Spontaneous P-parity breaking in dense baryon matter

A.A. Andrianov[a,b]*, V.A. Andrianov[b]† and D. Espriu[a]‡

[a] *Departament d'Estructura i Constituents de la Matèria and Institut de Ciències del Cosmos, Universitat de Barcelona, Diagonal 647, 08028 Barcelona, Spain*

[b] *V.A. Fock Department of Theoretical Physics, St. Petersburg State University, ul. Ulianovskaya, 198504, St.Petersburg, Russia*

Abstract

We investigate how a large baryon density may induce spontaneous parity violation in the meson sector of QCD. The analysis is done for an idealized homogeneous and infinite nuclear matter when the influence of density can be examined with the help of constant chemical potential. We elaborate a novel mechanism of parity breaking based on interplay between lightest and heavy meson condensates which cannot be realized in the pion sector solely. The analysis at intermediate energy scales is done by using an effective Lagrangian that includes two scalar and pseudoscalar multiplets. We scan all possible sectors of meson physics and argue that the parity breaking phenomenon is rather typical than exotic when chemical potentials are large enough.

1 Introduction

The appearance of P- violation for sufficiently large values of temperature and/or the chemical potential has been attracting much interest during last decades to search it both in dense nuclear matter (in neutron/quark stars and heavy ion collisions at intermediate energies) and in strongly interacting quark-gluon matter ("quark-gluon plasma" in heavy ion collisions at very high energies). In particular, at finite baryon density it is a possibility conjectured by A.B.Migdal in [1] in nuclear physics long ago (and reviewed in [2], see also the recent development in [3]-[6]). One has also to mention a phenomenon of $(C)P$-parity breaking in meta-stable nuclear bubbles created in hot nuclear matter [7] and/or in the presence of a strong background magnetic field [8] which however theoretically is quite different in its origin and will not be linked to in the present paper.

In fact, some time ago it was proved quite rigorously in [9] that parity, P, and vector flavor symmetry could not undergo spontaneous symmetry breaking in a vector like theory such as QCD. This is thus a well established result in strong interactions at *zero* chemical potential. Finite baryon density however results in a manifest breaking of CP- invariance. The presence of a finite chemical potential leads to the presence of a *constant* imaginary zeroth-component of a vector field and the partition function of QCD is not anymore invariant under a CP-transformation. The conditions under which the results of [9] were proven (positivity of the measure) then do not hold anymore.

Here we explore the issue of P-parity breaking employing effective lagrangian techniques in the range of nuclear densities for which hadron phase persists and quark percolation does not occur yet.

A novel mechanism proposed in our papers [10, 11] is essentially based on interplay between lightest and heavy meson states and cannot be realized solely in the Goldstone boson (pion)

***e-mail**: andrianov@bo.infn.it
†**e-mail**: v.andriano@rambler.ru
‡**e-mail**: espriu@ecm.ub.es

sector.The analysis is done for an idealized homogeneous and infinite nuclear matter when the influence of density can be examined with the help of constant chemical potential.

2 Generalized sigma model

Let us consider a model with two multiplets of scalar/pseudoscalar fields $H_j = \sigma_j \mathbf{I} + i\hat{\pi}_j, j = 1, 2$ with $\hat{\pi}_j \equiv \pi_j^a \tau^a$ with τ^a being a set of Pauli matrices. In the exact *chiral limit* a scalar system associated with the bosonized QCD must be symmetric in respect to global $SU(2)_L \times SU(2)_R$ rotations. These two chiral multiplets are thought of as representing the two lowest-lying radial states for a given J^{PC}. (the predecessors of models of such type are discussed in [12]). Let us define the effective potential of this generalized σ- model. First we write the most general Hermitian potential at zero μ using the chiral parameterization,

$$H_1(x) = \sigma_1(x)U(x) = \sigma_1(x)\xi^2(x); \quad H_2(x) = \xi(x)\big(\sigma_2(x) + i\hat{\pi}_2(x)\big)\xi(x). \tag{1}$$

This kind of parameterization preserves the parities of $\sigma_2(x)$ and $\hat{\pi}_2$ to be even and odd respectively (in the absence of SPB). It takes the following form,

$$\begin{aligned} V_{\text{eff}} &= -\sum_{j,k=1}^{2} \sigma_j \Delta_{jk} \sigma_k - \Delta_{22}(\pi_2^a)^2 + \lambda_2 \Big((\pi_2^a)^2\Big)^2 + (\pi_2^a)^2 \Big((\lambda_3 - \lambda_4)\sigma_1^2 + \lambda_6 \sigma_1 \sigma_2 + 2\lambda_2 \sigma_2^2\Big) \\ &\quad + \lambda_1 \sigma_1^4 + \lambda_2 \sigma_2^4 + (\lambda_3 + \lambda_4)\sigma_1^2 \sigma_2^2 + \lambda_5 \sigma_1^3 \sigma_2 + \lambda_6 \sigma_1 \sigma_2^3, \end{aligned} \tag{2}$$

with 9 real constants Δ_{jk}, λ_A . QCD bosonization rules indicate that $\Delta_{jk} \sim \lambda_A \sim N_c$. The neglected terms will be suppressed by inverse power of the chiral symmetry breaking (CSB) scale $\Lambda \simeq 1.2$GeV. If we assume the v.e.v. of H_j to be of the order of the constituent mass $0.2 \div 0.3$ GeV, it is reasonable to neglect these terms. Let us now investigate the hypothetical appearance of a non-zero v.e.v. of pseudoscalar fields. In order not to break the charge conservation, we must expect, if at all, only a neutral condensate represented by a solution with $\pi_2^a = \delta^{a0}\rho$. The conditions to have an extremum are derived from the first variation of the effective potential (2),

$$\begin{aligned} 2(\Delta_{11}\sigma_1 + \Delta_{12}\sigma_2) &= 4\lambda_1 \sigma_1^3 + 3\lambda_5 \sigma_1^2 \sigma_2 + 2(\lambda_3 + \lambda_4)\sigma_1 \sigma_2^2 + \lambda_6 \sigma_2^3 \\ &\quad + \rho^2 \Big(2(\lambda_3 - \lambda_4)\sigma_1 + \lambda_6 \sigma_2\Big), \\ 2(\Delta_{12}\sigma_1 + \Delta_{22}\sigma_2) &= \lambda_5 \sigma_1^3 + 2(\lambda_3 + \lambda_4)\sigma_1^2 \sigma_2 + 3\lambda_6 \sigma_1 \sigma_2^2 + 4\lambda_2 \sigma_2^3 + \rho^2 \Big(\lambda_6 \sigma_1 + 4\lambda_2 \sigma_2\Big), \\ 0 &= 2\pi_2^a \Big(-\Delta_{22} + (\lambda_3 - \lambda_4)\sigma_1^2 + \lambda_6 \sigma_1 \sigma_2 + 2\lambda_2 \sigma_2^2 + 2\lambda_2 \rho^2\Big). \end{aligned} \tag{3}$$

To avoid spontaneous parity breaking in normal vacuum phase of QCD, it is *necessary and sufficient* to impose,

$$(\lambda_3 - \lambda_4)\sigma_1^2 + \lambda_6 \sigma_1 \sigma_2 + 2\lambda_2 \sigma_2^2 > \Delta_{22}. \tag{4}$$

Since QCD in normal conditions does not lead to parity breaking, the low-energy model must necessarily fulfill (4).

The sufficient conditions follow from the positivity of the second variation $\hat{V}^{(2)}$ for a non-trivial solution of the two first equations (3) at $\rho = 0$.

The required conditions are given by tr $\left\{\hat{V}^{(2)}\right\} > 0$ and Det$\hat{V}^{(2)} > 0$. For positive matrices it means that $V_{11}^{(2)\sigma} > 0; \quad V_{22}^{(2)\sigma} > 0; \quad V_{aa}^{(2)\pi} > 0$. The last relation determines the mass squared of π' meson and thereby must be positive according to the inequality (4).

The two set of conditions, namely those presented in eq. (4) represent restrictions that the symmetry breaking pattern of QCD imposes on its low-energy effective realization. At vanishing chemical potential, of course.

3 Finite chemical potential

Finite density is transmitted to the boson sector via $\Delta\mathcal{L} = -(\bar{q}_R H_1 q_L + \bar{q}_L H_1^\dagger q_R)$, where $q_{L,R}$ are assumed to be constituent quarks. Then the one-loop contribution to $V_{\rm eff}$ is,

$$\begin{aligned}\Delta V_{\rm eff}(\mu) &= \frac{\mathcal{N}}{2}\Theta(\mu-|H_1|)\Bigg[\mu|H_1|^2\sqrt{\mu^2-|H_1|^2}-\frac{2\mu}{3}(\mu^2-|H_1|^2)^{3/2}\\ &\quad -|H_1|^4\ln\frac{\mu+\sqrt{\mu^2-|H_1|^2}}{|H_1|}\Bigg]\left(1+O\left(\frac{\mu^2}{\Lambda^2};\frac{|H_1|^2}{\Lambda^2}\right)\right); \qquad \mathcal{N}\equiv\frac{N_cN_f}{4\pi^2}, \qquad (5)\end{aligned}$$

where μ is the chemical potential. The higher-order contributions of chiral expansion in $1/\Lambda^2$ are not considered. This effective potential is normalized to reproduce the baryon density for quark matter $\rho_B = -\frac{1}{3}\partial_\mu \Delta V_{\rm eff}(\mu) = \frac{N_cN_f}{9\pi^2}p_F^3 = \frac{N_cN_f}{9\pi^2}(\mu^2-|\langle H_1\rangle|^2)^{3/2}$, where the quark Fermi momentum is $p_F = \sqrt{\mu^2-|\langle H_1\rangle|^2}$. Normal nuclear density is $\rho_B \simeq 0.17\ {\rm fm}^{-3} \simeq (1.8\ {\rm fm})^{-3}$ that corresponds to the average distance 1.8 fm between nucleons in nuclear matter.

The conditions for a minimum of the effective potential are thus modified. For instance (3) is modified to,

$$\begin{aligned}&2(\Delta_{11}\sigma_1+\Delta_{12}\sigma_2) = 4\lambda_1\sigma_1^3+3\lambda_5\sigma_1^2\sigma_2+2(\lambda_3+\lambda_4)\sigma_1\sigma_2^2+\lambda_6\sigma_2^3 \qquad (6)\\ &+\rho^2\Big(2(\lambda_3-\lambda_4)\sigma_1+\lambda_6\sigma_2\Big)+2\mathcal{N}\Theta(\mu-\sigma_1)\left[\mu\sigma_1\sqrt{\mu^2-\sigma_1^2}-\sigma_1^3\ln\frac{\mu+\sqrt{\mu^2-\sigma_1^2}}{\sigma_1}\right].\end{aligned}$$

The possibility of SPB is controlled by the inequality (4); in order to approach a SPB phase transition we have to diminish the l.h.s. of inequality (4) and therefore we need to have (assuming that the inequality indeed holds at $\mu=0$), $\partial_\mu\Big[(\lambda_3-\lambda_4)\sigma_1^2+\lambda_6\sigma_1\sigma_2+2\lambda_2\sigma_2^2\Big] < 0$. This inequality is a *necessary* condition that has to be satisfied by the model at zero chemical potential for it to be potentially capable of yielding SPB. Thus P-violation is not exceptional but rather typical for admissible values of low-energy parameters of our model.

Let us now leave the case $\mu \simeq 0$ and examine the possible existence of a critical point where the strict inequality (4) does not hold and instead for $\mu > \mu_{crit}$,

$$(\lambda_3-\lambda_4)\sigma_1^2+\lambda_6\sigma_1\sigma_2+2\lambda_2\Big(\sigma_2^2+\rho^2\Big) = \Delta_{22}. \qquad (7)$$

After substituting (7) into the second Eq.(3) one finds that

$$\lambda_5\sigma_1^2+4\lambda_4\sigma_1\sigma_2+\lambda_6\Big(\sigma_2^2+\rho^2\Big) = 2\Delta_{12}, \qquad (8)$$

where we have taken into account that $\sigma_1 \neq 0$. Together with (7) it completely fixes the v.e.v.'s of the scalar fields $\sigma_{1,2}$.

Let us now try to determine the critical value of the chemical potential, namely the value where $\rho(\mu_c)=0$. Combining our equations one gets,

$$(4\lambda_2\Delta_{12}-\lambda_6\Delta_{22})r^2+(2\lambda_6\Delta_{12}-4\lambda_4\Delta_{22})r+2(\lambda_3-\lambda_4)\Delta_{12}-\lambda_5\Delta_{22}=0; \quad r\equiv\frac{\sigma_2}{\sigma_1}. \qquad (9)$$

In order for a SPB phase to exist this equation has to possess real solutions. If $4\lambda_2\Delta_{12}-\lambda_6\Delta_{22}\neq 0$ the SPB phase is bounded by two critical points corresponding to second order transitions. If, on the contrary, $4\lambda_2\Delta_{12}-\lambda_6\Delta_{22}=0$ there is only one solution corresponding to a second order transition.

4 The physical spectrum in the SPB phase

Once a condensate for π_2^0 appears spontaneously the vector $SU(2)$ symmetry is broken to $U(1)$ and two charged π' mesons are expected to possess zero masses (in the chiral limit). Calculations of the matrix of second variation of the effective potential gives positive masses for two scalar and four pseudoscalar mesons, whereas the doublet of charged of π mesons remain massless. Quantitatively the mass spectrum can be obtained only after kinetic terms are normalized. We take the general kinetic term symmetric under $SU(2)_L \times SU(2)_R$ global rotations to be $\mathcal{L}_{kin} = \frac{1}{4}\sum_{j,k=1}^{2} A_{jk}\mathrm{tr}\left\{\partial_\mu H_j^\dagger \partial^\mu H_k\right\}$. After selecting out the v.e.v. $\langle H_1\rangle = \langle\sigma_1\rangle \equiv \bar\sigma_1$ one can separate the bare Goldstone boson action with the chiral parameterization (1) and expansion around a vacuum configuration $U = 1 + i\hat\pi/F_0 + \cdots$, $\xi = 1 + i\hat\pi/2F_0 + \cdots$ and use the v.e.v.'s $\sigma_j \equiv \bar\sigma_j + \Sigma_j$, $\hat\pi = \tau_3\rho + \hat\Pi$. Then the quadratic part looks as follows,

$$\begin{aligned}\mathcal{L}^{(2)}_{kin} &= \frac{1}{2}\sum_{j,k=1}^{2} A_{jk}\left[\partial_\mu\Sigma_j\partial^\mu\Sigma_k + \frac{1}{F_0^2}\bar\sigma_j\bar\sigma_k\partial_\mu\pi^a\partial^\mu\pi^a\right] \qquad (10)\\ &+\frac{1}{F_0}\sum_{j=1}^{2}A_{j2}\left[-\rho\partial_\mu\Sigma_j\partial^\mu\pi^0 + \bar\sigma_j\partial_\mu\pi^a\partial^\mu\Pi^a\right] + \frac{1}{2}A_{22}\left[\frac{\rho^2}{F_0^2}\partial_\mu\pi^0\partial^\mu\pi^0 + \partial_\mu\Pi^a\partial^\mu\Pi^a\right],\end{aligned}$$

which shows the mixture between light and heavy pseudoscalar states and, in the SPB phase, also between scalar and pseudoscalar states.

Let us define $F_0^2 = \sum_{j,k=1}^{2} A_{jk}\bar\sigma_j\bar\sigma_k$, $\zeta \equiv \frac{1}{F_0}\sum_{j=1}^{2} A_{j2}\bar\sigma_j$. In the symmetric phase $\rho = 0$ one diagonalizes the Lagrangian by shifting the pion field,

$$\mathcal{L}^{(2)}_{kin,\pi} = \frac{1}{2}\partial_\mu\tilde\pi^a\partial^\mu\tilde\pi^a + \frac{1}{2}(A_{22}-\zeta^2)\partial_\mu\Pi^a\partial^\mu\Pi^a, \qquad A_{22}-\zeta^2 = \frac{\bar\sigma_1^2\det A}{F_0^2} > 0, \qquad (11)$$

wherefrom, taking into account the second variation of the effective potential , one finds the masses of the pion triplets. The physical spectrum of the pseudoscalar mesons in the normal phase is, $m_\pi^2 = 0$, $m_{\pi'}^2 = \dfrac{\mathcal{F}_1}{(A_{22}-\zeta^2)}$, where $\mathcal{F}_1 = -\Delta_{22} + (\lambda_3-\lambda_4)\sigma_1^2 + \lambda_6\sigma_1\sigma_2 + 2\lambda_2\sigma_2^2$, .

In the SPB phase the situation is more involved: pseudoscalar states mix with scalar ones. In particular, diagonalization is different for neutral and charged pions because the vector isospin symmetry is broken: $SU(2)_V \to U(1)$. Namely $\tilde\pi^\pm = \pi^\pm + \zeta\Pi^\pm$, $\quad \tilde\pi^0 = \pi^0 + \frac{F_0^2}{F_0^2 + A_{22}\rho^2}\Big(\zeta\Pi^0 - \frac{\rho}{F_0}\sum_{j=1}^{2} A_{j2}\partial_\mu\Sigma_j\Big)$. In this way SPB induces mixing of both massless and heavy neutral pions with scalars. The (partially) diagonalized kinetic term has the following form,

$$\begin{aligned}\mathcal{L}^{(2)}_{kin} &= \partial_\mu\tilde\pi^\pm\partial^\mu\tilde\pi^\mp + \frac{1}{2}\Big(1 + \frac{A_{22}\rho^2}{F_0^2}\Big)\partial_\mu\tilde\pi^0\partial^\mu\tilde\pi^0 + (A_{22}-\zeta^2)\partial_\mu\Pi^\pm\partial^\mu\Pi^\mp \qquad (12)\\ &+\frac{1}{2}(A_{22} - \frac{F_0^2}{F_0^2 + A_{22}\rho^2}\zeta^2)\partial_\mu\Pi^0\partial^\mu\Pi^0\\ &+\frac{1}{2}\sum_{j,k=1}^{2}\frac{A_{jk}F_0^2 + \rho^2\det A\delta_{1j}\delta_{1k}}{F_0^2 + A_{22}\rho^2}\partial_\mu\Sigma_j\partial^\mu\Sigma_k - \frac{F_0\rho}{F_0^2 + A_{22}\rho^2}\zeta\partial_\mu\Pi^0\sum_{j=1}^{2}A_{j2}\partial^\mu\Sigma_j.\end{aligned}$$

We see that even in the massless pion sector the isospin breaking $SU(2)_V \to U(1)$ occurs: neutral pions become less stable with a larger decay constant. Another observation is that in the charged meson sector the relationship between massless π and π' remain the same as in the symmetric phase. Further diagonalization $\Pi^0, \Sigma_1, \Sigma_2$ fields leads to mixes neutral pseudoscalar and scalar states $\tilde\Pi^0, \tilde\Sigma_1, \tilde\Sigma_2$. Therefore genuine mass states do not possess a definite parity in decays.

5 P-violation in models with discrete $Z_2 \times Z_2$ symmetry

As a relevant example we now examine models with residual discrete chiral symmetry (after the breaking $SU(2)_L \times SU(2)_R \to SU(2)_V$) under independent reflections $H_1 \to -H_1$ and/or $H_2 \to -H_2$. Then $\lambda_5 = \lambda_6 = 0, A_{12} = 0, \Delta_{12} = 0$, but $\lambda_4 \neq 0$. One can always fix $A_1 = A_2$ redefining the other parameters.

Let us now see how the general relations in the previous section are realized in this model. The analysis of Eqs. (3), (4) and (6) as well as the positivity of the second variation matrix leads to conclusion that in the symmetric phase the only solution compatible with the very possibility of P-breaking is $\sigma_2 = 0$. In the SPB phase for these models the constraint (8) $2\lambda_4\sigma_1\sigma_2 = \Delta_{12} = 0$ has also a unique solution $\sigma_2 = 0$. Therefore $\sigma_2 = 0$ everywhere. As for σ_1 we get for $\mu = 0$, $\sigma_1^2 = \frac{\Delta_{11}}{2\lambda_1}$.

The generalized sigma model described here has the same symmetry properties as QCD itself. Let us now make a closer contact with QCD by an appropriate choice of the low energy constants in this effective lagrangian. This will allow us to estimate the typical scales of P-breaking directly from meson spectroscopy.

Just to get a feeling of the possible scales involved, let us make a tentative choice $F_0 =$ 100MeV and use the units F_0 further on. As well let's take $m_{p,1} = 0, \quad m_{s,1} = 0.7\text{GeV} = 7F_0, \quad m_{p,2} = 1.3\text{GeV} = 13F_0, \quad m_{s,2} = 1.5\text{GeV} = 15F_0$ in a fair agreement with particle phenomenology [17]. Then, according to the relation $F_0^2 = A_1\sigma_1^2$, one can put, $A_{11} = \frac{1}{9} = A_{22}, \quad \sigma_1 = 300\text{MeV} = 3F_0$. Furthermore, from the definitions of masses,

$$m_{s,1}^2 = \frac{2\Delta_{11}}{A_{11}} = \frac{4\lambda_1\sigma_1^2}{A_{11}}; \quad m_{s,2}^2 - m_{p,2}^2 = \frac{2\lambda_4\sigma_1^2}{A_{11}}, \tag{13}$$

one finds $\Delta_{11} \simeq 2.7F_0^2, \quad \lambda_1 \simeq 0.15, \quad \lambda_4 \simeq 0.35$.

Taking $\sigma_{1,crit} \simeq 1.8F_0$ and the previously estimated value $\lambda_1 = 0.15$ one finds that SPB occurs at $p_F \simeq 3.9F_0 = 1.44 p_{F,nuclear}$ which corresponds to dense nuclear matter with $\varrho_{B,crit} \simeq 0.5\text{fm}^{-3} \simeq 3\varrho_{B,nuclear}$. The phase transition occurs at $\mu_c \simeq 4.3F_0 > \sigma_1$. From the definition of the mass of π' one finds $\lambda_3 \simeq 3.6$ and $\Delta_{22} \simeq 11F_0^2$. Thus we see that the possibility of SPB emerges naturally for reasonable values of the meson physics parameters and low-energy constants. At this critical point the masses of scalar mesons are $m_{s,1} \simeq 1.7F_0, \; m_{s,2} \simeq 4.5F_0$.

6 Beyond the chiral limit: light u, d quarks

If the current quark masses $m_q \simeq m_u \simeq m_d$ does not vanish (but remain small) they can be incorporated as the averages of external scalar sources $M_j(x) = s_j(x) + i\tau^a p_j^a(x)$, namely, $< M_j(x) >= -\frac{1}{2} d_j m_q$ We will consider two-flavor case and retain the softest terms linear in H_j and m_q thereby neglecting (in principle relevant) terms cubic in scalar fields. With an appropriate choice of external scalar sources only one type of Hermitian structures is possible, $\sum_{j=1,2} \text{tr}(M_j^\dagger H_j + H_j^\dagger M_j)$, *i.e.* we add the two new terms to our effective potential (2),

$$-\frac{1}{2} m_q \text{tr}\Big[d_1(H_1 + H_1^\dagger) + d_2(H_2 + H_2^\dagger)\Big]. \tag{14}$$

Making use of our chiral parametrization of the fields H_j through the chiral field U,

$$U = \cos\frac{|\pi_1^a|}{F_0} + i\frac{\tau^a\pi_1^a}{|\pi_1^a|}\sin\frac{|\pi_1^a|}{F_0}, \tag{15}$$

one derives the following extension of the effective potential (2),

$$\Delta V_{\text{eff}}(m_q) = 2m_q\left[-(d_1\sigma_1 + d_2\sigma_2)\cos\frac{|\pi_1^a|}{F_0} + d_2\frac{\pi_1^a\pi_2^a}{|\pi_1^a|}\sin\frac{|\pi_1^a|}{F_0}\right]. \tag{16}$$

The presence of light quark masses does not remove the possibility of spontaneous P-parity breaking. Their main effect consists in making pions massive while the spontaneous breaking of isospin vector symmetry $SU_V(2) \longrightarrow U_V(1)$ which entails P-parity breaking generates truly massless Goldstone bosons, namely two massless charged pion-like states. Neglecting a small mixure of heavy and light states one deals with the pseudoscalar sector of light pseudoscalar masses,

$$
\begin{aligned}
m^2_{\tilde{\pi}^0} &= \left(1+\frac{A_{22}\rho^2}{F_0^2}\right)^{-1} 2m_q \left(\frac{d_1\sigma_1+d_2\sigma_2}{F_0^2}\cos\frac{\langle\pi^0\rangle}{F_0} - \frac{d_2\rho}{F_0^2}\sin\frac{\langle\pi^0\rangle}{F_0}\right), \\
m^2_{\tilde{\pi}^\pm} &= 0, \qquad\qquad (17)\\
m^2_{\Pi^\pm} &= 2m_q\frac{\cos\frac{\langle\pi^0\rangle}{F_0}}{A_{22}-\zeta^2}\left(\frac{d_2^2}{d_1\sigma_1+d_2\sigma_2} - 2\zeta\frac{d_2}{\langle\pi^0\rangle}\tan\frac{\langle\pi^0\rangle}{F_0} + A_{22}\frac{d_1\sigma_1+d_2\sigma_2}{\langle\pi^0\rangle^2}\tan^2\frac{\langle\pi^0\rangle}{F_0}\right).
\end{aligned}
$$

Thus in the SPB one registers two massless charged pseudoscalars and three light pseudoscalars with masses linear in the current quark mass. These equations represent the generalization of the Gell-Mann-Oakes-Renner relation in the phase with broken parity. We notice that the masses of neutral and charged pseudoscalars don't coincide just realizing the spontaneous breaking of isospin symmetry.

One can also guess that the manifest breaking of $SU(2)$ symmetry due to different masses of u and d quarks will supply the Goldstone bosons $\tilde{\pi}^{\pm}$ with tiny masses proportional to the difference $m_u - m_d$.

7 Conclusions

Thus our main results. Using an effective quark-meson lagrangian for low-energy QCD that retains the two lowest lying states in the scalar and pseudoscalar sectors parity breaking seems to be quite a realistic possibility in nuclear matter at moderate densities. We have found the necessary and sufficient conditions for a phase where parity is spontaneously broken to exist. Salient characteristics of this phase would be the spontaneous breaking of the vector isospin symmetry $SU(2)_V$ down to $U(1)$ and the generation two additional very light charged pseudoscalar mesons (massless in the limit $m_u = m_d$). We also find a strong mixing between scalar and pseudoscalar states that translate spontaneous parity breaking into meson decays. The mass eigenstates will decay both in odd and even number of pions simultaneously. Isospin breaking can also be visible in decay constants.

While the experiments at the RHIC at BNL and at the future LHC at CERN focus on the study of high temperatures the program for GSI at large double-ring accelerator SIS 100/300, namely, the experiment at FAIR on the compressed baryon matter (CBM) will concentrate on the investigation of highest baryon densities at still moderate temperatures. Therefore our model predictions are quite relevant to interpret these future experiments.

8 Acknowledgments

This work was supported by research grants FPA2007-66665, 2005SGR00564, 2007PIV10046 and grants RFBR. It is also supported by the Consolider-Ingenio 2010 Program CPAN (CSD2007-00042). We acknowledge the partial support of the EU RTN networks FLAVIANET and EN-RAGE . A.A. is very grateful to the Organizing Committee of the 15th International Seminar on High Energy Physics QUARKS'2008 for hospitality and support and also to Prof. V.A Rubakov for many interesting discussions.

References

[1] A.B. Migdal, Zh. Eksp. Teor. Fiz. **61**, 2210 (1971) [Sov. Phys. JETP **36**,1052 (1973)]; R.F. Sawyer, Phys. Rev. Lett. **29**,382 (1972); D.J. Scalapino, Phys. Rev. Lett. **29**,386 (1972); G. Baym, Phys. Rev. Lett.**30**,1340 (1973); A.B. Migdal, O.A. Markin and I.N. Mishustin, Sov. Phys. JETP, **39**, 212 (1974).

[2] A.B. Migdal, Rev. Mod. Phys. **50**,107 (1978); D. Bailin and A. Love, Phys. Rep. **107**, 325 (1984); C.-H. Lee, Phys. Rep. **275**,197 (1996); M. Prakash, I.Bombaci,M. Prakash, P. J. Ellis, J. M. Lattimer and R. Knorren,Phys. Rep. **280**,1 (1997).

[3] G.E. Brown, M. Rho, Phys. Rep. **363**,85 (2002); D. Toublan and J. B. Kogut, Phys. Lett. B **564**, 212 (2003); M. Frank, M. Buballa and M. Oertel, Phys. Lett. B **562**,221,(2003); O. Scavenius, Á. Mócsy, I.N. Mishustin and D.H. Rischke, Phys.Rev. C **64**, 045202 (2001).

[4] V. Bernard, Ulf-G. Meissner and I. Zahed, Phys. Rev **D36**,819 (1987); T. Hatsuda and T. Kunihiro, Phys. Rep., **247**, 221 (1994); M. Buballa, Phys. Rept. **407**,205 (2005); D.N. Walters and S. Hands, Nucl. Phys. Proc. Suppl. **140**,532(2005).

[5] M. Alford, K. Rajagopal and F. Wilczek, Phys. Lett. **B422**, 247 (1998); R. Rapp, T. Schaefer, E. V. Shuryak and M. Velkovsky, Phys. Rev. Lett. **81**,53 (1998).

[6] A. Barducci, R. Casalbuoni, G. Pettini, and L. Ravagli, Phys. Rev. D **69**,096004 (2004); D. Ebert and K.G. Klimenko, J.Phys. **G32**,599 (2006); Eur.Phys.J. **C46**,771 (2006).

[7] D. Kharzeev, R.D. Pisarski and M.H.G. Tytgat, Phys. Rev. Lett. **81**,512 (1998); D. Kharzeev and R.D. Pisarski, Phys. Rev. D **61**111901(R) (2000); D. Kharzeev, Phys. Lett. B **633**,260 (2006); D. Kharzeev and A. Zhitnitsky , arXiv: 0706.1026 [hep-ph] .

[8] D.E. Kharzeev, L.D. McLerran and H.J.Warringa, arXiv:0711.0950 [hep-ph].

[9] D. Weingarten, Phys. Rev. Lett. **51**, 1830 (1983); C. Vafa and E. Witten, Phys. Rev. Lett. **53**, 535 (1984); S. Nussinov, Phys. Rev. Lett. **52,** 966 (1984); D. Espriu, M. Gross and J.F. Wheater, Phys. Lett. **B 146**, 67 (1984); for a review see, S. Nussinov and M. Lambert, Phys. Rept. **362**,193(2002).

[10] A.A. Andrianov, V.A. Andrianov and S.S.Afonin, J. Math. Sci. **143**,2697 (2007).

[11] A.A. Andrianov and D.Espriu, Phys. Lett. **B** , (2008)

[12] An oldest, pre-QCD attempt to give the Lagrangian description for two multiplets of scalar and pseudoscalar mesons was done in [13] with only a few vertices and a CSB pattern not well compatible with QCD. However we have been rather inspired by our previous works on extended quark models [14, 15] where two different schemes with linear and non-linear realization of chiral symmetry were adopted for how to incorporate heavy pions and scalar mesons within an effective quark model with quark self-interaction. In particular in [14] one can find all types of vertices in the effective potential (2) with coupling constants specific for the model. A certain resemblance can be also found with the model [16] where two $SU(3)_f$ multiplets have been associated with two-quark and four-quark meson states although we don't share the attitude in [16] on the dominating of four-quark component in radially excited mesons.

[13] I. Bars and M.B. Halpern, Phys. Rev. D7 (1973) 3043.

[14] A.A. Andrianov and V.A. Andrianov, Nucl. Phys. Proc. Suppl. 39BC (1995) 257; A.A. Andrianov, V.A. Andrianov and V.L. Yudichev, Theor. Math. Phys. 108, 1069 (1996) 1069.

[15] A.A. Andrianov, D. Espriu and R. Tarrach, Nucl. Phys. B533 (1998) 429; Nucl. Phys. Proc. Suppl. 86 (2000) 275.

[16] D. Black, A.H. Fariborz, S. Moussa, S. Nasri and J. Schechter, Phys. Rev. D 64, 014031 (2001); A.H. Fariborz, R. Jora and J. Schechter, Phys. Rev. D 72, 034001 (2005) .

[17] W.M. Yao *et al.* [Particle Data Group], J. Phys. **G33**,1 (2006).