Direct CPV and extraction of angle α: $B \to \pi\pi, \rho\rho, \rho\pi, K\pi$ decays –

Updated talk in Repino

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PLAN

- importance of FSI phases in $B \to M_1 M_2$ decays
- $B \to \pi\pi$ and $B \to \rho\rho$ branching ratios: absence of color suppression of $B \to \pi^0\pi^0$ decay probability
- phenomenology
- FSI phases for $\pi\pi$ and $\rho\rho$ from experimental data
- a model for FSI phases for B decays into 2 light mesons
- origin of the FSI phases difference
- "description" of CP asymmetries in $B \to \pi^+\pi^-$ decay, UT angle α
- prediction of large CP asymmetries in $B \to \pi^0\pi^0$ decay
- conclusions

why FSI are interesting

1. to predict/explain ratios of branching ratios,
$\pi\pi, \rho\rho$ is very spectacular example

2. to study strong interactions

3. DCPV:
$C \sim \sin\alpha \sin\delta$,
so: to forsee in what decays CPV is large,
or even to detemine α from the measured value of C

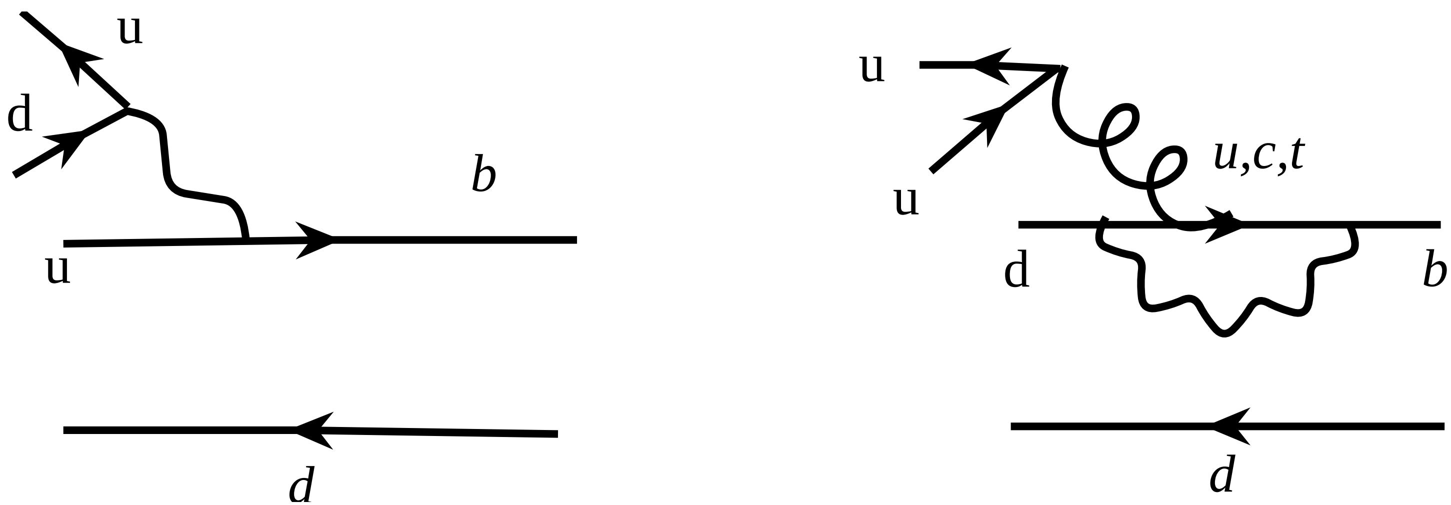
branching ratios

Mode	$\mathrm{Br}(10^{-6})$	Mode	$\mathrm{Br}(10^{-6})$
$B_d \to \pi^+\pi^-$	5.2 ± 0.2	$B_d \to \rho^+\rho^-$	23.1 ± 3.3
$B_d \to \pi^0\pi^0$	1.3 ± 0.2	$B_d \to \rho^0\rho^0$	0.7 ± 0.3
$B_u \to \pi^+\pi^0$	5.7 ± 0.4	$B_u \to \rho^+\rho^0$	18.2 ± 3.0

C-averaged branching ratios of $B \to \pi\pi$ and $B \to \rho\rho$ decays.

no color suppression (naive factor $1/3^2/2 = 1/18$ in decay probability) of $\pi^0\pi^0$ mode

diagrams



"t-convention" for penguin amplitudes:

$$(V_{ub}V_{ud}^* + V_{cb}V_{cd}^* + V_{tb}V_{td}^*) f(m_c/M_W) = 0$$

is subtracted from decay amplitudes;

in this convention CKM phases difference of T and P amplitudes is $\alpha \approx 90^o$ that is why they do not interfere in C-averaged decay probabilities

phenomenology

$$
\begin{aligned}
M_{\bar{B}_d \to \pi^+\pi^-} &= \frac{G_F}{\sqrt{2}} |V_{ub}V_{ud}^*| m_B^2 f_\pi f_+(0) \left\{ e^{-i\gamma} \frac{1}{2\sqrt{3}} A_2 e^{i\delta_2^\pi} + \right. \\
&+ \left. e^{-i\gamma} \frac{1}{\sqrt{6}} A_0 e^{i\delta_0^\pi} + \left| \frac{V_{td}^* V_{tb}}{V_{ub} V_{ud}^*} \right| e^{i\beta} P e^{i(\delta_P^\pi + \tilde{\delta}_0^\pi)} \right\} ,
\end{aligned}
$$

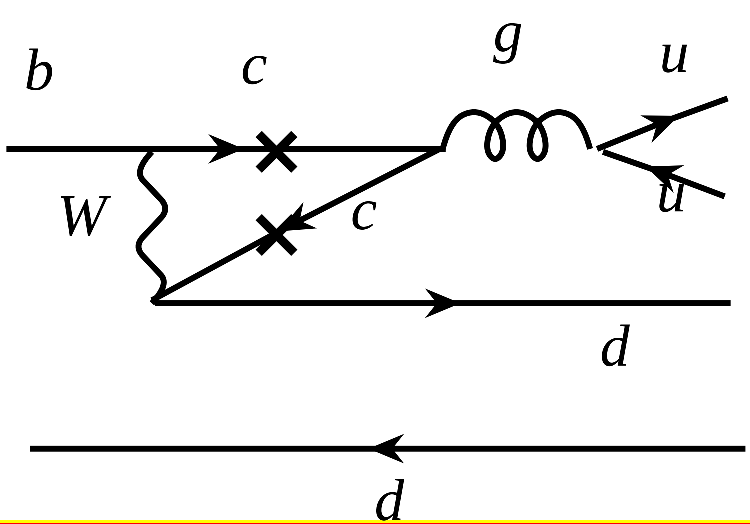
$$
\begin{aligned}
M_{\bar{B}_d \to \pi^0\pi^0} &= \frac{G_F}{\sqrt{2}} |V_{ub}V_{ud}^*| m_B^2 f_\pi f_+(0) \left\{ e^{-i\gamma} \frac{1}{\sqrt{3}} A_2 e^{i\delta_2^\pi} - \right. \\
&- \left. e^{-i\gamma} \frac{1}{\sqrt{6}} A_0 e^{i\delta_0^\pi} - \left| \frac{V_{td}^* V_{tb}}{V_{ub} V_{ud}^*} \right| e^{i\beta} P e^{i(\delta_P^\pi + \tilde{\delta}_0^\pi)} \right\} ,
\end{aligned}
$$

$$
M_{\bar{B}_u \to \pi^-\pi^0} = \frac{G_F}{\sqrt{2}} |V_{ub}V_{ud}^*| m_B^2 f_\pi f_+(0) \left\{ \frac{\sqrt{3}}{2\sqrt{2}} e^{-i\gamma} A_2 e^{i\delta_2^\pi} \right\} .
$$

why all δ are different

There are meson resonances in s-channel with $I = 0$, but not with $I = 2$ - a trivial reason why δ_2 and δ_0 are different.

penguin amplitude has 2 strong phases:
δ_P^π which comes from zig-zag diagram and depends on quarks distribution in pion; estimate of imaginary part of charmed penguin gives $\delta_P^\pi \approx 10^o$.



The reason why $\tilde{\delta}_0^\pi$ differs from δ_0^π is more involved: both originate from rescattering of light hadrons at large distances.

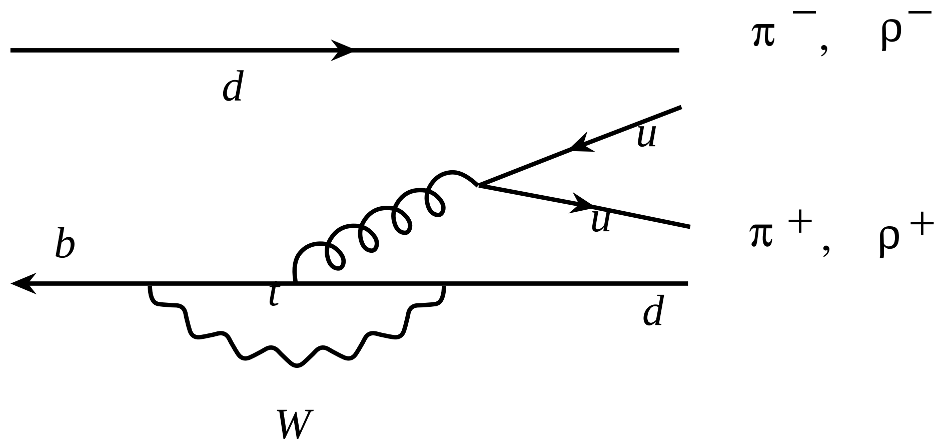
I will demonstrate below that large part of δ_0^π comes from $\rho^+\rho^-$ intermediate state, being proportional to $\sqrt{(\mathrm{Br} B_d \to \rho^+\rho^-)_T/(\mathrm{Br} B_d \to \pi^+\pi^-)_T} \approx \sqrt{(\mathrm{Br} B_d \to \rho^+\rho^-)/(\mathrm{Br} B_d \to \pi^+\pi^-)} \approx 2.1$

quite opposite production of vector mesons by penguin operator is suppressed:
$$\sqrt{(\mathrm{Br} B_d \to \rho^+\rho^-)_P/(\mathrm{Br} B_d \to \pi^+\pi^-)_P} \approx$$
$$\sqrt{(\mathrm{Br} B_u \to K^{0*}\rho^+)/(\mathrm{Br} B_u \to K^0\pi^+)} \approx 0.76$$

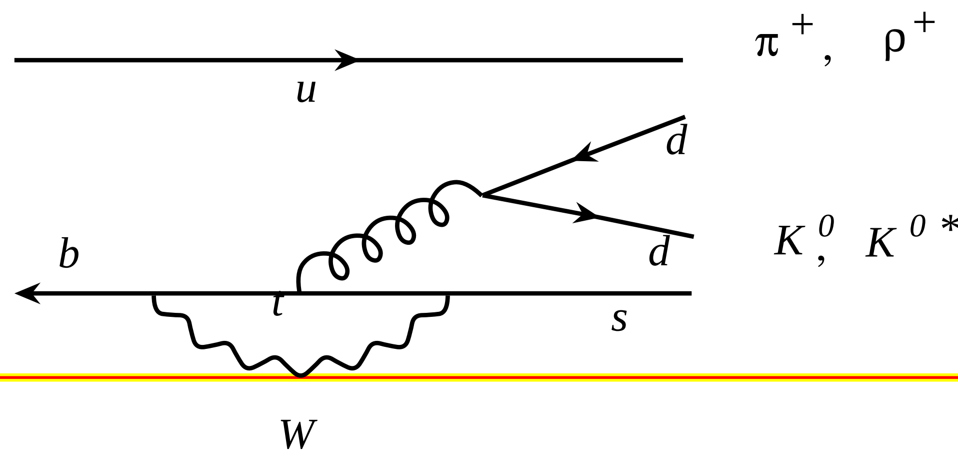
(On quarks: $\bar{u}\gamma_\mu u$ is a sum of left and right currents; the latter after Fierz transformation generate operator $\bar{u}_R d_L$ which can not produce vector ρ-meson from vacuum.)

that is why $(\tilde{\delta}_0^\pi)_{\rho\rho} \approx 1/2.8(\delta_0^\pi)_{\rho\rho}$

Thus it was shown that all four δ's are different.



d
b
t
W
u
u
d
π⁻, ρ⁻
π⁺, ρ⁺



u
b
t
W
d
d
s
π⁺, ρ⁺
K⁰, K⁰*

penguins

$$\mathrm{Br}(B_d \to \rho^+\rho^-)_P = \left(\frac{f_\rho}{f_{K^*}}\right)^2 \left[\lambda\sqrt{\eta^2 + (1-\rho)^2}\right]^2 \frac{\tau_{B_d}}{\tau_{B_u}} \mathrm{Br}(K^{0*}\rho^+) \approx$$

$$\approx 0.34 \cdot 10^{-6}$$

$$\mathrm{Br}(B_d \to \pi^+\pi^-)_P = \left(\frac{f_\pi}{f_K}\right)^2 \left[\lambda\sqrt{\eta^2 + (1-\rho)^2}\right]^2 \frac{\tau_{B_d}}{\tau_{B_u}} \mathrm{Br}(K^0\pi^+) \approx$$

$$\approx 0.59 \cdot 10^{-6} \quad .$$

$(\delta_0 - \delta_2)^{\pi\pi}$ from experimental data

$B \to \pi\pi$ neglecting penguin
3 branching ratios $\Longrightarrow$ 3 parameters $A_0, A_2, |\delta_0 - \delta_2|$

$$\cos(\delta_0^\pi - \delta_2^\pi) = \frac{\sqrt{3}}{4} \frac{\mathrm{B}_{+-} - 2B_{00} + \frac{2}{3}\frac{\tau_0}{\tau_+}B_{+0}}{\sqrt{\frac{\tau_0}{\tau_+}B_{+0}}\sqrt{B_{+-} + B_{00} - \frac{2}{3}\frac{\tau_0}{\tau_+}B_{+0}}} \ ,$$

$|\delta_0^\pi - \delta_2^\pi| = 48^o$
$B \to \pi\pi$ taking penguin into account
tree-penguin interference term cancels in C-averaged Br's for $\alpha = 90^o$; P^2 term we extract from Br$(K^0\pi^+)$: Br$(B_d \to \pi^+\pi^-)_P \approx 0.59 \cdot 10^{-6}$; subtracting it from experimental data we get: $|\delta_0^\pi - \delta_2^\pi| = 37^o \pm 10^o$

recent fit of $B \to \pi\pi, \pi K$ data (Chiang, Zhou): $\delta_0^\pi - \delta_2^\pi = 40^o \pm 7^o$

$(\delta_0 - \delta_2)^{\rho\rho}$ from experimental data

there are 3 polarization amplitudes for 2 vector mesons production in B decays making complete analysis highly nontrivial. Fortunately ρ-mesons produced in B decays are almost completely longitudinal polarized - analysis goes just like for π-mesons:
$|\delta_0^\rho - \delta_2^\rho| = 20^o + 8^o - 20^o$, one sigma from zero (unlike pion case)
This difference of FSI phases is responsible for different patterns of $B \to \pi\pi$ and $B \to \rho\rho$ decay probabilities.

We want to understand why FSI phases are large in $B \to \pi\pi$ amplitudes but small in $B \to \rho\rho$ amplitudes.

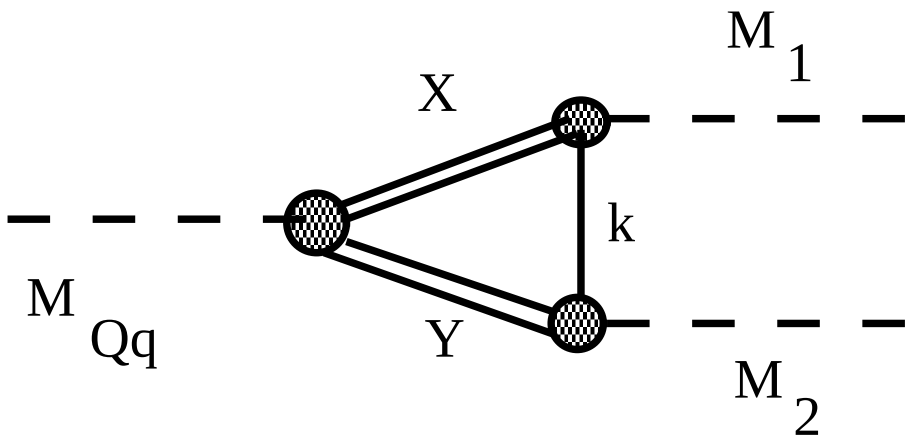
which intermediate states matter

$b \to u\bar{u}d$ decay produce mostly 3 isotropically oriented jets of light mesons, each having about 1.5 GeV energy. In e^+e^- annihilation at 3 GeV c.m. energy average charged particles (pions) multiplicity is about 4 - so, taking π^0's into account in B-mesons decays to light quarks in average 9 "pions" are produced, flying in 3 widely separated directions (or almost isotropically, taken transverse momentum into account). Branching ratio of such decays is large, about 10^{-2}. However such states *NEVER* rescatter into 2 pions or 2 ρ-mesons.

Which intermediate states will transform into two mesons final state we easily understand studing inverse process of 2 light mesons scattering at 5 GeV c.m. energy. In this process 2 jets of particles moving in the directions of initial particles are formed. Energy of each jet is $M_B/2$, while its invariant mass squared is not more than $M_B\Lambda_{QCD}$.

Following these arguments in the calculation of the imaginary parts of decay amplitudes we will take two particle intermediate states into account, to which branching ratios of B-mesons are maximal.

model for FSI phases



$$\int dk_0 dk_z = 1/(2 \cdot M_B^2) \int ds_X ds_Y$$

Hadron dynamics: integrals over s rapidly decrease with s increase since only low mass clusters contribute into amplitude of 2 meson production. In this way we get:

$$M_{\pi\pi}^I = M_{XY}^{(0)I} (\delta_{\pi X} \delta_{\pi Y} + i T_{XY \to \pi\pi}^{J=0})$$

$\rho\rho$

Since Br$B \to \rho\rho$ is large it contributes a lot in FSI phase of $B \to \pi\pi$ decay; NOT VICE VERSA!
$B \to \rho\rho \to \pi\pi$ chain with the help of unitarity relation; for small t we can trust elementary π-meson exhange in t-channel.

$$\mathrm{Im}M(B \to \pi\pi) = \int \frac{d\cos\theta}{32\pi} M(\rho\rho \to \pi\pi) M^*(B \to \rho\rho)$$

Introducing f/f $exp(t/\mu^2)$ for $\mu^2 = 2m_\rho^2$ we obtain:

$\delta_0^\pi(\rho\rho) = 15^o \ , \delta_2^\pi(\rho\rho) = -5^o \ \ , \delta_0^\pi(\rho\rho) - \delta_2^\pi(\rho\rho) = 20^o$
and half of experimentally observed phase difference is explained.

Let us stress, that $\delta_I^\pi(\rho\rho) \sim 1/M_B \to 0$

It is remarkable that FSI phases generated by $B \to \pi\pi \to \rho\rho$ chain are damped by $\mathrm{Br}(B \to \rho^+\rho^-, \rho^+\rho^0)/\mathrm{Br}(B \to \pi^+\pi, \pi^+\pi^0)$ ratios and are a few degrees:

$$\delta_0^\rho(\pi\pi) - \delta_2^\rho(\pi\pi) \approx 4^o$$

$\pi\pi$

for $\pi\pi$ intermediate state we take Regge model expression for $T_{\pi\pi\to\pi\pi}$, which takes into account pomeron, ρ and f trajectories exchange. Pomeron exchange produce imaginary T and do not contribute to phase shifts as far as it is critical, $\alpha_P(0)=1$. However, for the amplitude of the supercritical P exchange we have:
$T \sim (s/s_0)^{\alpha_P(t)}(1+exp(-i\pi\alpha_P(t)))/(-\sin(\pi\alpha_P(t))) =$
$(s/s_0)^{(1+\Delta)}(i+\Delta\pi/2)$, where in the last expression $t=0$ was substituted and $\alpha_P(0)=1+\Delta, \Delta \approx 0.1$ was used.

$$\delta_0^{\pi}(\pi\pi) = 5.0^o \ , \ \delta_2^{\pi}(\pi\pi) = 0^o$$

πa_1

πa_1 intermediate state should be accounted for as well. Large branching ratio of $B_d \to \pi^{\pm} a_1^{\mp}$-decay ($\mathrm{Br}(B_d \to \pi^{\pm} a_1^{\mp}) = (40 \pm 4) * 10^{-6}$) is partially compensated by small $\rho\pi a_1$ coupling constant (it is $1/3$ of $\rho\pi\pi$ one):

$$\delta_0^{\pi}(\pi a_1) = 4^o \; , \; \delta_2^{\pi}(\pi a_1) = -2^o \quad ,$$

where we assume that the sign of πa_1 contribution into phases difference is the same as that of the elastic channel.

Finally: $\delta_0^{\pi} = 23^o \; , \quad \delta_2^{\pi} = -7^o \quad , \quad \delta_0^{\pi} - \delta_2^{\pi} = 30^o \quad$,

and the accuracy of this number is not high.

δ_P^π in Regge model

charmed mesons intermediate states:
$B \to \bar{D}D, \bar{D}^*D, \bar{D}D^*, \bar{D}^*D^* \to \pi\pi$.
In Regge model all these amplitudes are described at high energies by exchanges of $D^*(D_2^*)$-trajectories.
An intercept of these exchange-degenerate trajectories can be obtained from masses of $D^*(2007)$ – 1^- and $D_2^*(2460)$ – 2^+ resonances, assuming linearity of Regge-trajectories: $\alpha_{D^*}(0) = -1$ and the slope $\alpha'_{D^*} \approx 0.5 GeV^{-2}$.
However since at large t the slope growth to universal value $\approx 1 GeV^2$ it is natural to suppose that for small t it diminishes and as a result $\alpha_{D^*}(0) = -0.7$ or even larger.

$$T_{D\bar{D}\to\pi\pi}(s,t) = -\frac{2e^{-i\pi\alpha(t)}}{\pi} g_0^2 \Gamma(1-\alpha_{D^*}(t))(s/s_{cd})^{\alpha_{D^*}(t)} \quad ,$$

$s_{cd} \approx 2.2 GeV^2$, the sign of the amplitude is fixed by the unitarity in the t-channel (close to the D^*-resonance). The constant g_0^2 is determined by the width of the $D^* \to D\pi$ decay: $g_0^2/(16\pi) = 6.6$. For $\alpha_{D^*}(0) = -0.7$, $\alpha'_{D^*} \approx 0.5 GeV^{-2}$ we obtain:

$\delta_P^\pi \approx -5^o$. A smallness of the phase is due to the low intercept of D^*-trajectory. The sign of δ_P is negative - opposite to the positive sign which was obtained in perturbation theory.

Since $D\bar{D}$-decay channel constitutes only $\approx 10\%$ of all two-body charm-anticharm decays of B_d-meson taking these channels into account we can easily get

$$\delta_P \sim -10^o \ \ ,$$

which may be very important for the interpretation of the experimental data on direct CP asymmetry C_{+-}.

CPV in $B_d(\bar{B}_d) \to \pi^+\pi^-$

$$\begin{aligned}
C_{+-} &= -\frac{\tilde{P}}{\sqrt{3}} \sin\alpha [\sqrt{2} A_0 \sin(\delta_0 - \tilde{\delta}_0 - \delta_P) + A_2 \sin(\delta_2 - \tilde{\delta}_0 - \delta_P)] \\
/ & \quad [\frac{A_0^2}{6} + \frac{A_2^2}{12} + \frac{A_0 A_2}{3\sqrt{2}} \cos(\delta_0 - \delta_2) - \\
- & \quad \sqrt{\frac{2}{3}} A_0 \tilde{P} \cos\alpha \cos(\delta_0 - \tilde{\delta}_0 - \delta_P) - \\
- & \quad \frac{A_2 \tilde{P}}{\sqrt{3}} \cos\alpha \cos(\delta_2 - \tilde{\delta}_0 - \delta_P) + \tilde{P}^2] \quad ,
\end{aligned}$$

where

$$\tilde{P} \equiv \left| \frac{V_{td}^* V_{tb}}{V_{ub} V_{ud}^*} \right| P \quad .$$

$$\frac{\mathrm{Br}(B_u \to K^0\pi^+)}{\mathrm{Br}(B_u \to \pi^0\pi^+)} = \frac{f_K^2 P^2 |V_{ts}^* V_{tb}|^2}{f_\pi^2 \frac{3}{8} A_2^2 |V_{ud}^* V_{ub}|^2} \ ,$$

$$\frac{A_0}{A_2} = 0.80 \pm 0.09 \ , \frac{\tilde{P}}{A_2} = 0.21(0.04) \ ,$$

$C_{+-} \approx -0.56 \sin((\delta_0 + \delta_2)/2 - \tilde{\delta}_0 - \delta_P)$
Here are experimental results:
$C_{+-}^{Belle} = -0.55(0.09)$, $C_{+-}^{BABAR} = -0.16(0.11)$, now $C_{+-}^{BABAR} = -0.21(0.09)$, 2.5 standard deviations difference.

"Our" values: $\delta_0 = 37^o, \delta_2 = 0^o$, neglecting small values of $\tilde{\delta}_0$ and δ_P: $C_{+-} \approx -0.18$.

With nonperturbative penguin phase $\delta_P = -10^o$, $\delta_0 = 30^o, \delta_2 = -7^o$ we obtain $C_{+-} \approx -0.21$.

It is instructive to compare the obtained numbers with the value of C_{+-} which follows from the asymmetry $A_{CP}(K^+\pi^-)$ if $d \leftrightarrow s$ symmetry is supposed :

$$\begin{aligned} C_{+-} &= \left(\frac{f_\pi}{f_K}\right)^2 A_{CP}(K^+\pi^-)\frac{\Gamma(B\to K^+\pi^-)}{\Gamma(B\to\pi^+\pi^-)}\frac{\sin(\beta+\gamma)}{\sin(\gamma)}\left|\frac{V_{td}}{V_{ts}\lambda}\right| = \\ &= 1.2^{(-2)}(-0.093\pm 0.015)\frac{19.8}{5.2}\frac{\sin 82^o}{\sin 60^o}0.87 = -0.24\pm 0.04 \quad . \end{aligned}$$

S_{+-}

from experimental data (now BABAR and Belle agree in S_{+-}): $S_{+-}^{exper} = -0.62 \pm 0.09$.
Neglecting the penguin contribution we get:

$$S_{+-} = \sin 2\alpha^{\mathrm{T}} \ \ , \alpha^{\mathrm{T}} = 108^o \pm 3^o \ \ .$$

Penguin shifts the value of α. The accurate formula looks like:

$$\begin{aligned} S_{+-} &= [\sin 2\alpha(\frac{A_0^2}{6} + \frac{A_2^2}{12} + \frac{A_0 A_2}{3\sqrt{2}}\cos(\delta_0 - \delta_2)) - \\ &- \frac{A_2\tilde{P}}{\sqrt{3}}\sin\alpha\cos(\delta_2 - \tilde{\delta}_0 - \delta_P) - \sqrt{\frac{2}{3}}A_0\tilde{P}\sin\alpha\cos(\delta_0 - \tilde{\delta}_0 - \\ &/ \ \ [\frac{A_0^2}{6} + \frac{A_2^2}{12} + \frac{A_0 A_2}{3\sqrt{2}}\cos(\delta_0 - \delta_2) - \sqrt{\frac{2}{3}}A_0\tilde{P}\cos\alpha\cos(\delta_0 - \tilde{\delta} \end{aligned}$$

$$- \quad \frac{A_2 \tilde{P}}{\sqrt{3}} \cos\alpha \cos(\delta_2 - \tilde{\delta}_0 - \delta_P) + \tilde{P}^2],$$

taking $\delta_0 = 30^o, \delta_2 = -7^o$ and neglecting $\tilde{\delta}_0$ and δ_P we get:
$\alpha_{\pi\pi} = 88^o \pm 4^0(ex) \pm 10^0(th; P; d - s)$
UT fit: $\alpha^{CKMfitter} = (99.0^{+4.0}_{-9.4})^o$, $\alpha^{UTfit} = (93 \pm 4)^o$

The relative smallness of penguin contribution to $B \to \rho\rho$ decay amplitudes allow us to determine α with better theoretical accuracy: $(S_{+-}^{exper})^{\rho\rho} = -0.06 \pm 0.18$

$$(\alpha)_{\rho\rho} = 87^o \pm 5^0(ex) \pm 3^0(th; P; d - s) \quad .$$

CPV in $B_d(\bar{B}_d) \to \pi^0\pi^0$

From analogous formulas we get predictions for large CPV in this decay:

$C_{00} \approx -0.60, \quad S_{00} = 0.70 \pm 0.15$

experimental measurement of C_{00} has large uncertainty:

$C_{00}^{exper} = -0.36(0.32)$, while S_{00} is still not measured.

$\rho\pi$

$Br B_d(\bar{B}_d) \to \to \rho^{\pm}\pi^{\mp}$	$A_{\mathrm{CP}}^{\rho\pi}$	$C_{\rho\pi}$	$\Delta C_{\rho\pi}$	$S_{\rho\pi}$	$\Delta S_{\rho\pi}$
$(23.1 \pm 2.7)10^{-6}$	-0.13 ± 0.04	0.01 ± 0.07	0.37 ± 0.08	0.01 ± 0.09	-0.04 ± 0.10

Consideration similar to that for $\pi\pi$ and $\rho\rho$ decays was performed.
Small penguin and high accuracy of $S_{\rho\pi}$ allow record accuracy:

$$\alpha_{\rho\pi} = 84^o \pm 3^o(\exp) \pm 3^o(\mathrm{theor}) \quad .$$

Conclusions

- a model of FSI in $B \to M_1 M_2$ decays is suggested;
- the model explains the absence of color suppression of $B \to \pi^0\pi^0$ decay;
- relatively small $B \to \pi^+\pi^-$ branching ratio is the reason why $B \to \rho^0\rho^0$ mode remains small;
- $B \to \pi^+\pi^-$: we can not reproduce C_{+-} value measured by Belle, while BABAR result is much more acceptable;
- $\alpha = 88^o \pm 4^0(exper) \pm 5^0(theor)$ from $S_{+-}^{exper} = -0.62 \pm 0.09$
- and predict almost maximal CPV in $B(\bar{B}) \to \pi^0\pi^0$ decays:
  $C_{00} \approx -0.60 \quad S_{00} \approx 0.70$