Study of charmonium distribution amplitudes

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Abstract

Charmonium distribution amplitudes are the key ingredient of any hard exclusive process with charmonium production. Study of these amplitudes can lead to a better understanding of charmonium production. This report is devoted to the study of the leading twist light cone distribution amplitudes of 1S and 2S charmonium mesons (in particular $J/\Psi, \eta_c, \Psi'$, η_c'). The moments of distribution amplitudes have been calculated within three approaches: potential models, nonrelativistic QCD and QCD sum rules. Using the results obtained within these approaches the models for the distribution amplitudes of the leading twist have been proposed.

1 Introduction

Charmonium distribution amplitudes (DA) are universal nonperturbative objects that parametrize nonperturbative affects of the hadronization of partons into charmonium mesons in hard exclusive processes [1]. The universality of DAs and the variety of the processes where these functions can be used make the study of charmonium DAs to be a very important task. However, despite the fact that charmonium DAs are very important in understanding hard exclusive processes with charmonium production there is a very limited knowledge of the properties of these functions. In this report study of $1S$ and $2S$ states charmonium DAs is discussed. The models for the DAs are built and applied to the study of double charmonium production. The results that are presented in this report were first obtained in papers [2, 3, 4].

Let us begin from the definition of DAs of S-wave charmonium states. There is one leading twist light cone wave function (DA) of $^1S_0$ meson $\phi_0(\xi,\mu)$ and there are two leading twist DAs of $^3S_1$ meson $\phi_L(\xi,\mu)$, $\phi_T(\xi,\mu)$. The function $\phi_L(\xi,\mu)$ is twist two DA of longitudinally polarized $^3S_1$ meson. The function $\phi_T(\xi,\mu)$ is twist two DA of transversely polarized $^3S_1$ meson. These DAs can be defined as follows [1]

$$\begin{aligned}
\langle 0|\bar{Q}(z)\gamma_\alpha\gamma_5[z,-z]Q(-z)|P(p)\rangle_\mu &= if_\eta p_\alpha \int_{-1}^{1} d\xi\, e^{i(pz)\xi}\phi_0(\xi,\mu), \\
\langle 0|\bar{Q}(z)\gamma_\alpha[z,-z]Q(-z)|V(\epsilon_{\lambda=0},p)\rangle_\mu &= f_L p_\alpha \int_{-1}^{1} d\xi\, e^{i(pz)\xi}\phi_L(\xi,\mu), \\
\langle 0|\bar{Q}(z)\sigma_{\alpha\beta}[z,-z]Q(-z)|V(\epsilon_{\lambda=\pm1},p)\rangle_\mu &= f_T(\mu)(\epsilon_\alpha p_\beta - \epsilon_\beta p_\alpha) \int_{-1}^{1} d\xi\, e^{i(pz)\xi}\phi_T(\xi,\mu),
\end{aligned}$$

where the following designations are used: x_1, x_2 are the parts of momentum of the whole meson carried by quark and antiquark correspondingly, $\xi = x_1 - x_2$, p is the momentum of corresponding meson, μ is the energy scale at which DAs are difined. The factor $[z,-z]$, makes the matrix elements to be gauge invariant [1]. The dependence of the DAs $\phi_{0,L,T}(x,\mu)$ on scale μ can be found in [1].

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Commonly, charmonium mesons are considered as a nonrelativistic bound states of quark-antiquark pair. At leading order approximation in relative velocity of quark-antiquark pair $^1S_0$ and $^3S_1$ mesons cannot be distinguished. So within this approximation $^1S_0$ and $^3S_1$ mesons have identical DAs at scale $\mu \sim M_c$

$$\phi_0(\xi,\mu) = \phi_L(\xi,\mu) = \phi_T(\xi,\mu) = \phi(\xi,\mu). \tag{1}$$

One can expect that in the case of $2S$ mesons corrections to this approximation can be large. However, present accuracy does not allow one to distinguish DAs $\phi_{0,L,T}(x,\mu)$. For this reason the approximation (1) will be used.

In this report DAs will be parameterized by their moments at some scale: $\langle \xi^n_{0,L,T} \rangle$. It is worth noting that, the DAs $\phi_{0,L,T}(\xi,\mu)$ are ξ-even. Thus all odd moments $\langle \xi^{2k+1}_{0,L,T} \rangle$ equal zero and one needs to calculate only even moments.

2 Study of charmonium distribution amplitudes.

There are two approaches to the study of DAs. The first one is a functional approach. It is based on Bethe-Salpeter equation. The second approach is an operator approach. It is based on the possibility to parameterize DA by matrix elements of some QCD operators. In this report these matrix elements will be studied within NRQCD or QCD sum rules.

2.1 Functional approach.

It is known that in the center mass frame Bether-Salpeter equation of nonrelativistic system can be reduced to Schrodinger equation. Now the question arises if the solution of Schrodinger equation is known how it is possible to find DA. The answer to this question is given by Brodsky-Huang-Lepage (BHL) [5] which can be written [2] as

$$\phi(\xi) \quad \sim \quad (1-\xi^2) \int dt \; \psi\left(t + \frac{\xi^2 M_c^2}{1-\xi^2}\right). \tag{2}$$

Here M_c is a quark mass in potential model. If the DA is known than it is not difficult to find the moments. It should be noted here that the larger the power of the moment the larger the contribution coming from the end point regions ($x \sim 0$ and $x \sim 1$) to this moment. From formula (2) one sees that the motion of quark-antiquark pair in these regions is relativistic and cannot be considered reliably in the framework of potential models. Thus it is not possible to calculate higher moments within potential models. Due to this fact the calculations have been restricted by few first moments.

2.2 Operator approach: NRQCD.

As was noted above: it is possible to parameterize DA by matrix elements of some QCD operators. For nonrelativistic system such as charmonium these operators can be expended in relative velocity. At leading order approximation one can get simple formula that allows one to connect the moments of DA with matrix element of NRQCD operators

$$\langle \xi^n \rangle = \frac{\langle v^n \rangle}{n+1}, \quad \langle v^{2k} \rangle = \frac{\langle 0|\chi^+((i\overleftrightarrow{\mathbf{D}})^2)^k \psi|\eta_c(p)\rangle}{\langle 0|\chi^+\psi|\eta_c(p)\rangle}. \tag{3}$$

Recently the matrix elements of NRQCD operators was studied in paper [6] where the following results were obtained $\langle v^n \rangle = \gamma^n$. The constant γ can be expressed through the value of the matrix element $\langle v^2 \rangle$. Substituting the last formula into (3) one has the following expression for

$\langle\xi^n\rangle$	Buchmuller-Tye model	Cornell model	NRQCD	QCD sum rules
$\langle\xi^2\rangle_{1S}$	0.086	0.084	0.075 ± 0.011	0.070 ± 0.007
$\langle\xi^2\rangle_{2S}$	0.16	0.16	0.22 ± 0.14	$0.18\ ^{+0.05}_{-0.07}$
$\langle\xi^4\rangle_{1S}$	0.020	0.019	0.010 ± 0.003	0.012 ± 0.002
$\langle\xi^4\rangle_{2S}$	0.042	0.046	0.085 ± 0.110	$0.051\ ^{+0.031}_{-0.031}$
$\langle\xi^6\rangle_{1S}$	0.0066	0.0066	0.0017 ± 0.0007	0.0032 ± 0.0009
$\langle\xi^6\rangle_{2S}$	0.015	0.016	0.039 ± 0.077	$0.017\ ^{+0.016}_{-0.014}$

Table 1: The moments of DAs of $1S$ and $2S$ charmonium states obtained within different approaches. In the second and third columns the moments calculated in the framework of Buchmuller-Tye and Cornell potential models are presented. In the fourth column NRQCD predictions for the moments are presented. In last column contains the results obtained within QCD sum rules.

the moments at leading order approximation in relative velocity $\langle\xi^n\rangle = \gamma^n/(n+1)$. It is not difficult to show that this result for the moments can be reproduced by the following DA

$$\phi(\xi) = \frac{1}{2\gamma}\theta(\gamma - |\xi|), \tag{4}$$

which can be considered as a DA of nonrelativistic meson at leading order approximation in relative velocity.

2.3 Operator approach: QCD sum rules.

Another approach to the calculation of the moments is based on QCD sum rules [7]. The application of QCD sum rules for the study of DAs was developed by Chernyak and Zhitnitsky [1, 8]. In this report this approach will not be considered in detail. The details can be found in papers [2, 3, 4]. Here it should be noted that the main advantage of QCD sum rules in comparison to the approaches considered above is that in the framework of QCD sum rules one does not treat quarkonium as a nonrelativistic system. This allows one to avoid the main source of uncertainty – the relativistic corrections. For this reason, QCD sum rules is the most accurate approach to the calculation of the moments.

2.4 Numerical results and models for the distribution amplitudes.

Numerical results of the calculation are collected in Table I. As it was shown in papers [2, 3, 4] these results can be represented by the following models of DAs:

$$\begin{aligned}\phi_{1S}(\xi, \mu \sim m_c) &\sim (1-\xi^2)\ \mathrm{Exp}\left[-\frac{\beta}{1-\xi^2}\right] \\ \phi_{2S}(\xi, \mu \sim m_c) &\sim (1-\xi^2)(\alpha+\xi^2)\ \mathrm{Exp}\left[-\frac{\beta}{1-\xi^2}\right]\end{aligned} \tag{5}$$

For $1S$ charmonium state the constant β can vary within the interval 3.8 ± 0.7. For $2S$ charmonium state the constants α and β can vary within the intervals $0.03^{+0.32}_{-0.03}$ and $2.5^{+3.2}_{-0.8}$ correspondingly.

In papers [2, 3, 4] it was shown that due to evolution DAs have some interesting properties. The first one is that due to the radiative corrections nonrelativistic QCD velocity scaling rules are violated in hard exclusive processes. Next property can be formulated as follows: it is not difficult to show that at scale $\mu \sim m_c$ distribution amplitudes (5) are the DAs of nonrelativistic system. This results from the fact that functions (5) strongly suppress end point regions

$|\xi| \sim 1$ where the motion is relativistic. However, at larger scales due to the evolution there appears relativistic tail. From the NRQCD perspective this means that the role of higher order NRQCD operators is greatly enhanced by radiative corrections. The last property consists in the improvement of the accuracy of any model at scales larger than m_c. For instance, at scale $\mu \sim m_c$ the result for the second moment of $1S$ state found above is $\langle \xi_L^2 \rangle = 0.070 \pm 0.007$. At scale $\mu = 10$ GeV this result transforms to $\langle \xi_L^2 \rangle = 0.123 \pm 0.005$. In the case of 2S state, at scale $\mu \sim m_c$ we have $\langle \xi_L^2 \rangle = 0.18^{+0.05}_{-0.07}$. At scale $\mu = 10$ GeV this result transforms to $\langle \xi_L^2 \rangle = 0.19\ ^{+0.03}_{-0.04}$. Similar improvement takes place for higher moments.

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