarrow \infty$. Thus, we can conclude that the defect of the anomalous dimension, $\delta\gamma$, can be identified with the well-known cusp anomalous dimension [16].

Generalized renormalization It is known that the renormalization of the Wilson operators with obstructions (cusps, or self-intersections) cannot be performed by the ordinary R -operation alone, but requires an additional renormalization factor depending on the cusp angle [17, 18, 19, 16]:

$$Z_{p^+} = \left[\left\langle 0 \left| \mathcal{P} \exp \left[ig \int d\zeta^\mu \hat{A}_\mu^a(\zeta) \right] \right| 0 \right\rangle \right]^{-1} \quad . \quad (16)$$

Using this statement as a hint, we compute the extra renormalization constant associated with the soft counter term [11] and show that it can be expressed in terms of a vacuum expectation value of a specific gauge link. Hence, in order to cancel the anomalous dimension defect $\delta\gamma$, we introduce the counter term

$$R \equiv \Phi(p^+, n^- | 0) \Phi^\dagger(p^+, n^- | \xi) \quad , \quad (17)$$

where

$$\Phi(p^+, n^- | \xi) = \left\langle 0 \left| \mathcal{P} \exp \left[ig \int_{\Gamma_{\text{cusp}}} d\zeta^\mu t^a A_\mu^a(\xi + \zeta) \right] \right| 0 \right\rangle \quad (18)$$

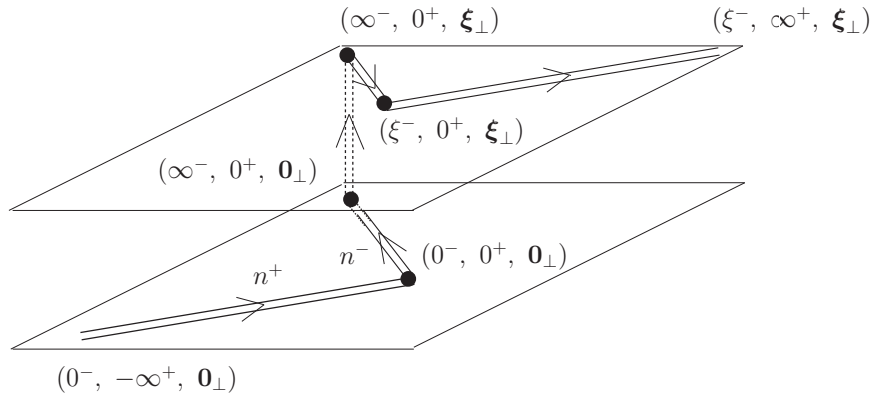


Figure 2: Integration contour for the additional cusp-dependent renormalization factor.

and evaluate it along the non-smooth, off-the-light-cone integration contour Γ_{cusp} , depicted in Fig. 2.

The one-loop gluon virtual corrections, contributing to the UV divergences of the soft factor R , are shown in Fig. 1(c,d). For the UV divergent term we obtain

$$\Sigma_R^{\text{UV}} = -\frac{\alpha_s}{\pi} C_F 2 \left(\frac{1}{\epsilon} \ln \frac{\eta}{p^+} - \gamma_E + \ln 4\pi \right) \quad (19)$$

and observe that this expression is equal, but with opposite sign, to the unwanted term in the UV singularity, related to the cusped contour, calculated above.

Therefore, we propose to redefine the conventional TMD PDF and absorb the soft counter term in its definition:

$$f_{q/q}^{\text{mod}}(x, \mathbf{k}_\perp) = \frac{1}{2} \int \frac{d\xi^- d^2 \xi_\perp}{2\pi(2\pi)^2} e^{-i\mathbf{k}_\perp \cdot \xi_\perp} \langle q(p) | \bar{\psi}(\xi^-, \xi_\perp) [\xi^-, \xi_\perp; \infty^-, \xi_\perp]^\dagger [\infty^-, \xi_\perp; \infty^-, \infty_\perp]^\dagger \gamma^+ \times [\infty^-, \infty_\perp; \infty^-, \mathbf{0}_\perp] [\infty^-, \mathbf{0}_\perp; 0^-, \mathbf{0}_\perp] \psi(0^-, \mathbf{0}_\perp) | q(p) \rangle \cdot R(p^+, n^-). \quad (20)$$

One immediately verifies that the integration over the transverse momentum \mathbf{k}_\perp yields the integrated PDF:

$$\int d^{\omega-2} \mathbf{k}_\perp f_{i/a}^{\text{mod}}(x, \mathbf{k}_\perp; \mu, \eta) = f_{i/a}(x, \mu), \quad (21)$$

which obeys the DGLAP equation (3). The anomalous dimension of the modified TMD PDF (20) is equal to the anomalous dimension of the corresponding operator with the smooth gauge connector, according to the anomalous dimensions (AD) sum rule, which can be formulated in the following symbolic form

$$\text{AD} \frac{1}{2} \int \frac{d\xi^- d^2 \xi_\perp}{2\pi(2\pi)^2} e^{-i\mathbf{k}^+ \xi^- + i\mathbf{k}_\perp \cdot \xi_\perp} \langle p | \bar{\psi}(\xi) \gamma^+ [\xi, 0]_{\text{direct link}} \psi(0) | p \rangle = \text{AD} \frac{1}{2} \int \frac{d\xi^- d^2 \xi_\perp}{2\pi(2\pi)^2} e^{-i\mathbf{k}^+ \xi^- + i\mathbf{k}_\perp \cdot \xi_\perp} \langle p | \bar{\Psi}(\xi | \infty) \gamma^+ \Psi(0 | \infty) | p \rangle \Phi(p^+, n^- | 0^-, \mathbf{0}_\perp) \Phi^\dagger(p^+, n^- | \xi^-, \xi_\perp). \quad (22)$$

This sum rule is based on the following considerations, based on the probabilistic interpretation of PDF's in terms of their anomalous dimensions (alias the RG properties) of corresponding operators. The distribution functions cannot be calculated from first principles, but their evolution can. In particular, we have the DGLAP equation for the integrated PDF's and the two-quark UV anomalous dimension for the TMD PDF's, where the quark fields are separated by a non-light-like distance. The requirement that the off-the-light-cone two-quark matrix element should have an anomalous dimension equal to that of the corresponding quantity with

the smooth gauge connector in order to respect the probabilistic interpretation, is tantamount to the anomalous dimensions sum rule. Therefore, the RG properties can serve to define the necessary condition for the PDF to be a number density. The generalized TMD PDF indeed obeys this condition.

It is interesting to note that the additional soft counter term R can be treated within Mandelstam's explicitly gauge-invariant formalism and appears there as an "intrinsic Coulomb phase" [20] stemming from the long-range interactions of a colored quark, created initially at the "point" $-\infty^+$ together with its oppositely color-charged counterpart, then travelling along the plus light-cone direction to the origin, where it is affected by a hard collision with the photon, thus changing its route and going along the minus direction to $+\infty^-$. From this point of view, the soft counter term can be treated as that part of the TMD PDF which accumulates the residual effects of the primordial separation of two oppositely color-charged particles, created at light-cone infinity and being unrelated to the existence of external color sources.

Conclusions The anomalous dimension of the TMD PDF in the light-cone gauge was calculated in the 1-loop order. It was shown explicitly, how the transverse semi-infinite gauge link eliminates the dependence from the different pole prescriptions in the gluon light-cone-propagator. An anomalous dimension sum rule (ADSR) was introduced, which allows to study the possible structure of gauge links in the TMD PDF on the basis of their UV renormalization properties, starting from the smooth connector which provides the simplest way of gauge-invariance restoration and obeys simple and well-known RG properties. A generalized renormalization procedure of the TMD PDF's was proposed, based on the renormalization of Wilson exponentials with cusped gauge contours.

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