Condensates in QCD from Dual Models

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Outline of the talk

- Mass Spectra and Condensates in CPT
- AdS/CFT with flavours
- Condensates from Holography
- Mass Spectra from Holography

Condensate in Field-Theoretical Models

The Chiral Perturbation Theory result of *Smilga and Shushanov[1997]* is: for weak fields,

$$\langle \bar{q}q \rangle_H = \langle \bar{q}q \rangle_0 \left(1 + \frac{|eH| \ln 2}{16\pi^2 f_\pi^2} \right),$$

and for strong fields

$$\langle \bar{q}q \rangle_H \sim |eH|^{\frac{3}{2}} e^{-\frac{\pi}{2}\sqrt{\frac{\pi}{2\alpha_s |eH|}}}$$

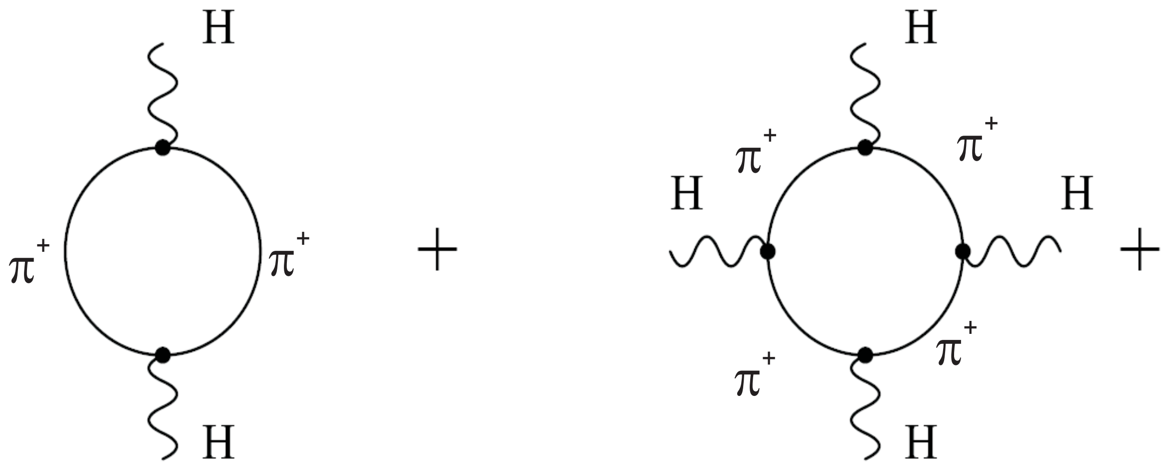
Condensate in Nambu—Jona-Lasinio model was calculated by *Klevansky, Lemmer[1989]*:

$$\langle \bar{q}q \rangle_H = \left(1 + c\frac{e^2 H^2}{(\langle \bar{q}q \rangle_0)^4} \right).$$

Important point: CPT result is non-analytic like $\sqrt{F^2}$. This is the signature of the massless pions in loops, and is an essential feature of chiral symmetry breaking, absent from NJL model, which is analytic.

Condensate via Euler—Heisenberg

What are the diagrams from which the linear result comes from? These are resummed one-loop diagrams with pions in the loops.



These diagrams contain IR singularities in the chiral limit. When resummed and differentiated over $m_q \sim m_\pi^2$, they yield a finite answer linear in H.

Beyond the Leading Order

Result by *Agasian, Shushpanov[1999]* Next-order corrections behave like

$$\langle\overline{\psi}\psi\rangle_{(H)}|_{NLO} = -\langle\overline{\psi}\psi\rangle_{(0)}\frac{(eH)^2}{(4\pi f_\pi)^4}\left[(\bar{l}_6-\bar{l}_5)\left(\ln\frac{eH}{\mu^2}+C\right)-\frac{160(4\pi)^4}{3}d^r(\mu)\right]$$

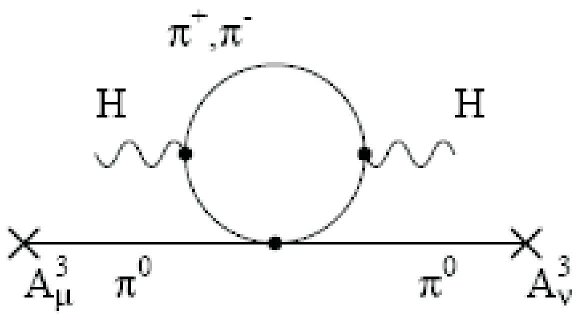
Where does the $\ln$ come from? Mass of pion in chiral limit is modified by terms like $\frac{F^2}{f_\pi^2}$. By analog with final-temperature massless theories, a power expansion is substituted by a mixed power-logarithm expansion.

m_{π^0}

Important: we stress here that m_{π^0} and m_{π^+} behave themselves drastically differently. Here we speak about m_{π^0}.

$$m_\pi^2(H) = m_\pi^2(H)\left(1 - \frac{eH\ln 2}{16\pi^2 f_\pi^2}\right)$$

This result has the same Euler—Heisenberg nature as the result for conbdensates. It comes from resummed diagrams like this



where $\pi^\pm$ are running in the loops

AdS_5 Geometry

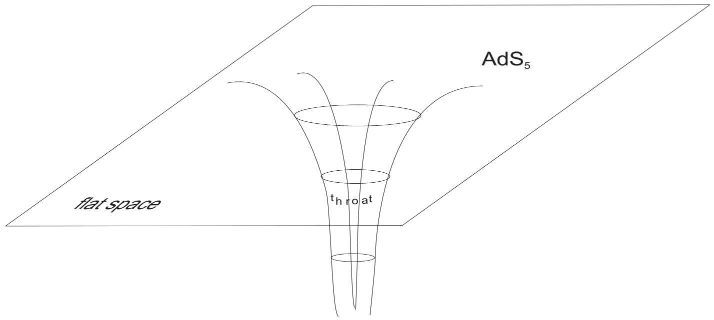
For review see *Gubser et al. [1999]*.
There is pack of N_c copies $D3$ branes, all placed into the same place in ten-dimensional spacetime. The branes being heavy act as a source term for (super)gravity equations of motion. When solved, they yield the $AdS_5 \times S^5$ metric with equal radii of the sphere and the AdS part

$$ds^2 = R^2 \left(\frac{dz^2 + dx^2}{z^2} + d\Omega_5^2 \right).$$

For $y = \frac{R^2}{z}$ the metric is re-written as

$$ds^2 = \left(1 + \frac{R^4}{y^4}\right)^{-\frac{1}{2}} dx^2 + \left(1 + \frac{R^4}{y^4}\right)^{\frac{1}{2}} \left(dy^2 + y^2 d\Omega_5^2\right)$$

which is illustrated in the figure below: it is flat at $y \to \infty$, and looks like a “throat” at $y \to 0$



Basics on AdS/CFT correspondence

For review see *D'Hoker, Freedman[2002]*, *Gubser et al. [1999]*. The AdS/CFT conjecture is:

$$Z_{SYM}[J] = Z_{string}[\Phi]$$

where partition function $Z_{SYM}[J]$ is calculated in presence of four-dimensional currents J, coupled to some operators O

$$Z_{SYM}[J] \equiv \langle e^{-(S+\sum JO_J)} \rangle,$$

and $Z_{string}[\Phi_{\partial AdS}]$ is defined as partition function of supergravity, where each field Φ^J in supergravity has its counterpart J — a current on the gauge theory side, which is related to the boundary value of the five-dimensional field as

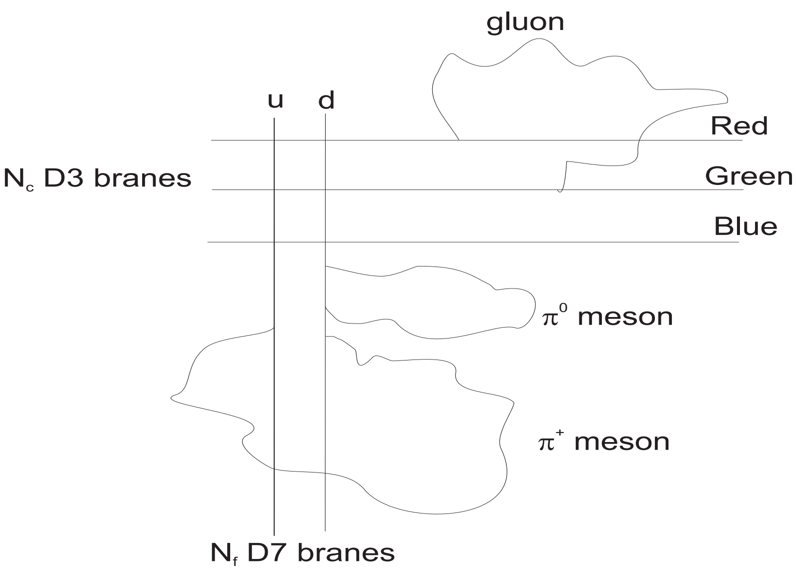
$$\Phi^J_{\partial AdS} \sim J.$$

Some examples of correspondence:

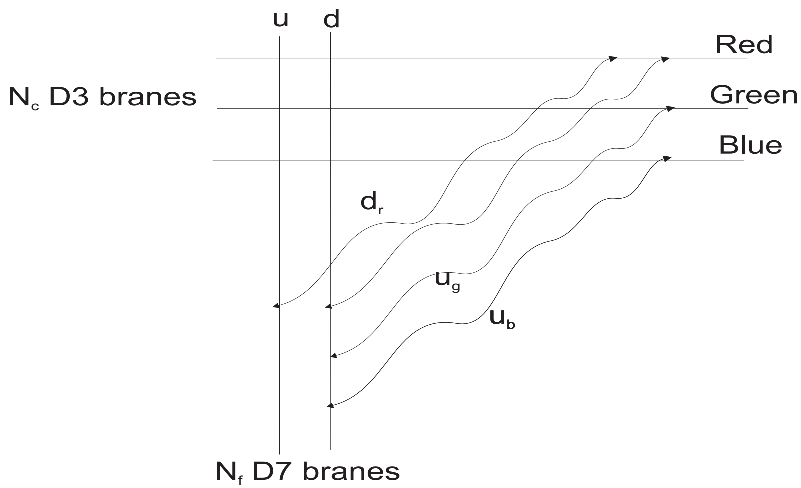
- Dilaton field ϕ in supergravity is dual to operator $\mathrm{tr} F^2$ in SYM.
- Graviton field $h_{\mu\nu}$ is dual to energy-momentum current $T_{\mu\nu}$ in SYM.

AdS/CFT with flavours

For review see: *Aharony, [2002]*; *Mateos [2007]*. General idea of introducing flavour into AdS/CFT is illustrated in the two pictures below:



(a)



(b)

We require $N_c \gg N_f$, for otherwise the stack of N_f $D7$ branes will deform the metric essentially. This is known as “quenched approximation”.

Holography with Flavours, External Fields

- *Karch, Katz [2002]*: Adding flavour to AdS/CFT.
- *Babington et al. [2004]*: Constable—Myers deformation.
- *Filev et al.[2007]*: Flavoured large N gauge theory in an external magnetic field.
- *Erdmenger, Meyer, Shock [2007]*: Pure AdS background in external fields.
- *Bergman et al. [2008]*: Phase transitions in Sakai/Sugimoto models due to electromagnetic fields:
- *Johnson, Kundu [2008]*: External Fields and Chiral Symmetry Breaking in the Sakai-Sugimoto Model.
- *Kim et al.[2008]*: Pair production in Sakai—Sugimoto model.

For a review see *Erdmenger et al. [2007]*.

Why Constable—Myers?

- There is no spontaneous chiral symmetry breaking in pure AdS, and the condensate is zero. However, there will be a condensate at non-zero Kalb—Ramond field B (see *Erdmenger, Meyer, Shock [2007]*).
- There is spontaneous symmetry breaking in AdS deformed a la Constable—Myers.

There are several reasons to study Constable—Myers background as a candidate for non-AdS/non-CFT correspondence, for it provides:

- Spontaneous CSB
- Conformal symmetry breaking
- Supersymmetry breaking

Constable—Myers Geometry

The Constable —Myers metric is organized as

$$ds^2 = H^{-\frac{1}{2}} \left(\frac{w^4+b^4}{w^4-b^4}\right)^{\frac{\delta}{4}} dx^2 + H^{\frac{1}{2}} \left(\frac{w^4+b^4}{w^4-b^4}\right)^{\frac{2-\delta}{4}} \frac{w^4-b^4}{w^4} \sum_{i=1}^{6} dw_i^2,$$

where $w^2 = \rho^2 + L^2$, $\rho^2 = w_1^2 + w_2^2 + w_3^2 + w_4^2$, $L^2 = w_5^2 + w_6^2$,

$$H = \left(\frac{w^4+b^4}{w^4-b^4}\right)^{\delta} - 1$$

and the dilaton is

$$e^{2\phi} = e^{2\phi_0} \left(\frac{w^4+b^4}{w^4-b^4}\right)^{\Delta}$$

and a C_4 form field

$$C_{(4)} = -\frac{1}{4} H^{-1} dx_0 \wedge dx_1 \wedge dx_2 \wedge dx_3,$$

with conditions imposed upon deformation parameters $\Delta^2 + \delta^2 = 10,\ \delta = \frac{1}{2b^4}$.

D3/D7 Model in Constable—Myers

$D7$ brane does not change the metric in the quenched approximation. The dynamics of the brane is described by a Dirac—Born—Infeld action

$$S_{D7} = \mu_7 \int d^8\xi \sqrt{\det_{\alpha,\beta}\left(2\pi\alpha' B_{\alpha\beta} + 2\pi\alpha' F_{\alpha\beta} + g_{\mu\nu}\frac{\partial X^\mu}{\partial\xi^\alpha}\frac{\partial X^\nu}{\partial\xi^\beta}\right)} + \int d^8\xi C_4 \wedge F \wedge B$$

D3 and D7 branes run as follows:

	0	1	2	3	4	5	6	7	8	9
D3	+	+	+	+	-	-	-	-	-	-
D8	+	+	+	+	+	+	+	+	-	-

$\xi_1 \ldots \xi_8$ — internal coordinates of the world-volume. Let us search for an embedding like $w_5 = w(\rho), w_6 = 0$, where $\rho = \sqrt{w_1^2 + w_2^2 + w_3^2 + w_4^2}$. With such an Ansatz and in the metric given above, the DBI action is organized as

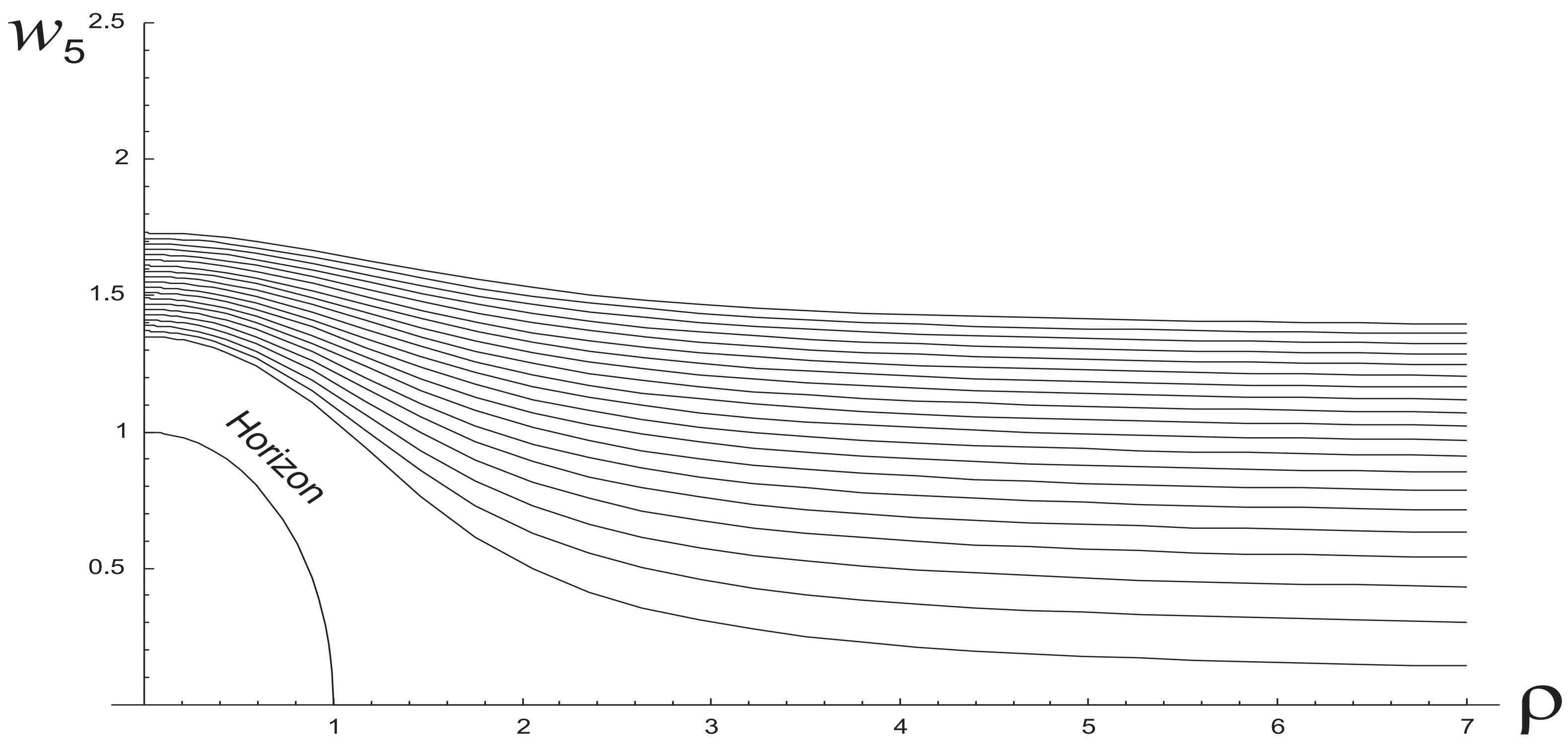
$$S = -\mu_7 \int d^8\xi G(\rho, w)\sqrt{1 + w'^2(\rho)}\sqrt{1 + B^2/g_{11}^2} + S_{\text{Chern—Simons}},$$

where $G(\rho, w) = \rho^3 \frac{((\rho^2+w^2)^2+b^4)((\rho^2+w^2)^2-b^4)}{(\rho^2+w^2)^4} e^{2\phi}$. The equations of motion will look like

$$\frac{d}{d\rho}\left(\frac{Gw'}{\sqrt{1+w'^2}}\sqrt{1 + B^2/g_{11}^2}\right) - \sqrt{1 + w'^2}\frac{d}{dw}\left(G\sqrt{1 + B^2/g_{11}^2}\right) = 0$$

Classical Solutions

Here we show how the classical solutions behave in our setting



Regular solutions of equations of motion

A family of embeddings of the spectator D7 brane into Constable—Myers background.
The asymptotes of solutions will yield us vacuum parameters.

Condensate and Quark Masses

General AdS/CFT prescription *(D'Hoker, Freedman [2002])*: For a 10-dimensional bulk field ϕ the solution to field equations is a combination of two modes

$$\phi = \begin{cases} z_0^{\Delta}, \text{normalizable} \\ z_0^{4-\Delta}, \text{non-normalizable} \end{cases}$$

The boundary values of the non-normalizable mode in the bulk are related to the field values on the boundary as

$$J(\vec{x}) = \lim_{z_0 \to 0} z_0^{4-\Delta} \phi(z_0, \vec{x})$$

and thus a correlator of fields

$$< O(x)_J O_J(0) >= \frac{\delta^2 S_{bulk}}{\delta J(x) \delta J(0)}.$$

The normalizable mode of the field ϕ_J is related to the VEV of operator O_J.

Dependence of condensate on mass

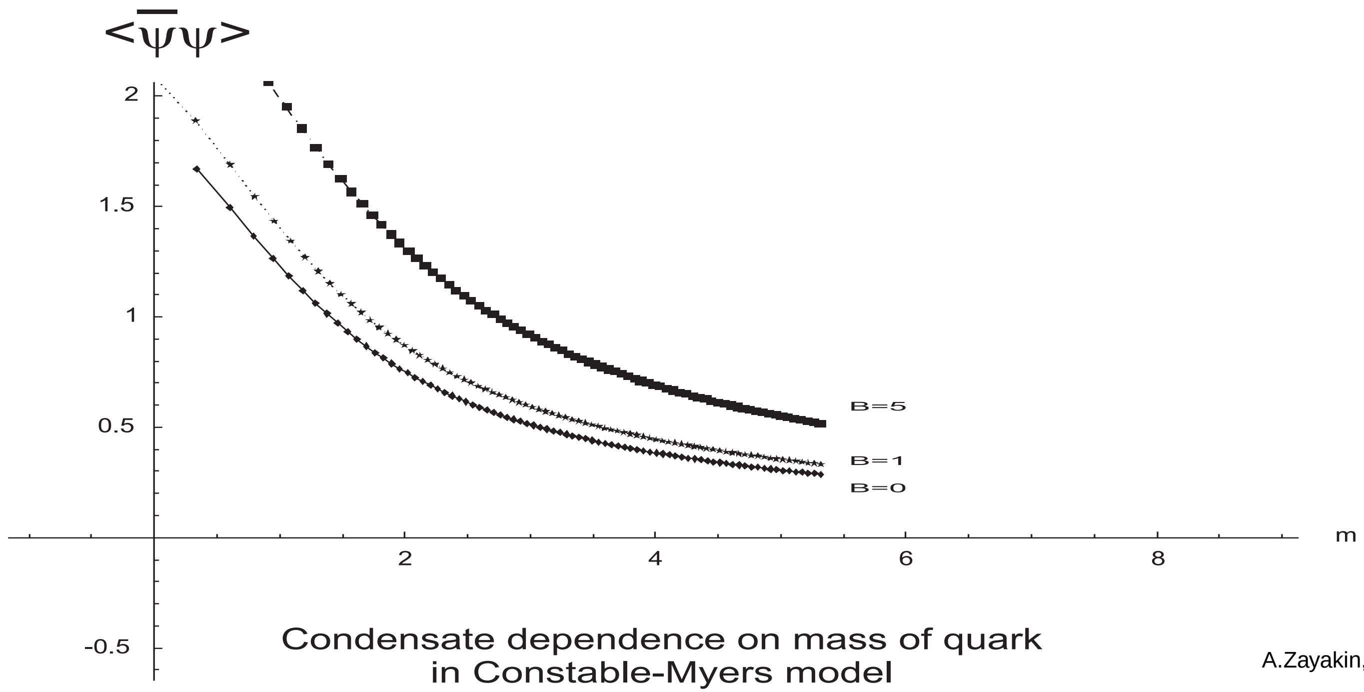
For D3/D7 the classical solution contains information on quark mass and vacuum condensate:

$$w(\rho) = m + \frac{c}{\rho^2}.$$

The parameters m and c correspond to quark mass and chiral condensate:

$$m_q = \frac{m}{2\pi\alpha'},$$
$$\langle \bar{q}q \rangle = \frac{c}{(2\pi\alpha')^3},$$

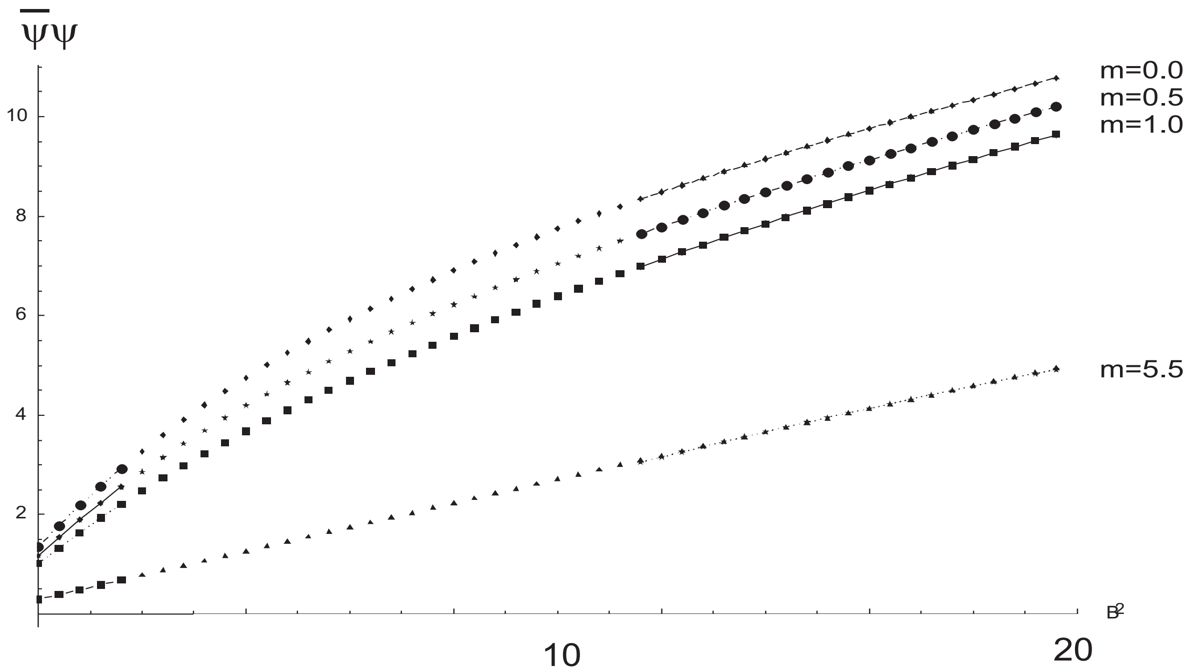
Physical solutions are found by imposing $w'(0) = 0$ and $w(0) = w_0 = const$. By studying a family of solutions one can obtain for any magnetic field a set of $(m, <\overline{\psi}\psi>)$ pairs.



Condensate dependence on mass of quark
in Constable-Myers model

Different regimes

Approximation by either linear or quadratic dependence is valid for large or small fields respec



Condensate dependence on the square of the magnetic field; continuous lines show approximation for small and large field limits:

$$\langle\overline{q}q\rangle_B \sim \begin{cases} \langle\overline{q}q\rangle_{B=0}(1+c_1(m)B^2),\ B \ll 1 \\ \langle\overline{q}q\rangle_{B=0}(1+c_2(m)B), B \gg 1. \end{cases}$$

Fluctuations

Meson fields are fluctuations around classical solutions. Their eigenvalues are meson masses. Following correspondence can be made:

Goldstone meson analogous to η' (massless at $N_c \to \infty$)	$\delta\phi$
Non-goldstone scalar meson	δL
Vector meson	δA_μ

The equations for e.g. $f(\rho) = \delta L(\rho)$ will look like:

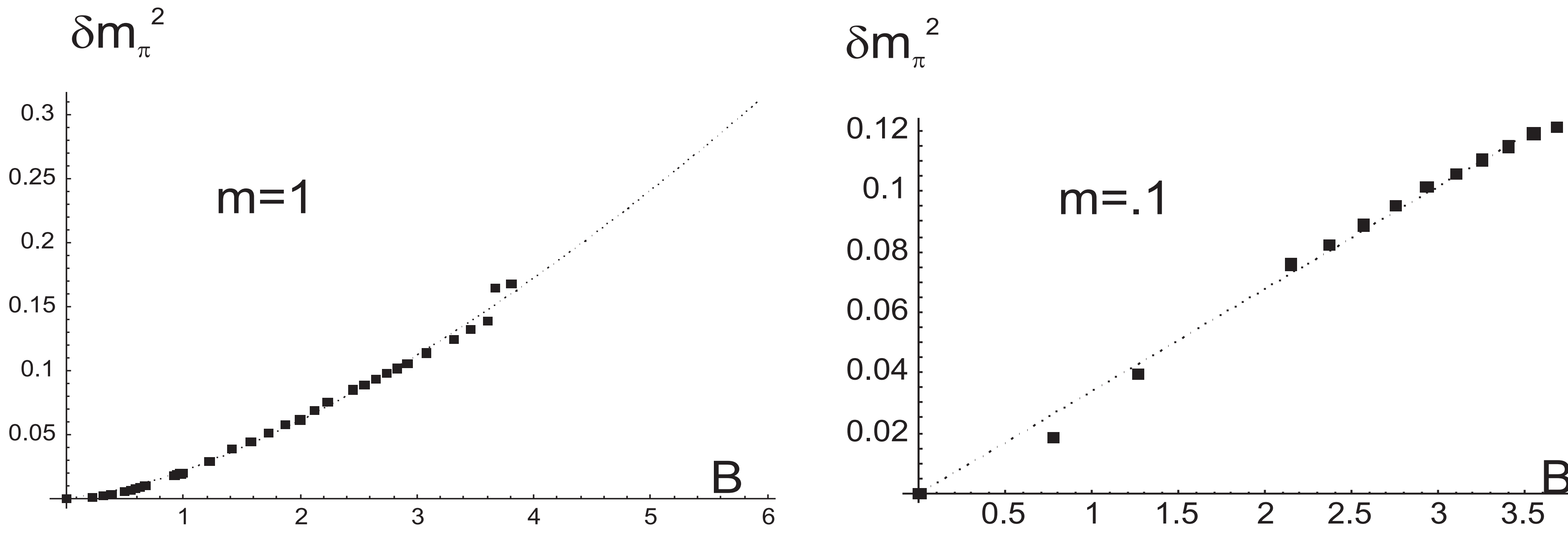
$$\frac{d}{d\rho}\left[\frac{G\sqrt{1+B^2/g_{11}^2}}{\sqrt{1+w'^2}}\partial_\rho f(\rho)\right] + M^2\frac{G\sqrt{1+B^2/g_{11}^2}}{\sqrt{1+w'^2}}H\left(\frac{(\rho^2+w^2)^2+b^4}{(\rho^2+w^2)^2-b^4}\right)^{\frac{1-\delta}{2}}\frac{(\rho^2+w^2)^2-b^4}{(\rho^2+w^2)^2}f(\rho)$$
$$-\sqrt{1+w'^2}\sqrt{1+B^2/g_{11}^2}\frac{4b^4\rho^3}{(\rho^2+w^2)^5}\left(\frac{(\rho^2+w^2)^2+b^4}{(\rho^2+w^2)^2-b^4}\right)^{\frac{\Delta}{2}}\left(2b^4-\Delta(\rho^2+w^2)^2\right)f(\rho) = 0.$$

where boundary conditions are:

$$\begin{cases} f'(0) = 0 \\ f(\rho)|_{\rho\to\infty} \to \frac{1}{\rho^2}. \end{cases}$$

Mass Spectra

Below we show $\frac{m_\pi^2(B)-m_\pi^2(0)}{m_\pi^2(0)}$, obtained numerically. Note that approximation of it by expected linear or quadratic dependence is not satisfactory, unlike some other power laws



Spectra of m_π^2 as functions of B. Continuous lines on the left ($m = 1$) and right ($m = 0.1$) plots show interpolation $\delta m_\pi^2 = \alpha B^{3/2}$ and $\delta m_\pi^2 = \alpha B^{1/2}$ respectively.

Conclusions

- Linear growth of condensate has not been found for small fields, but quadratic dependence observed in that domain.
- Linear growth of mass has not been observed as well.

Thus the model considered in this paper could perhaps restore some of the dynamics of Nambu—Jona—Lasinio model, rather than true non-perturbative QCD. More elaborated metrics must be used to overcome those problems.

Perspectives

- Meson decay constants from holography
- Validity check of Gell-Mann—Oakes—Renner relation in holography.
- Dependence on temperature, chemical potential, external electric and magnetic field.
- Search of more realistic backgrounds to reproduce QCD

Work in progress in these directions.

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