

The coupled channel analysis of D_s and B_s mesons

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Introduction

In most cases the quark model predicts meson states in good agreement with experiment. And we believe that we have a theory of bound states in QCD, working well in general. However, there are exclusions. Some states are shifted from the places prescribed by this theory. Some examples for mesons:

- a) pions and kaons
- b) some heavy-light mesons, like D_s
- c) some light scalar mesons, like a_0, f_0 .

In baryons, examples include $N(1440)$ (*Roper*), $\Lambda(1405)$, etc. We expect that in some of these cases the Channel Coupling interaction is operating and explains the strong shift of masses. Moreover, we suggest a new mechanism, called the chiral coupling of channels, which works for a bound light quark and can explain the shift of $\gtrsim 100$ MeV in a mass. It is important, that this new mechanism does not contain any fitting parameters and its predictions are fixed in some sense.

We use the quark-meson Lagrangian

$$L = i \int d^4x d^4y \psi^\dagger(x) \widehat{M}(x, y) \psi(y)$$

which includes both effect of confinement and chiral symmetry breaking.
Here

$$\widehat{M}(x, y) = M_S(x, y) e^{i\gamma_5 \widehat{\phi}(x, y)}$$

and in the limit of small $T_g \rightarrow 0$

$$M_S(x, y) \approx \sigma |\mathbf{x}| \delta^{(4)}(x - y), |\mathbf{x}| \gg T_g$$

To lowest order in $\widehat{\phi}$

$$\Delta L^{(1)} = \int \bar{\psi}(x) \sigma |\mathbf{x}| \gamma_5 \frac{\pi^A \lambda^A}{F_\pi} \psi(x) dt d^3\mathbf{x}$$

$$\pi^A \lambda^A = \sqrt{2} \begin{pmatrix} \frac{\eta^0}{\sqrt{6}} + \frac{\pi^0}{\sqrt{2}}, & \pi^+, & K^+ \\ \pi^-, & \frac{\eta^0}{\sqrt{6}} - \frac{\pi^0}{\sqrt{2}}, & K^0 \\ K^-, & \bar{K}_0, & -\frac{2\eta^0}{\sqrt{6}} \end{pmatrix}$$

For chiral transitions between heavy-light mesons we obtain the matrix element

$$U_{21} = \int \bar{\psi}_2(x) \frac{\sigma|\mathbf{x}|}{F_\pi} \gamma_5 \frac{\lambda^A e^{i\mathbf{k}\mathbf{x}}}{\sqrt{2\omega_\pi(\mathbf{k})V}} \psi_1(\mathbf{x}) d^3\mathbf{x}$$

and the corresponding perturbation width

$$w = 2\pi |U_{21}|^2 \delta(E_1 - E_2 - \omega) \frac{V d^3\mathbf{k}}{(2\pi)^3}$$

Let \hat{U} be any (local or nonlocal) operator, which causes a transition from channel i to channel j by the matrix element U_{ij} . Then the multichannel relativistic Green's function G_{ik} satisfies Hamiltonian-like equations (N – number of channels):

$$[(H_i - M)\delta_{ik} + U_{il}]G_{lk} = 1, \quad i, k, l = 1, \dots, N$$

For two channels one can reduce the problem to an effective one-channel equation

$$(H_1 - E)G_{11} - U_{12} \frac{1}{H_2 - E} U_{21} G_{11} = 1$$

Consider an unperturbed set of states $|n\rangle$ in channel 1 and $|m\rangle$ in channel 2. Then a nonlinear equation for the energy eigenvalue M is

$$M = M_1^{(n)} - \sum_m \langle n | U_{12} | m \rangle \frac{1}{M_2^{(m)} - M} \langle m | U_{21} | n \rangle$$

Calculation details

Equation for the mass shift of the i^{th} state:

$$m[i] = m^{(0)}[i] - \sum_f \frac{|\langle i | \hat{V} | f \rangle|^2}{E_f - m[i]}$$

We consider the following thresholds:

i	$D_s(0^+)$	$D_s(1^+)$	$D_s(2^+)$
f	$D(0^-) + K(0^-)$	$D^*(1^-) + K(0^-)$	$D^*(1^-) + K(0^-)$

i	$B_s(0^+)$	$B_s(1^+)$	$B_s(2^+)$
f	$B(0^-) + K(0^-)$	$B^*(1^-) + K(0^-)$	$B^*(1^-) + K(0^-)$

Masses of meson states used for calculation (MeV):

$$m_D = 1869, \quad m_D + m_K = 2363$$

$$m_{D^*} = 2010, \quad m_{D^*} + m_K = 2504$$

$$m_B = 5279, \quad m_B + m_K = 5773$$

$$m_{B^*} = 5325, \quad m_{B^*} + m_K = 5819$$

In what follows all equations are given for D , D_s mesons; changes for B , B_s mesons are obvious.

Notations:

$$E_f = \omega_D + \omega_K, \quad T_f = E_f - m_D - m_K,$$

$$\omega_K(\mathbf{p}) = \sqrt{\mathbf{p}^2 + m_K^2}, \quad \omega_D(\mathbf{p}) = \sqrt{\mathbf{p}^2 + m_D^2},$$

$$E_0 = m^{(0)}[D_s] - m_D - m_K, \quad \delta m = m[D_s] - m^{(0)}[D_s],$$

$$\Delta = E_0 + \delta m = m[D_s] - m_D - m_K$$

We neglect the DK interacting and the D -meson moving in the f -state, so

$$|f\rangle = \Psi_K(\mathbf{p}) \otimes \Psi_D(M_f), \quad |i\rangle = \Psi_{D_s}(M_i),$$

$$\Psi_K(\mathbf{p}) = \frac{e^{i\mathbf{p}\mathbf{r}}}{\sqrt{2\omega_K(\mathbf{p})}}, \quad \sum_f \rightarrow \int \frac{d^3\mathbf{p}}{(2\pi)^3} \sum_{M_f}$$

One may write the equation for the mass shift as

$$E_0 - \Delta = \mathcal{F}(\Delta)$$

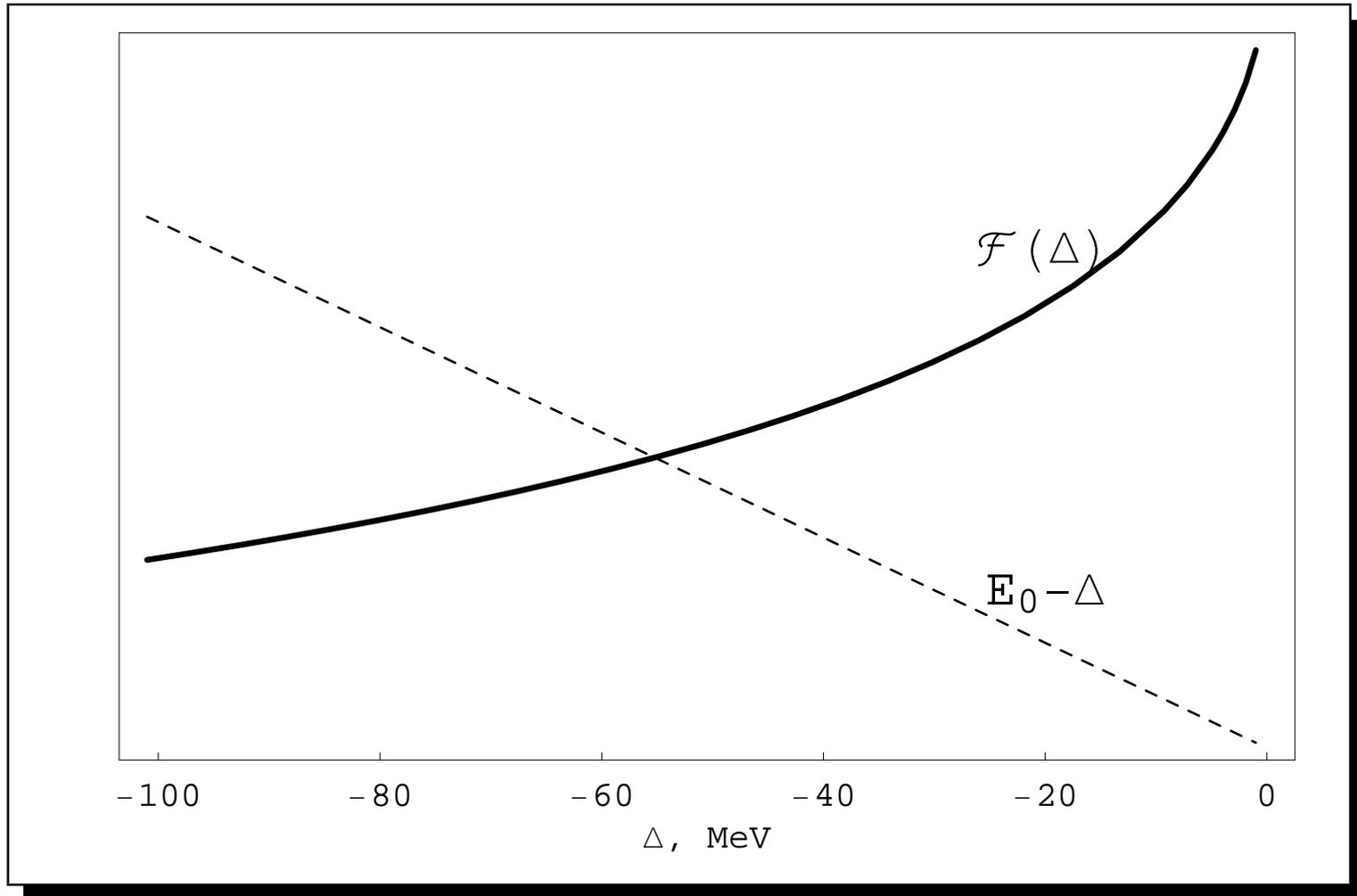
where

$$\mathcal{F}(\Delta) \stackrel{\text{def}}{=} \int \frac{d^3\mathbf{p}}{(2\pi)^3} \sum_{M_f} \frac{|\langle M_i | \hat{V} | \mathbf{p}, M_f \rangle|^2}{T_f(\mathbf{p}) - \Delta}$$

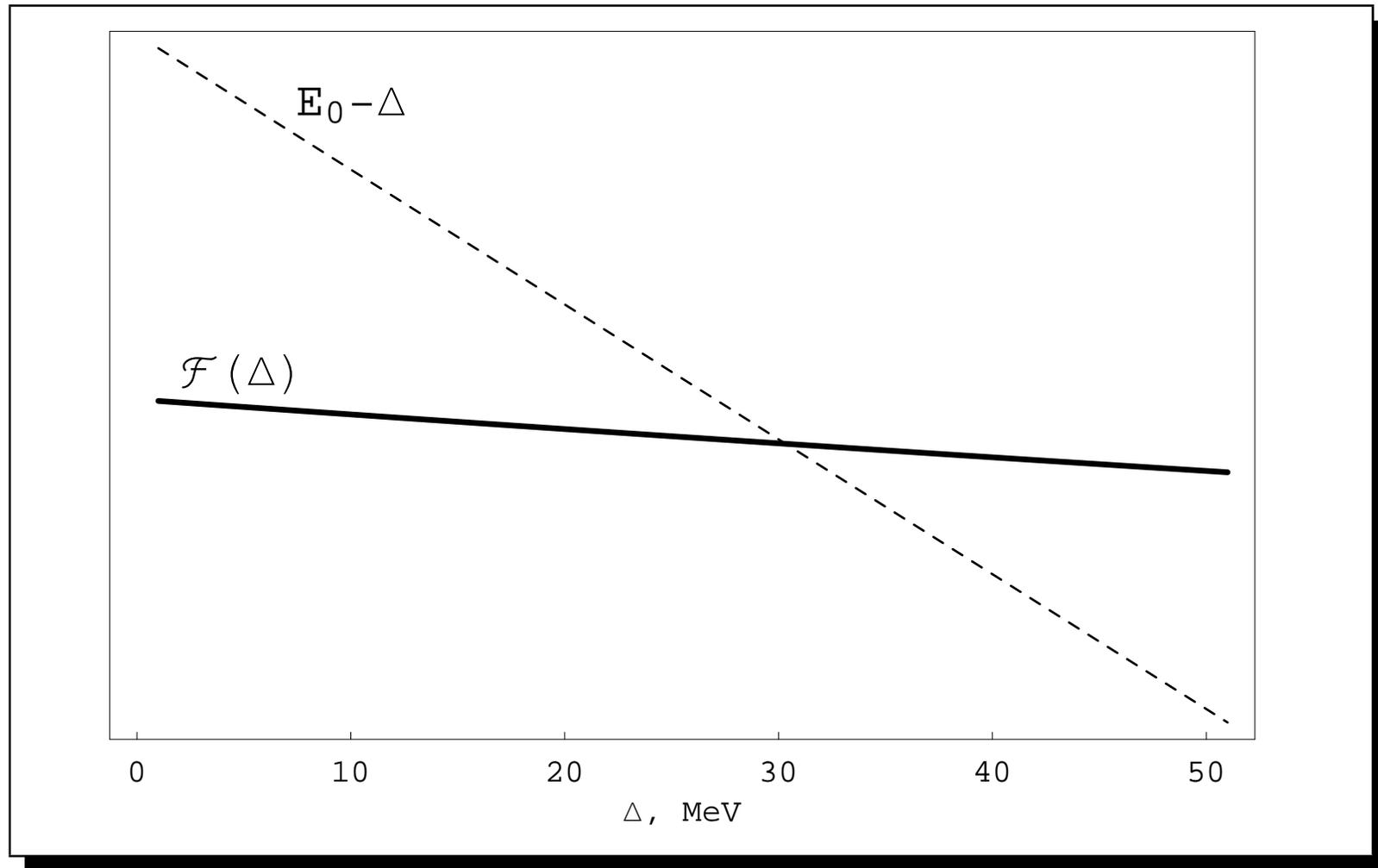
and

$$\langle M_i | \hat{V} | \mathbf{p}, M_f \rangle = - \int \Psi_{D_s}^\dagger(M_i) \sigma|\mathbf{r}| \gamma_5 \frac{\sqrt{2}}{f_K} \Psi_D(M_f) \frac{e^{i\mathbf{p}\mathbf{r}}}{\sqrt{2\omega_K(\mathbf{p})}} d^3\mathbf{r}$$

While solving this equation we have two possible situations: $E_0 < E_0^{\text{crit}}$ and $E_0 > E_0^{\text{crit}}$, where $E_0^{\text{crit}} = \mathcal{F}(-0)$.



If $E_0 < E_0^{\text{crit}}$, the equation has a **negative** real root $\Delta < 0$ and the resulting mass of the D_s meson proves to be under the threshold.



If $E_0 > E_0^{\text{crit}}$, the equation has a complex root $\Delta = \Delta' + i\Delta''$ with **positive** real part $\Delta' > 0$ and **negative** imaginary part $\Delta'' < 0$, so the resulting mass of the D_s meson proves to be over the threshold, the meson having a finite width $\Gamma = 2|\Delta''|$. But an analytic continuation of the $\mathcal{F}(\Delta)$ from the upper half-plane is required here.

In D and D_s mesons considered here we treat c -quark as heavy and static so the mesons are described by the following wave functions:

$$\begin{aligned} \Psi_D (J^-, M_f) &= C_{\frac{1}{2}, M_f - \frac{1}{2}; \frac{1}{2}, +\frac{1}{2}}^{J, M_f} \psi_q^{\frac{1}{2}, 0, M_f - \frac{1}{2}} \otimes |\bar{c} \uparrow\rangle + \\ &\quad + C_{\frac{1}{2}, M_f + \frac{1}{2}; \frac{1}{2}, -\frac{1}{2}}^{J, M_f} \psi_q^{\frac{1}{2}, 0, M_f + \frac{1}{2}} \otimes |\bar{c} \downarrow\rangle \\ \Psi_{D_s} (J_j^+, M_i) &= C_{j, M_i - \frac{1}{2}; \frac{1}{2}, +\frac{1}{2}}^{J, M_i} \psi_s^{j, 1, M_i - \frac{1}{2}} \otimes |\bar{c} \uparrow\rangle + \\ &\quad + C_{j, M_i + \frac{1}{2}; \frac{1}{2}, -\frac{1}{2}}^{J, M_i} \psi_s^{j, 1, M_i + \frac{1}{2}} \otimes |\bar{c} \downarrow\rangle \end{aligned}$$

where $\psi_{q,s}^{jlm}$ is the light quark wave function – Dirac bispinor.

The wave function of the light quark q being in the field of the heavy static antiquark \bar{Q} is the solution of the Dirac equation:

$$\psi_q^{jlm} = \begin{pmatrix} g(r)\Omega_{jlm} \\ (-1)^{\frac{1+l-l'}{2}} f(r)\Omega_{jl'm} \end{pmatrix}, \quad \int_0^\infty (f^2 + g^2) r^2 dr = 1$$

$$g' + \frac{1 + \kappa}{r} g - (E_q + m_q + U - V) f = 0$$

$$f' + \frac{1 - \kappa}{r} f + (E_q - m_q - U - V) g = 0$$

where the interaction between the quark and the antiquark is described by the linear scalar potential and the vector Coulomb potential:

$$U = \sigma r, \quad V = -\frac{\beta}{r}, \quad \beta = \frac{4}{3}\alpha_s$$

Introducing new dimensionless variables

$$x = r\sqrt{\sigma}, \quad \varepsilon_q = E_q/\sqrt{\sigma}, \quad \mu_q = m_q/\sqrt{\sigma}$$

and new dimensionless functions

$$g = \sigma^{3/4} \frac{G(x)}{x}, \quad f = \sigma^{3/4} \frac{F(x)}{x}, \quad \int_0^{\infty} (F^2 + G^2) dx = 1$$

we come to the following system of equations

$$G' + \frac{\varkappa}{x}G - \left(\varepsilon_q + \mu_q + x + \frac{\beta}{x} \right) F = 0$$

$$F' - \frac{\varkappa}{x}F + \left(\varepsilon_q - \mu_q - x + \frac{\beta}{x} \right) G = 0$$

which has been analysed further numerically.

We used the following parameters, as usual

$$\begin{aligned}\sigma &= 0.18 \text{ GeV}^2, & \alpha_s &= 0.39, \\ m_s &= 210 \text{ MeV}, & m_q &= 4 \text{ MeV}\end{aligned}$$

and obtained Dirac eigenvalues ε

\varkappa	$\bar{Q}_q, \mu_q = 0.01$	$\bar{Q}_s, \mu_s = 0.5$
-1	1.0026	1.28944
+1	1.7829	2.08607
-2	1.7545	2.08475

and eigenfunctions G, F .

For D , D_s mesons with given parity several quantum sets are possible:

D			
J^P	j	l	κ
0^-	$\frac{1}{2}$	0	-1
1^-	$\frac{1}{2}$	0	-1
1^-	$\frac{3}{2}$	2	+2

D_s			
J^P	j	l	κ
0^+	$\frac{1}{2}$	1	+1
1^+	$\frac{1}{2}$	1	+1
1^+	$\frac{3}{2}$	1	-2
2^+	$\frac{3}{2}$	1	-2
2^+	$\frac{5}{2}$	3	+3

Physical $D_s(1^+)$ states are mixed:

$$\begin{aligned}\Psi_{D_s} \left(1_L^+, M_i \right) &= -\sin \phi \cdot \Psi_{D_s} \left(1_{1/2}^+, M_i \right) + \cos \phi \cdot \Psi_{D_s} \left(1_{3/2}^+, M_i \right), \\ \Psi_{D_s} \left(1_H^+, M_i \right) &= \cos \phi \cdot \Psi_{D_s} \left(1_{1/2}^+, M_i \right) + \sin \phi \cdot \Psi_{D_s} \left(1_{3/2}^+, M_i \right)\end{aligned}$$

Note: we neglect possible mixing between $D(1_{1/2}^-)$, $D(1_{3/2}^-)$ states and also between $D_s(2_{3/2}^+)$, $D_s(2_{5/2}^+)$ states.

In what follows short notations for quark bispinors are used:

$$\psi_1(m_1) \stackrel{\text{def}}{=} \psi_s^{\frac{1}{2},1,m_1}, \quad \psi_2(m_2) \stackrel{\text{def}}{=} \psi_q^{\frac{1}{2},0,m_2}, \quad \psi_3(m_3) \stackrel{\text{def}}{=} \psi_s^{\frac{3}{2},1,m_3}$$

Using explicit expressions for spherical spinors

$$\Omega_{l+1/2,l,m} = \begin{bmatrix} \sqrt{\frac{j+m}{2j}} Y_{l,m-1/2} \\ \sqrt{\frac{j-m}{2j}} Y_{l,m+1/2} \end{bmatrix}, \quad \Omega_{l-1/2,l,m} = \begin{bmatrix} -\sqrt{\frac{j-m+1}{2j+2}} Y_{l,m-1/2} \\ \sqrt{\frac{j+m+1}{2j+2}} Y_{l,m+1/2} \end{bmatrix},$$

and expansion of a plane wave on spherical functions

$$e^{i\mathbf{p}\mathbf{r}} = 4\pi \sum_{l,m} i^l j_l(pr) Y_{l,m}^* \left(\frac{\mathbf{p}}{p} \right) Y_{l,m} \left(\frac{\mathbf{r}}{r} \right)$$

after long cumbersome transformations ...

we obtain

$$\begin{aligned} \left\| \mathcal{V}_{12} \right\|_{m_1, m_2} &= - \int \psi_1^\dagger(m_1) \sigma |\mathbf{r}| \gamma_5 \frac{\sqrt{2}}{f_K} \psi_2(m_2) \frac{e^{i\mathbf{p}\mathbf{r}}}{\sqrt{2\omega_K(\mathbf{p})}} d^3\mathbf{r} = \\ &= \frac{\sqrt{\sigma}}{f_K \sqrt{\omega_K(p)}} \Phi_0 \left(\frac{p}{\sqrt{\sigma}} \right) \sqrt{4\pi} Y_{0, m_1 - m_2}^* \left(\frac{\mathbf{p}}{p} \right) \end{aligned}$$

$$\begin{aligned} \left\| \mathcal{V}_{32} \right\|_{m_3, m_2} &= - \int \psi_3^\dagger(m_3) \sigma |\mathbf{r}| \gamma_5 \frac{\sqrt{2}}{f_K} \psi_2(m_2) \frac{e^{i\mathbf{p}\mathbf{r}}}{\sqrt{2\omega_K(\mathbf{p})}} d^3\mathbf{r} = \\ &= - \frac{\sqrt{\sigma}}{f_K \sqrt{\omega_K(p)}} \Phi_2 \left(\frac{p}{\sqrt{\sigma}} \right) \sqrt{\frac{4\pi}{5}} Y_{2, m_3 - m_2}^* \left(\frac{\mathbf{p}}{p} \right) \cdot \begin{bmatrix} -1 & +2 \\ -\sqrt{2} & +\sqrt{3} \\ -\sqrt{3} & +\sqrt{2} \\ -2 & +1 \end{bmatrix} \end{aligned}$$

where

$$\Phi_0(q) = \int_0^{\infty} j_0(qx)xdx \left[G_1(x)F_2(x) - F_1(x)G_2(x) \right],$$

$$\Phi_2(q) = \int_0^{\infty} j_2(qx)xdx \left[G_3(x)F_2(x) - F_3(x)G_2(x) \right]$$

Finally, introducing functions

$$\tilde{\mathcal{F}}_{0,2}(\Delta) = \frac{\sigma}{2\pi^2 f_K^2} \int_0^{\infty} \frac{p(T_f)\omega_D(T_f)dT_f}{T_f + m_D + m_K} \cdot \frac{\Phi_{0,2}^2\left(\frac{p(T_f)}{\sqrt{\sigma}}\right)}{T_f - \Delta}$$

$$\tilde{\Gamma}_{0,2}(T_f) = \frac{\sigma}{\pi f_K^2} \cdot \frac{p(T_f)\omega_D(T_f)}{T_f + m_D + m_K} \cdot \Phi_{0,2}^2\left(\frac{p(T_f)}{\sqrt{\sigma}}\right)$$

we obtain the following equations to determine meson masses and widths:

$$D_s(0^+) \quad E_0[0^+] - \Delta = \tilde{\mathcal{F}}_0(\Delta)$$

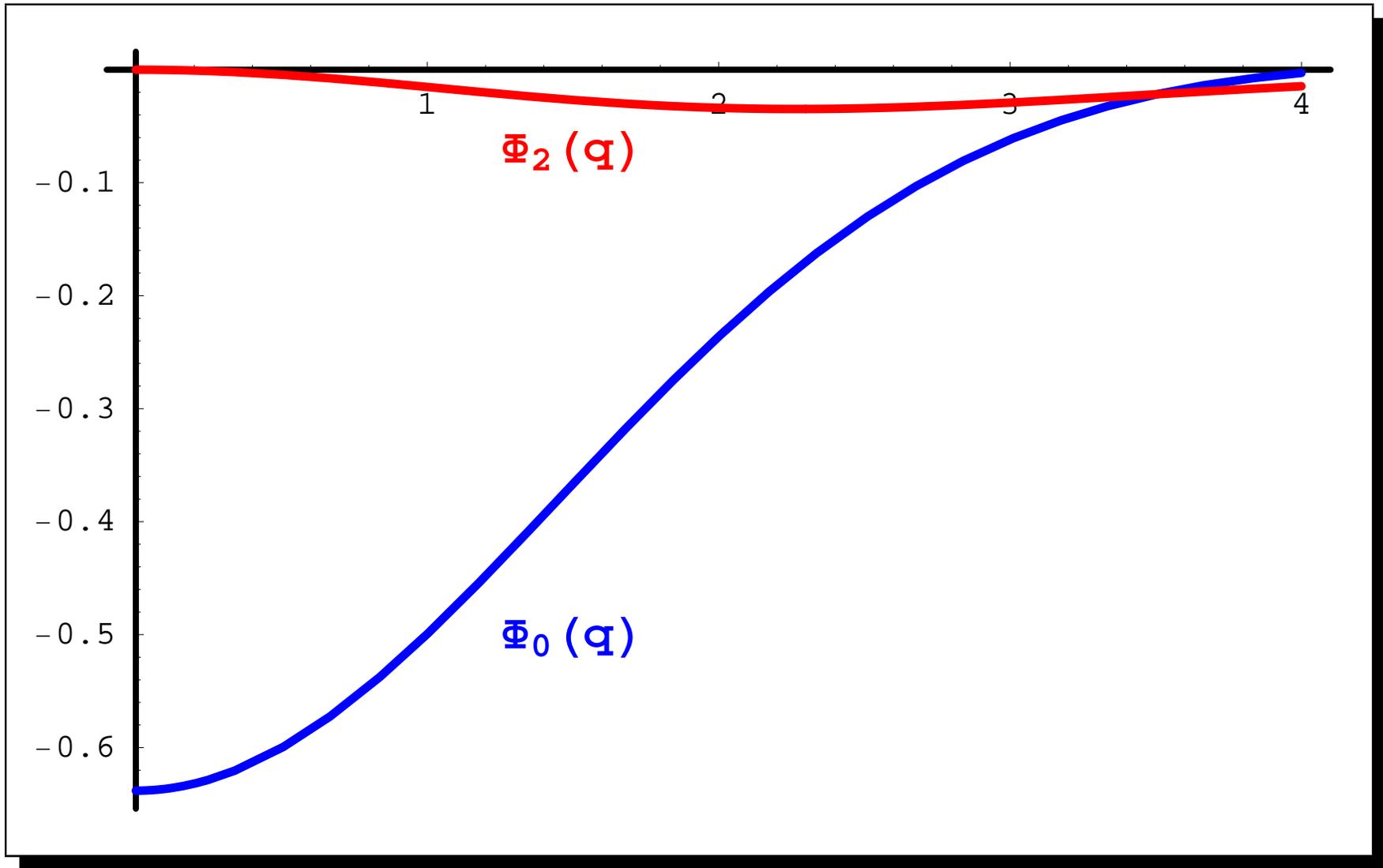
$$D_s(1_H^+) \quad E_0[1_H^+] - \Delta = \cos^2 \phi \cdot \tilde{\mathcal{F}}_0(\Delta) + \sin^2 \phi \cdot \tilde{\mathcal{F}}_2(\Delta)$$

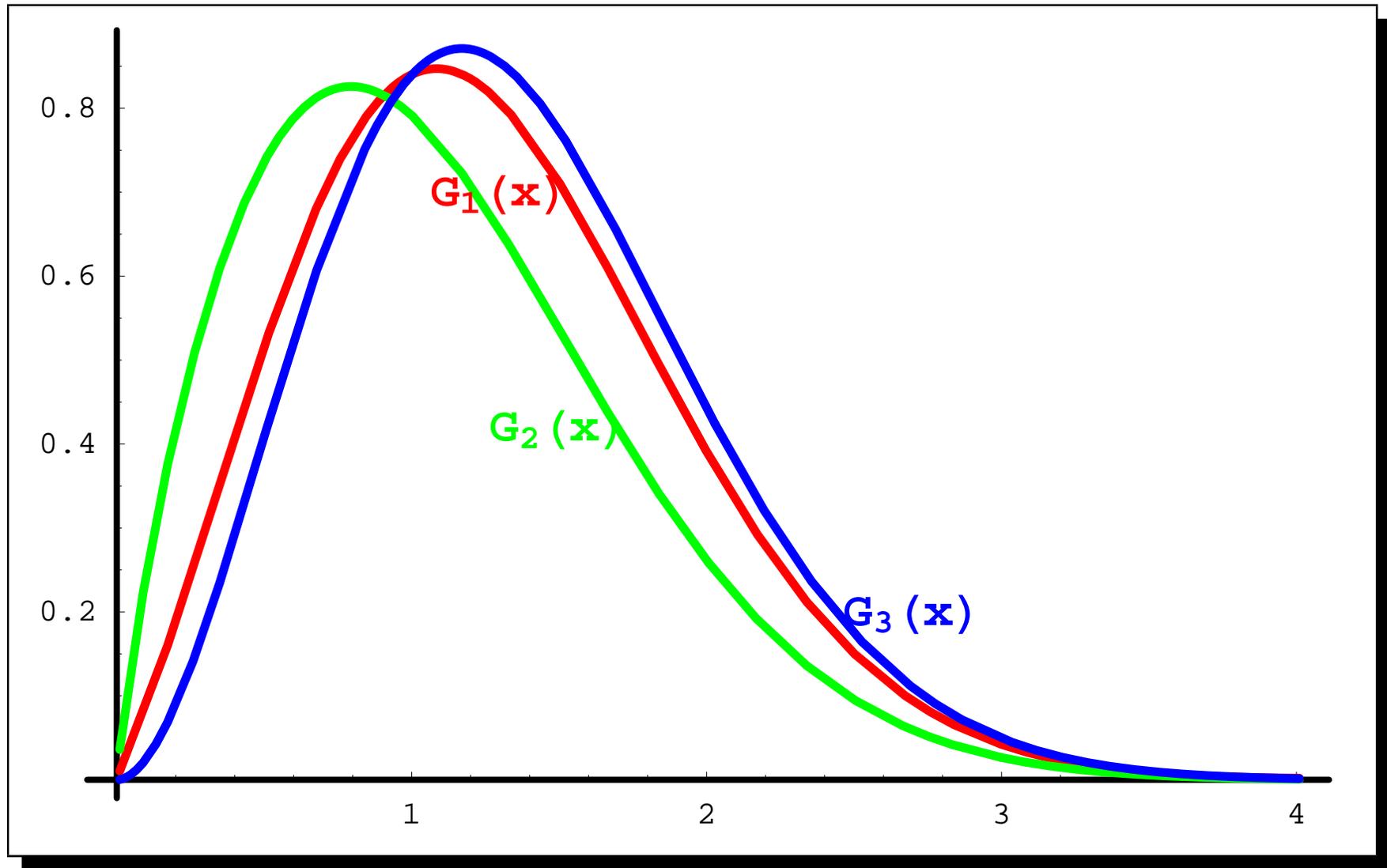
$$D_s(1_L^+) \quad E_0[1_L^+] - \Delta' = \sin^2 \phi \cdot \tilde{\mathcal{F}}_0(\Delta') + \cos^2 \phi \cdot \tilde{\mathcal{F}}_2(\Delta')$$

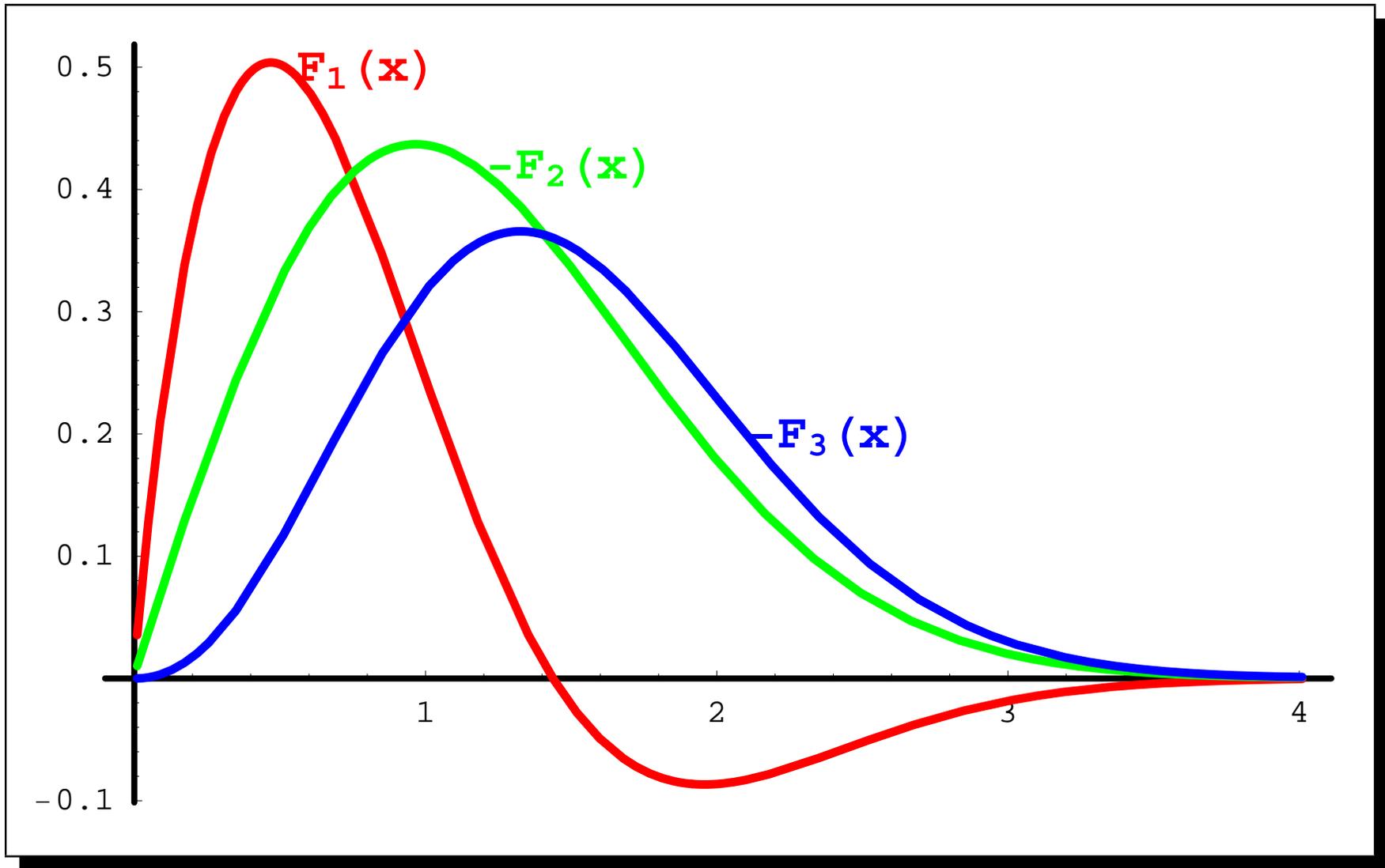
$$\Gamma[1_L^+] = \sin^2 \phi \cdot \tilde{\Gamma}_0(\Delta') + \cos^2 \phi \cdot \tilde{\Gamma}_2(\Delta')$$

$$D_s(2_{3/2}^+) \quad E_0[2_{3/2}^+] - \Delta' = \frac{3}{5} \cdot \tilde{\mathcal{F}}_2(\Delta')$$

$$\Gamma[2_{3/2}^+] = \frac{3}{5} \cdot \tilde{\Gamma}_2(\Delta')$$







Results and discussion

$D_s(0^+)$ -meson mass shift due to DK channel and $B_s(0^+)$ -meson mass shift due to BK channel. Thresholds: $m_D + m_K = 2363$ MeV, $m_B + m_K = 5772$ MeV.

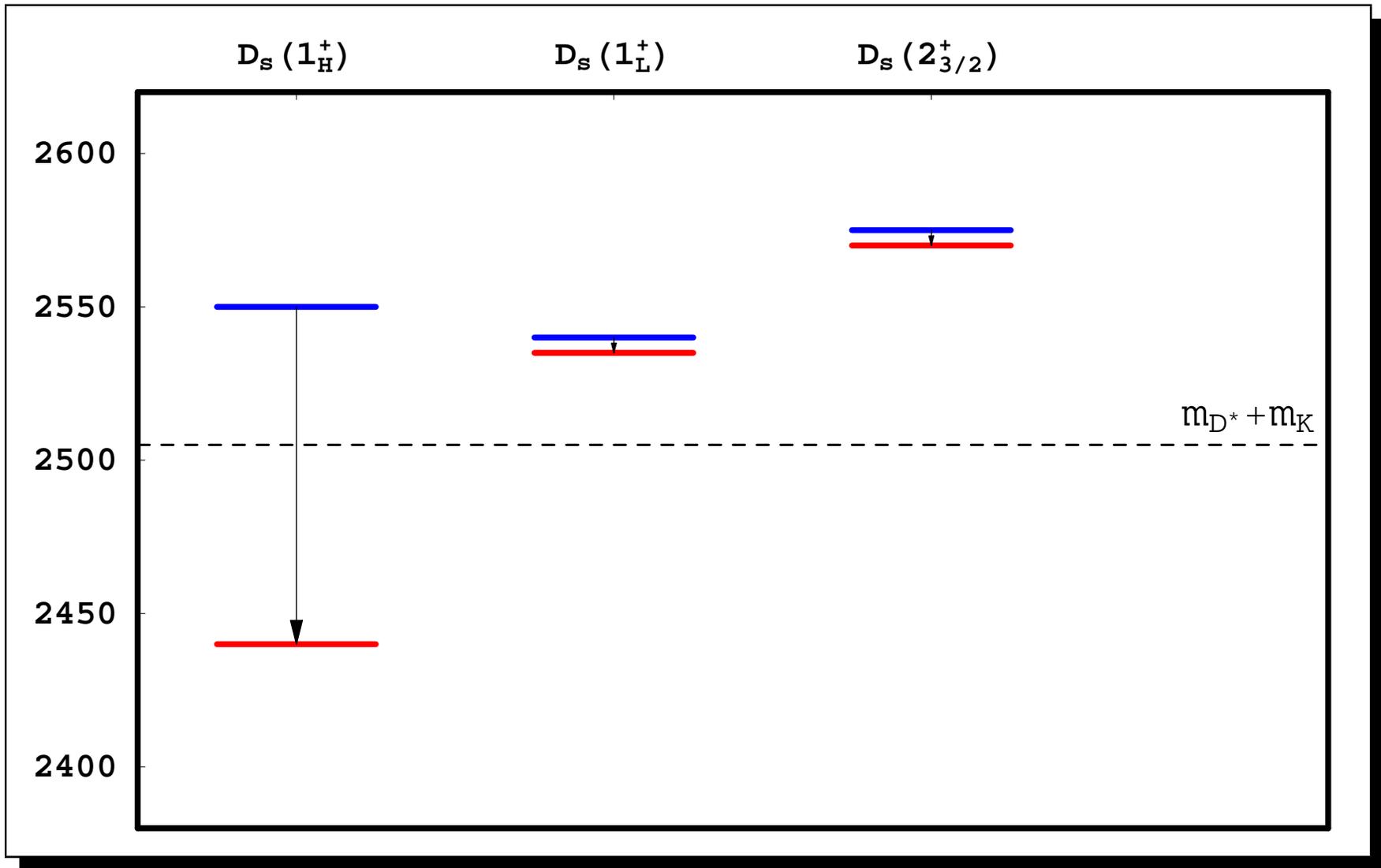
state	$m^{(0)}$	$m^{(\text{theor})}$	$m^{(\text{exp})}$	δm
$D_s(0^+)$	2475(20)	2330(20)	2317	-145
$B_s(0^+)$	5814(15)	5709(15)	not seen	-105

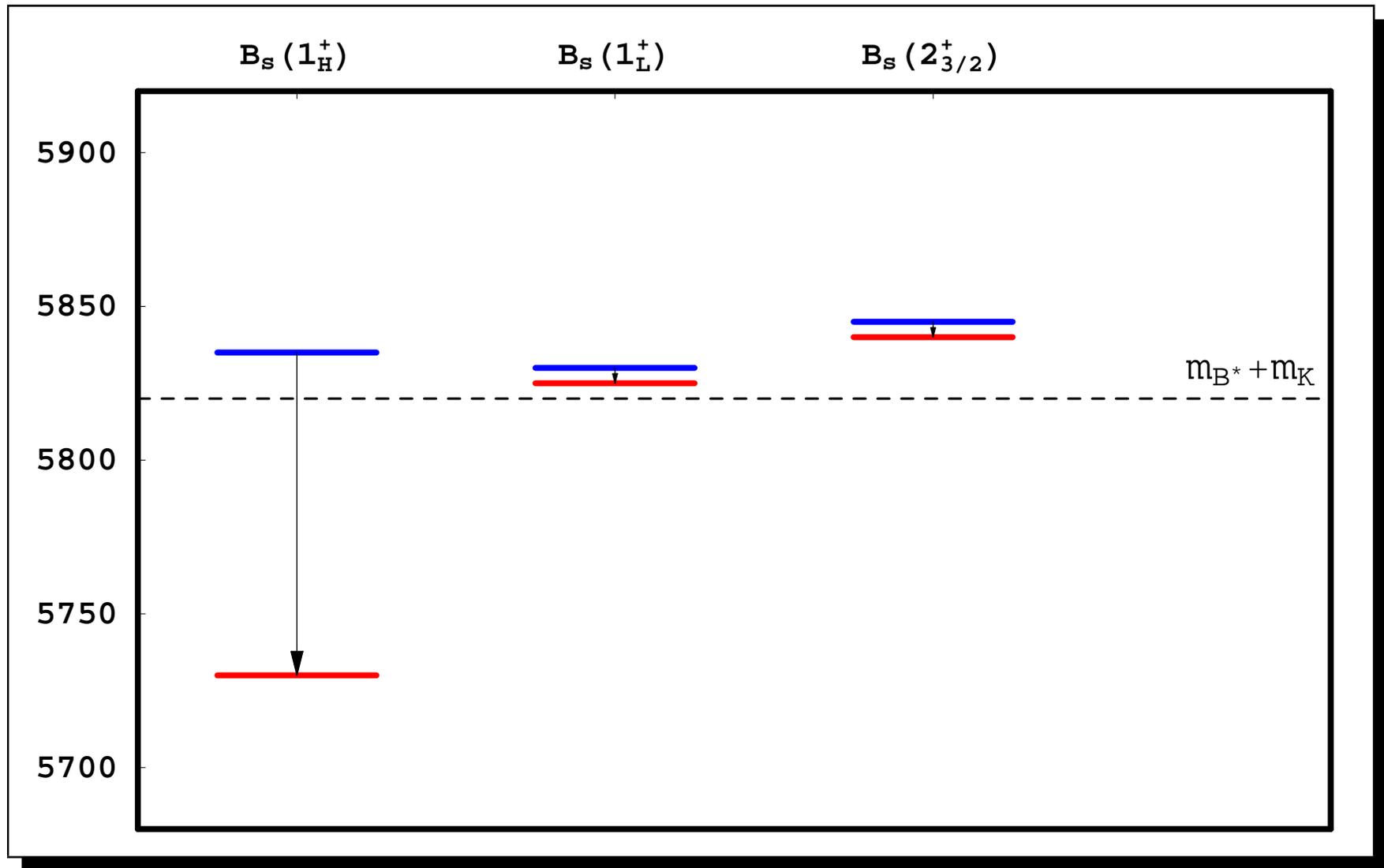
Results on D_s meson mass shift and partial width due to D^*K channel.
 Threshold $m_{D^*} + m_K = 2504$ MeV, mixing angle $\sim 4^\circ$.

state	$m^{(0)}$	$m^{(\text{theor})}$	$m^{(\text{exp})}$	δm	$\Gamma^{(\text{theor})}_{(D^*K)}$	$\Gamma^{(\text{exp})}_{(D^*K)}$
$D_s(1_H^+)$	2568(15)	2458(15)	2460	-110	\times	\times
$D_s(1_L^+)$	2537	2535	2535	-2	1.1	< 2.3
$D_s(2_{3/2}^+)$	2575	2573	2573	-2	0.03	not seen

Results on B_s meson mass shift and partial width due to B^*K channel.
 Threshold $m_{B^*} + m_K = 5819$ MeV, mixing angle $\sim 4^\circ$.

state	$m^{(0)}$	$m^{(\text{theor})}$	$m^{(\text{exp})}$	δm	$\Gamma^{(\text{theor})}_{(B^*K)}$	$\Gamma^{(\text{exp})}_{(B^*K)}$
$B_s(1_H^+)$	5835(15)	5727(15)	not seen	-108	\times	\times
$B_s(1_L^+)$	5830	5828	5829	-2	0.8	< 1.3
$B_s(2_{3/2}^+)$	5840	5838	5839	-2	$< 10^{-3}$	not seen





Summary

- Our method yields final masses and widths of D_s and B_s mesons in good agreement with experimental data (better than 20 MeV). The interaction is deduced from QCD and is almost parameter free.
- The suggested mechanism ensures strong shifts $\gtrsim 100$ MeV for $j = 1/2$ levels, while $j = 3/2$ levels remain almost *in situ*. Also the mechanism may lead to reordering of the initial levels due to chiral coupling.
- We believe that the chiral coupling might be the universal mechanism also responsible for baryon mass shifts.