Testing extra dimension in the stabilized RS1 model

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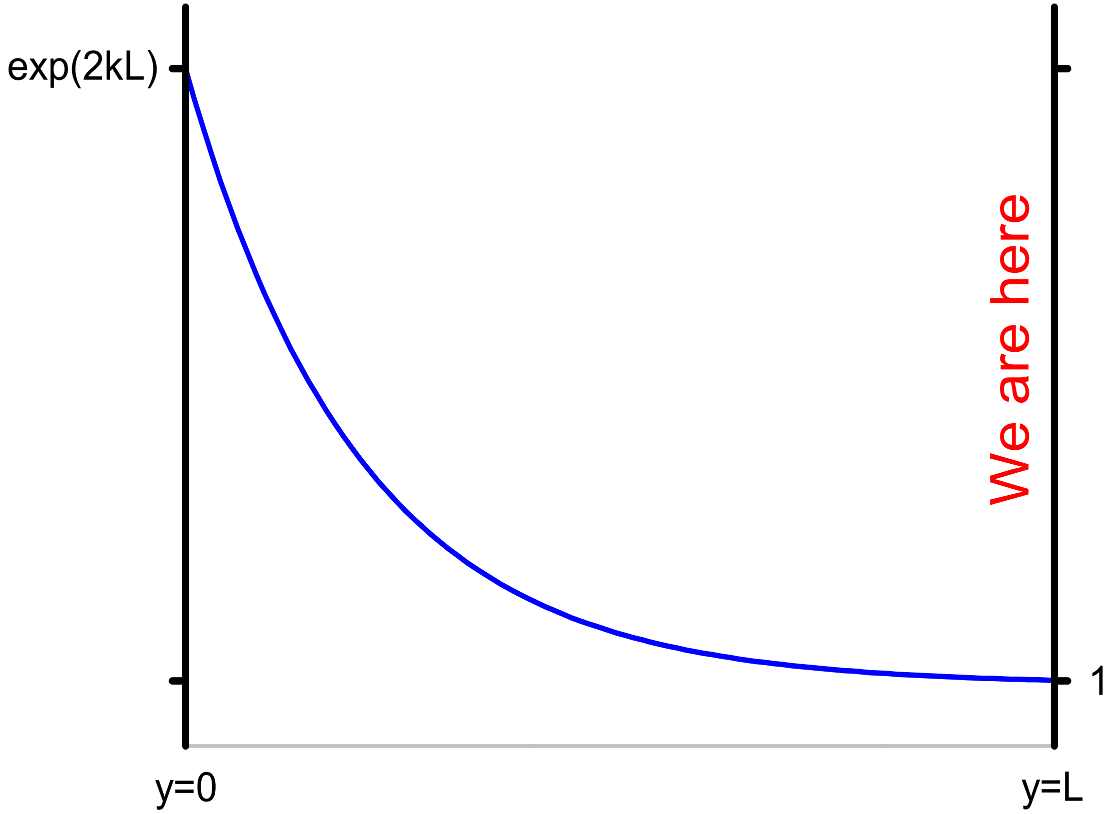
Randall-Sundrum model:

L. Randall and R. Sundrum,
“A large mass hierarchy from a small extra dimension,”
Phys. Rev. Lett. **83** (1999) 3370

Two branes with tension at the fixed points of the orbifold S^1/Z_2.
The RS solution for the metric:

$$ds^2 = g_{MN}dx^M dx^N = e^{-2ky+c}\,\eta_{\mu\nu}dx^\mu dx^\nu - dy^2, \quad 0 \le y \le L.$$

Galilean coordinates: $g_{\mu\nu} = diag(1, -1, -1, -1).$



Coordinates $\{x^\mu\}$ are Galilean for $c = 2kL$,

$$M_{Pl}^2 = \frac{M^3}{k}(e^{2kL} - 1)$$

The hierarchy problem can be solved, if $M \sim k \sim 1\,TeV$ and $kL \sim 35$.

Newtonian limit in the zero mode approximation:

$$V = -\frac{G_2 m}{r}\left(1 + \frac{e^{2kL}}{3}\right)$$

The coupling of the radion to matter is too strong and contradicts the experimental restrictions from classical gravity.

Stabilization mechanisms:

W. D. Goldberger and M. B. Wise,
"Modulus stabilization with bulk fields,"
Phys. Rev. Lett. **83** (1999) 4922

O. DeWolfe, D. Z. Freedman, S. S. Gubser and A. Karch,
"Modeling the fifth dimension with scalars and gravity,"
Phys. Rev. D **62** (2000) 046008

We take the latter model and choose such values of the parameters that the metric of the stabilized model is approximately that of the unstabilized.

The model

$$
\begin{aligned}
S = & - 2M^3 \int d^4x \int_{-L}^{L} dy R\sqrt{-g} + \\
& + \int d^4x \int_{-L}^{L} dy \left(\frac{1}{2} g^{MN} \partial_M \phi \partial_N \phi - V(\phi) \right) \sqrt{-g} - \\
& - \int_{y=0} \sqrt{-\tilde{g}} \lambda_1(\phi) d^4x + \int_{y=L} \sqrt{-\tilde{g}} (-\lambda_2(\phi) + L_{SM}) d^4x.
\end{aligned}
$$

A background solution

$$
ds^2 = e^{-2A(y)} \eta_{\mu\nu} dx^\mu dx^\nu - dy^2 \equiv \gamma_{MN}(y) dx^M dx^N
$$

$$
\phi(x, y) = \phi(y)
$$

Potential

$$V(\phi) = \frac{1}{8}\left(\frac{dW}{d\phi}\right)^2 - \frac{1}{24M^3}W^2(\phi)$$

$$W(\phi) = 24M^3k - u\phi^2$$

$$\lambda_1 = W(\phi) + \beta_1^2(\phi - \phi_1)^2$$

$$\lambda_2 = -W(\phi) + \beta_2^2(\phi - \phi_2)^2$$

Solutions for functions $A(y), \phi(y)$ are

$$\begin{aligned} \phi(y) &= \phi_1 e^{-u|y|}, \\ A(y) &= k(|y| - L) + \frac{\phi_1^2}{48M^3}(e^{-2u|y|} - e^{-2uL}). \end{aligned}$$

The separation distance in defined by the equation

$$L = \frac{1}{u}\ln\left(\frac{\phi_1}{\phi_2}\right)$$

$$\begin{aligned} g_{MN}(x,y) &= \gamma_{MN}(y) + \frac{1}{\sqrt{2M^3}} h_{MN}(x,y), \\ \phi(x,y) &= \phi_1 e^{-u|y|} + \frac{1}{\sqrt{2M^3}} f(x,y). \end{aligned}$$

$$L_{int} = -\frac{1}{2\sqrt{2M^3}} h^{\mu\nu} T_{\mu\nu}, \qquad T_{\mu\nu} = 2\frac{\delta L_{SM}}{\delta \gamma^{\mu\nu}} - \gamma_{\mu\nu} L_{SM},$$

Approximation $uL \ll 1$

$$A(y) = k(|y|-L) + \frac{\phi_1^2}{48M^3}(e^{-2u|y|} - e^{-2uL}) \approx \tilde{k}\,(|y| - L)\,, \quad \tilde{k} = k - \frac{\phi_1^2}{24M^3} u$$

The physical degrees of freedom of the model in the linear approximation were explicitly isolated in

E. E. Boos, Y. S. Mikhailov, M. N. Smolyakov and I. P. Volobuev, "Physical degrees of freedom in stabilized brane world models," Mod. Phys. Lett. A **21** (2006) 1431

They are:

tensor fields $b^n_{\mu\nu}(x)$, $n = 0, 1, \cdots$ with masses m_n ($m_0 = 0$) and wave functions in the space of extra dimension $\psi_n(y)$

and scalar fields $\varphi_n(x)$, $n = 1, 2, \cdots$ with masses μ_n and wave functions in the space of extra dimension $g_n(y)$.

The masses and the wave functions are not arbitrary, but rigidly defined by the model parameters.

$$L_{int} = -\frac{1}{2\sqrt{2M^3}}\left(\psi_0(L)b^0_{\mu\nu}(x)T^{\mu\nu} + \sum_{n=1}^{\infty}\psi_n(L)b^n_{\mu\nu}(x)T^{\mu\nu} + \frac{1}{2}\sum_{n=1}^{\infty} g_n(L)\varphi_n(x)T^\mu_\mu\right)$$

Tensor modes (the same as in the RS1 model with $k \to \tilde{k}$):

$$m_0 = 0, \qquad \psi_0(L) = \frac{\tilde{k}^{\frac{1}{2}}}{(e^{2\tilde{k}L} - 1)^{\frac{1}{2}}}$$

$$J_1\left(\frac{m_n}{\tilde{k}}\right) = 0, \qquad \psi_n(L) = -\sqrt{\tilde{k}}, \qquad m_1 \approx 3.83\tilde{k}$$

Scalar modes (in the limit $\beta_2 \to \infty$):

$$\left(1 + \alpha + \frac{u}{\tilde{k}}\right) J_\alpha\left(\frac{\mu_n}{\tilde{k}}\right) - \frac{\mu_n}{\tilde{k}} J_{\alpha-1}\left(\frac{\mu_n}{\tilde{k}}\right) = 0, \qquad \alpha = \sqrt{\left(1 + \frac{u}{\tilde{k}}\right)^2 + \frac{\phi_1^2}{6M^3}\frac{u^2}{\tilde{k}^2}}$$

$$g_n(L) = \frac{e^{-uL}u\phi_1\sqrt{2\tilde{k}}}{3M^{\frac{3}{2}}\sqrt{\mu_n^2 - \frac{\phi_1^2 u^2}{6M^3}}}$$

E. E. Boos, V. E. Bunichev, M. N. Smolyakov and I. P. Volobuev, "Testing extra dimensions below the production threshold of Kaluza-Klein excitations", arXiv:0710.3100 [hep-ph].

Integrating out the heavy tensor modes induces the interaction of the Standard Model fields of the form

$$L_T = \frac{1}{16M^3}\left(\sum_{n=1}^{\infty}\frac{\psi_n^2(L)}{m_n^2}\right)T^{\mu\nu}\Delta_{\mu\nu,\rho\sigma}T^{\rho\sigma},$$

$$\Delta_{\mu\nu,\rho\sigma} = \eta_{\mu\rho}\eta_{\nu\sigma} + \eta_{\mu\sigma}\eta_{\nu\rho} - \frac{2}{3}\eta_{\mu\nu}\eta_{\rho\sigma}$$

Integrating out the scalar modes induces the interaction of the form

$$L_S = \frac{1}{64M^3}\left(\sum_{n=1}^{\infty}\frac{g_n^2(L)}{\mu_n^2}\right)T_\mu^\mu T_\nu^\nu.$$

$$\frac{1}{16M^3}\sum_{n>0}\frac{\psi_n^2(L)}{m_n^2}\approx\frac{0.1246}{16M^3\tilde{k}}\approx\frac{0.91}{\Lambda_\pi^2 m_1^2}\,,$$

where

$$\frac{1}{\Lambda_\pi}=-\frac{\psi_1(L)}{\sqrt{8M^3}}=\frac{\sqrt{\tilde{k}}}{\sqrt{8M^3}},\qquad m_1=3.83\tilde{k}$$

Let us suppose that the lowest scalar mode has the mass of the order of $2\,\tilde{k}$. Such situation can be realized if

$$u\simeq 0.003\,\tilde{k},\qquad \frac{\phi_1^2 u^2}{6M^3}\simeq 2.24\,\tilde{k}^2$$

$$\frac{1}{64M^3}\sum_n\frac{g_n^2(L)}{\mu_n^2}\approx\frac{1}{64M^3\tilde{k}}(0.341+0.002)\approx\frac{0.63}{\Lambda_\pi^2 m_1^2}$$

$$L_{eff} = L_T + L_S = \frac{0.91}{\Lambda_\pi^2 m_1^2} T^{\mu\nu} \tilde{\Delta}_{\mu\nu,\rho\sigma} T^{\rho\sigma}$$

$$\tilde{\Delta}_{\mu\nu,\rho\sigma} = \eta_{\mu\rho}\eta_{\nu\sigma} + \eta_{\mu\sigma}\eta_{\nu\rho} - \left(\frac{2}{3} - \delta\right)\eta_{\mu\nu}\eta_{\rho\sigma}$$

where δ stands for the contribution of the scalar modes,

$$\delta \approx 0.69$$

Example:

$$\Lambda_\pi \simeq 2.8\, TeV, \quad m_1 \simeq 3.83\, TeV$$

(correspondingly, $M \simeq 1\, TeV$, $\tilde{k} \simeq 1\, TeV$, radion mass – $\mu_1 \simeq 2\, TeV$)

$$\frac{1}{\Lambda_\pi m_1} \approx 0.09\, TeV^{-2}$$

Corresponding cross-sections can be found in
E. E. Boos, V. E. Bunichev, M. N. Smolyakov and I. P. Volobuev, "Testing extra dimensions below the production threshold of Kaluza-Klein excitations", arXiv:0710.3100 [hep-ph].

$$\hat{\sigma}_{gg\to ZZ} = \frac{\kappa^2}{16\pi\cdot 32}\beta\left[1-\frac{2}{3}\beta^2+\frac{1}{10}\beta^4+3\delta^2\left(\frac{b(g)}{2g}\right)^2\left(1-\frac{2}{3}\beta^2+\beta^4\right)\right]\hat{s}^3$$

$$\hat{s} = x_1 x_2 s$$

$$\beta = \sqrt{1-\frac{4M_Z^2}{\hat{s}}}$$

$$\kappa = \frac{0.91}{\Lambda_\pi^2 m_1^2}$$

Various processes with this Lagrangian were calculated with the help of the CompHEP program

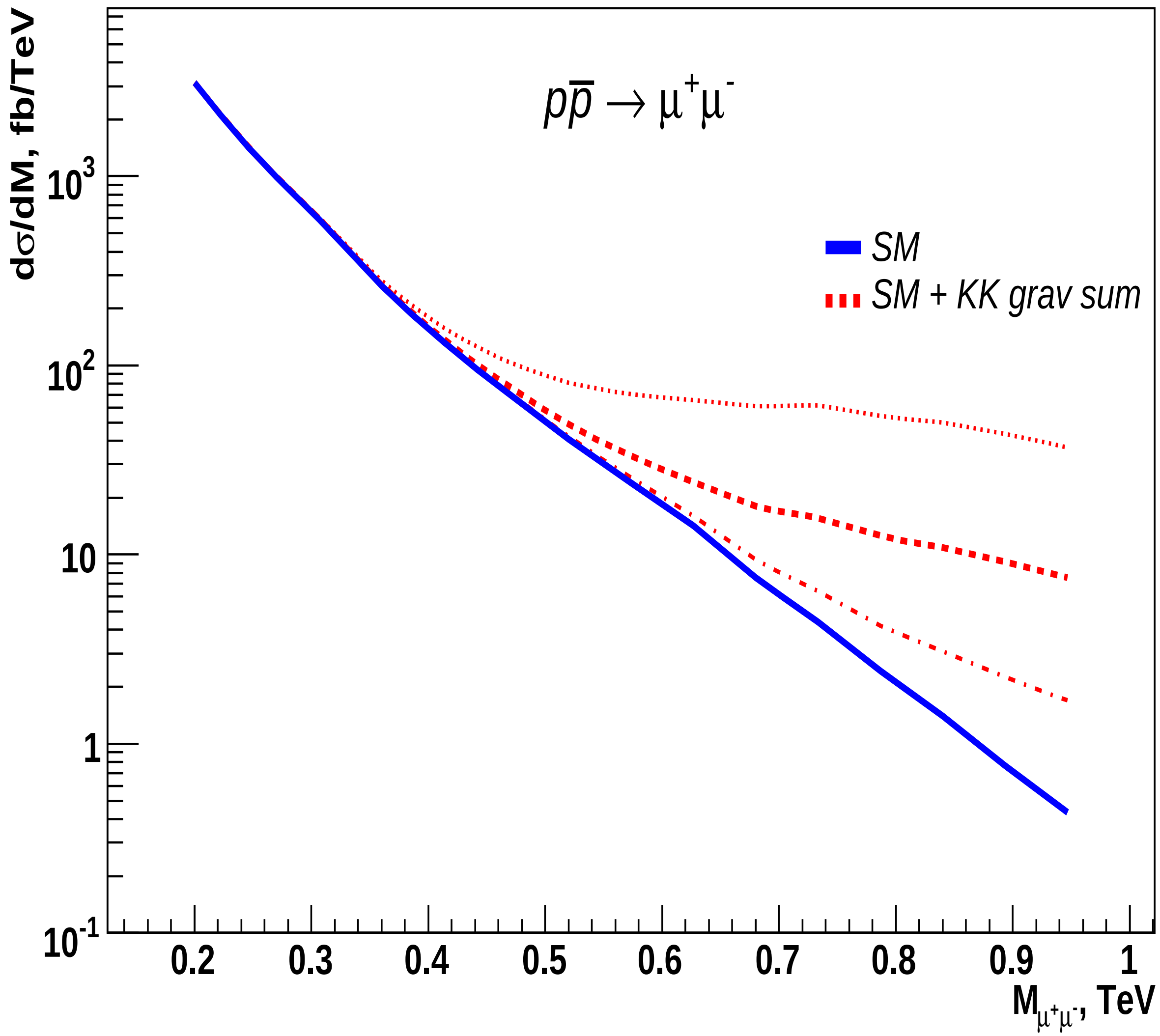


Figure 1: Dilepton invariant mass distribution for parameter $\frac{1}{\Lambda_\pi m_1} \times (1TeV)^2$=1.2 (dash-dotted line), 2 (dashed line), 3 (dotted line) for the Tevatron

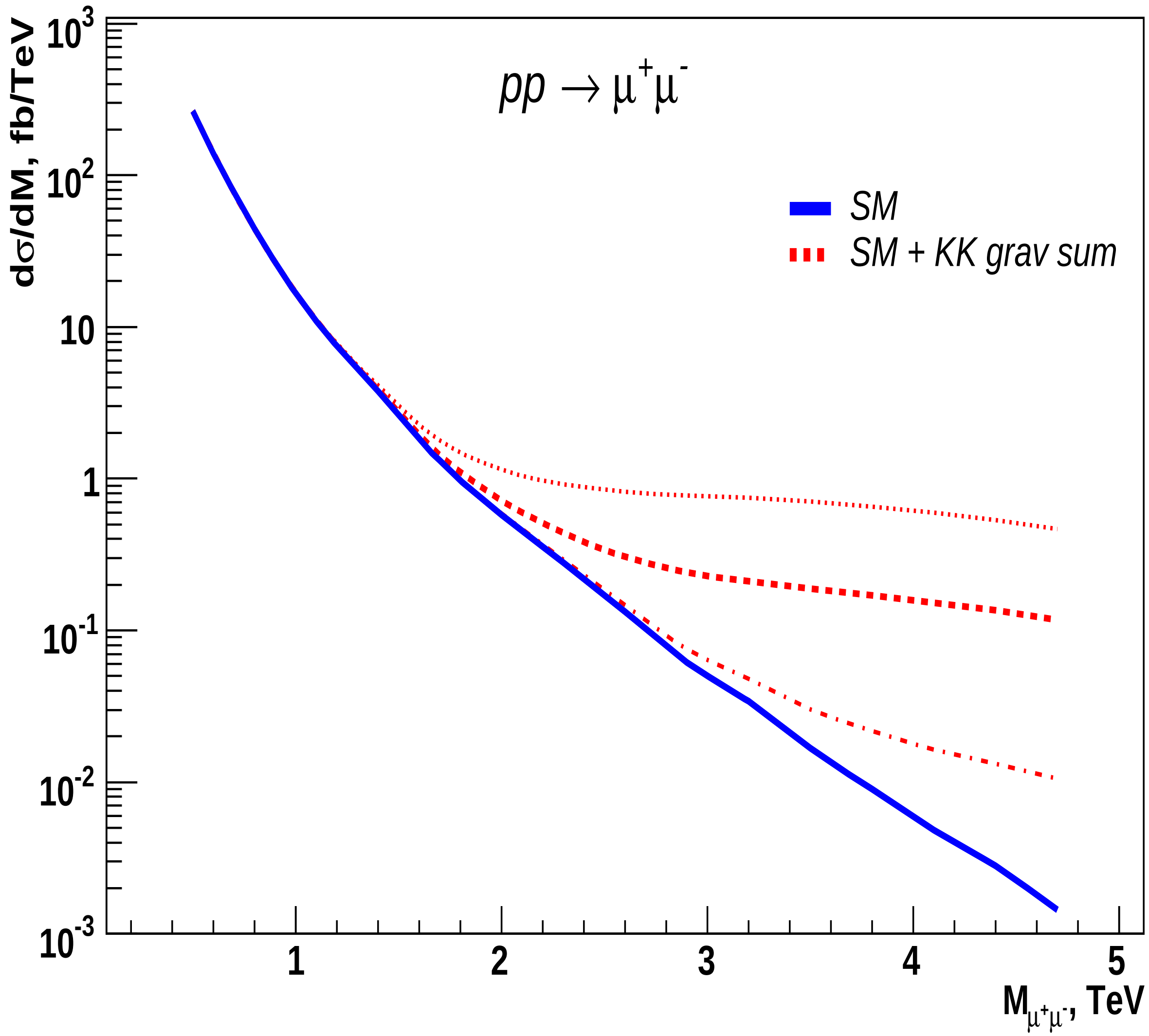


Figure 2: Dilepton invariant mass distribution for parameter $\frac{1}{\Lambda_\pi m_1} \times (1TeV)^2$=0.055 (dash-dotted line), 0.1 (dashed line), 0.15 (dotted line) for the LHC

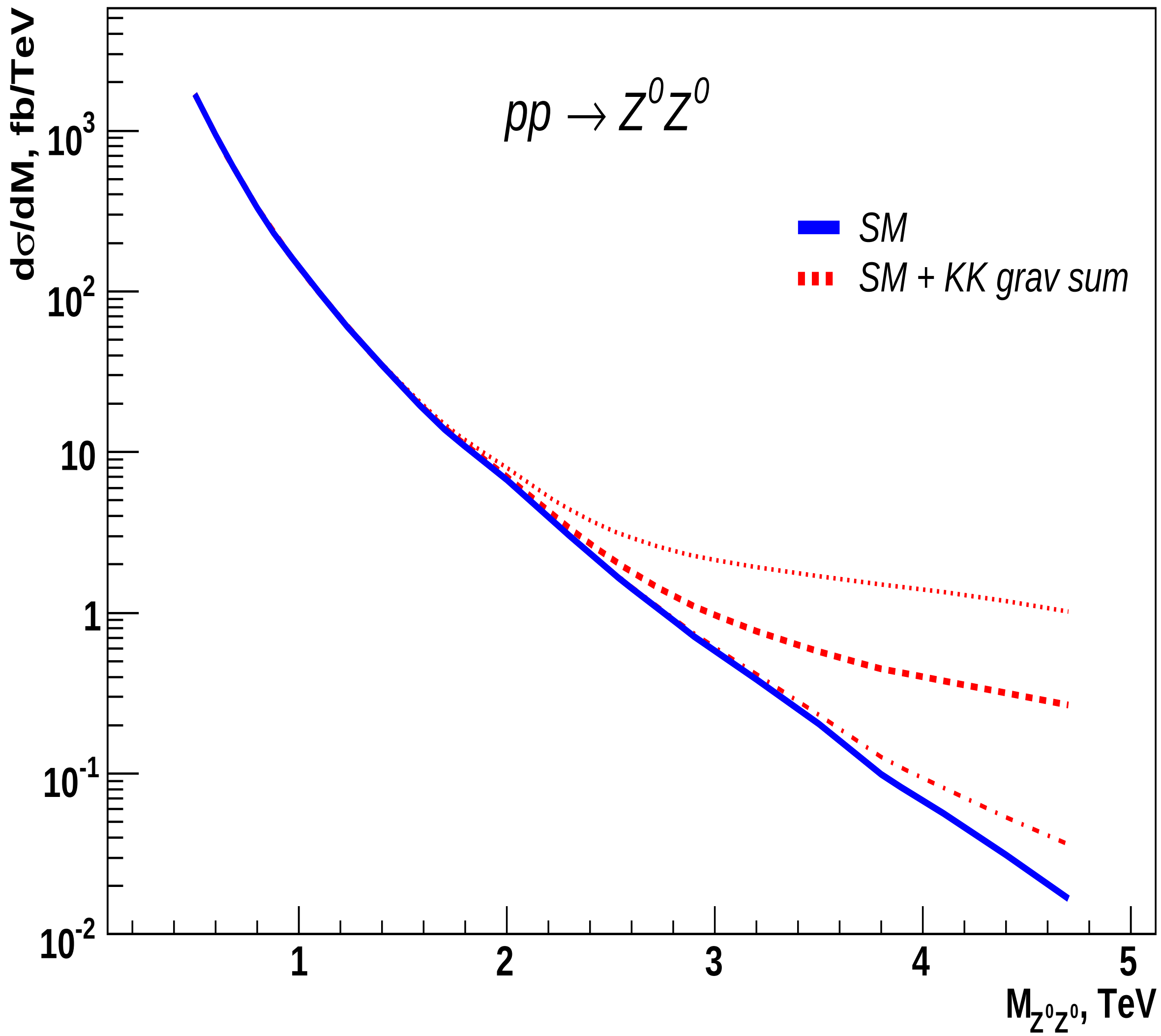


Figure 3: Z-boson pair invariant mass distribution for parameter $\frac{1}{\Lambda_\pi m_1} \times (1TeV)^2$=0.055 (dash-dotted line), 0.1 (dashed line), 0.15 (dotted line) for the LHC

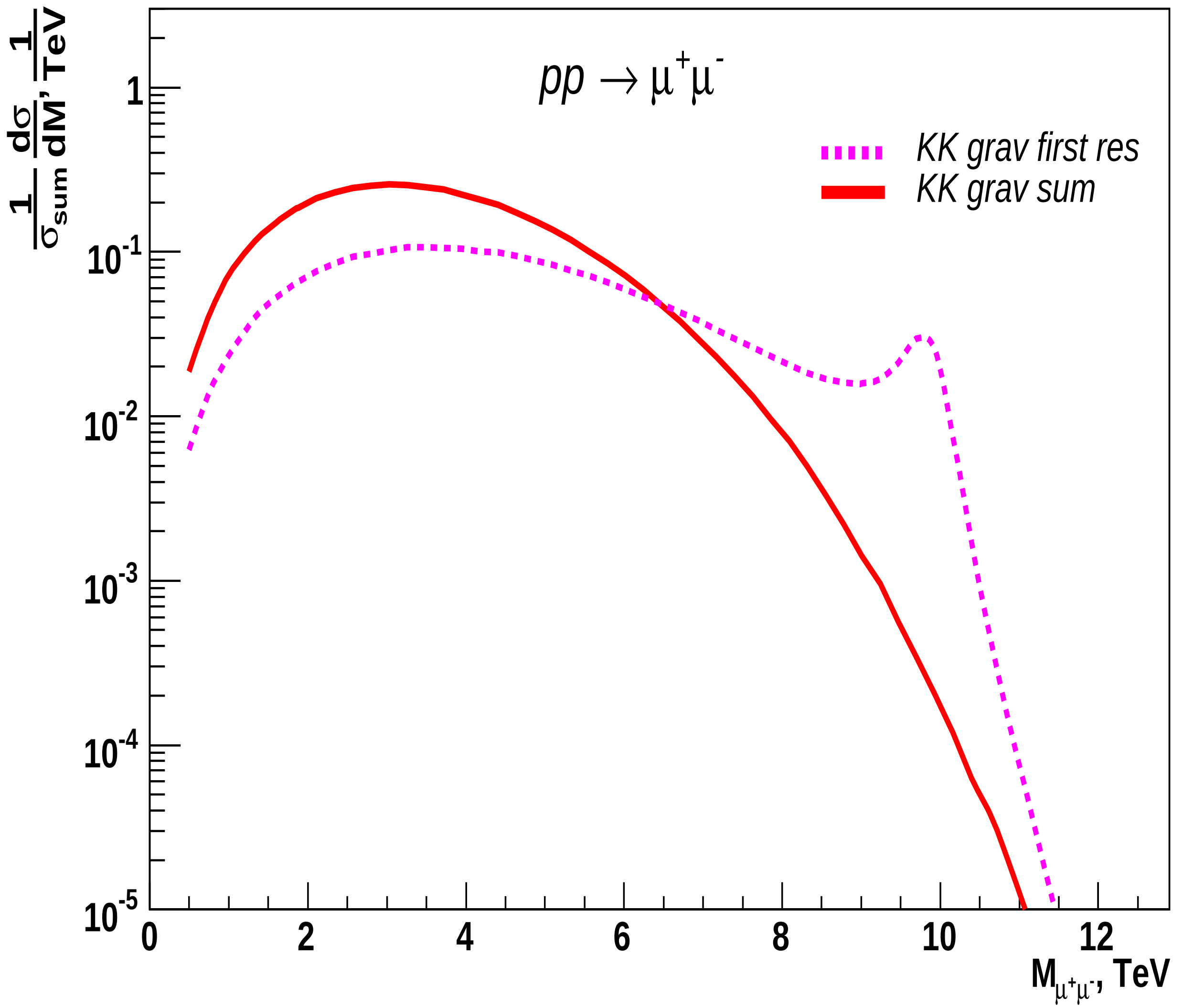


Figure 4: The normalized dilepton invariant mass distribution from the sum of KK tower states including the first KK mode (solid line) and from the first KK mode only (dashed line) for $m_1 = 10\ TeV$, $\Gamma_{res} = 0.5\ TeV$ for the LHC

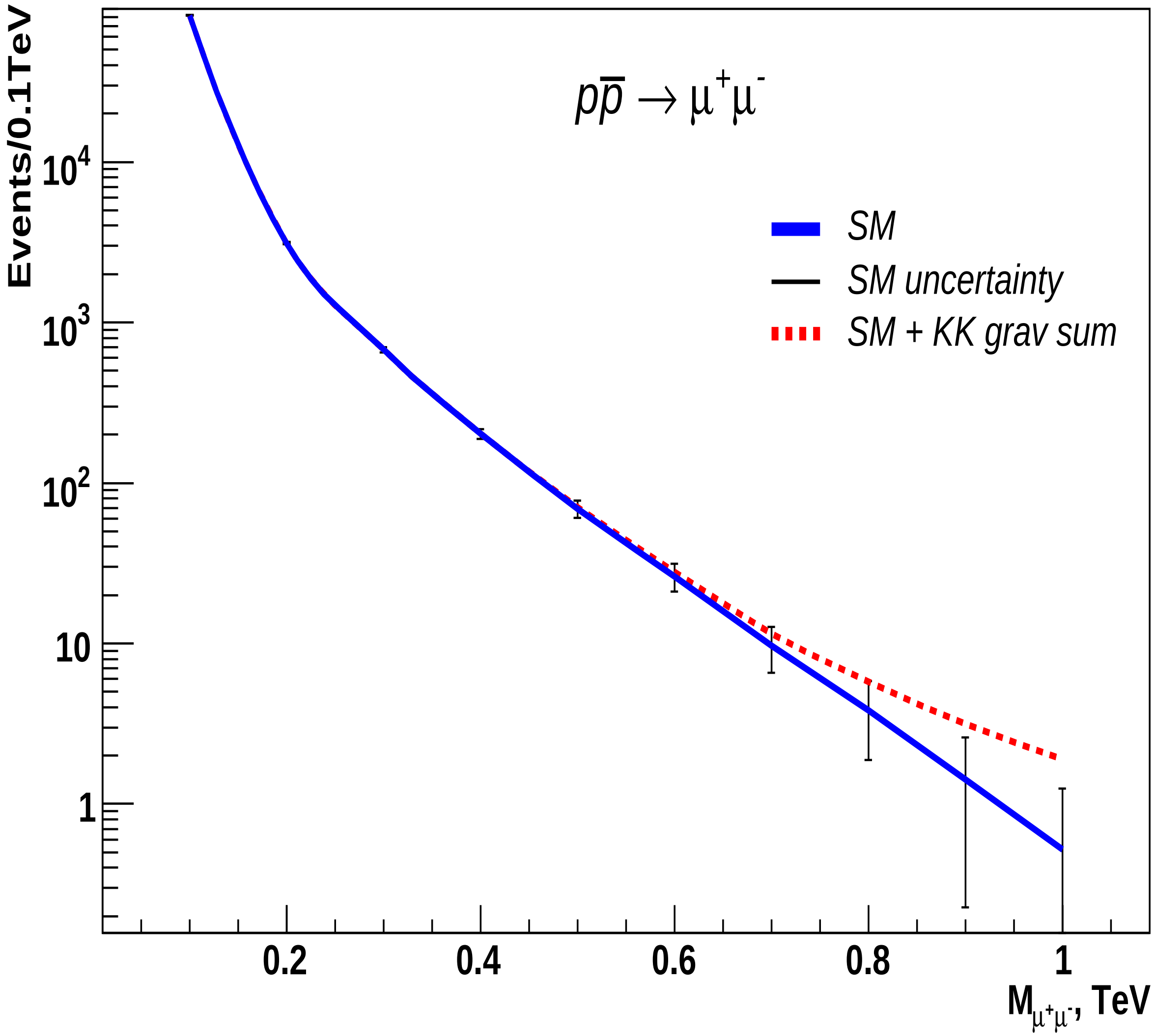


Figure 5: Dilepton invariant mass distribution for 95% CL parameter $\frac{1}{\Lambda_\pi m_1} \times (1TeV)^2$=1.2 for the Tevatron ($L = 10fb^{-1}$)

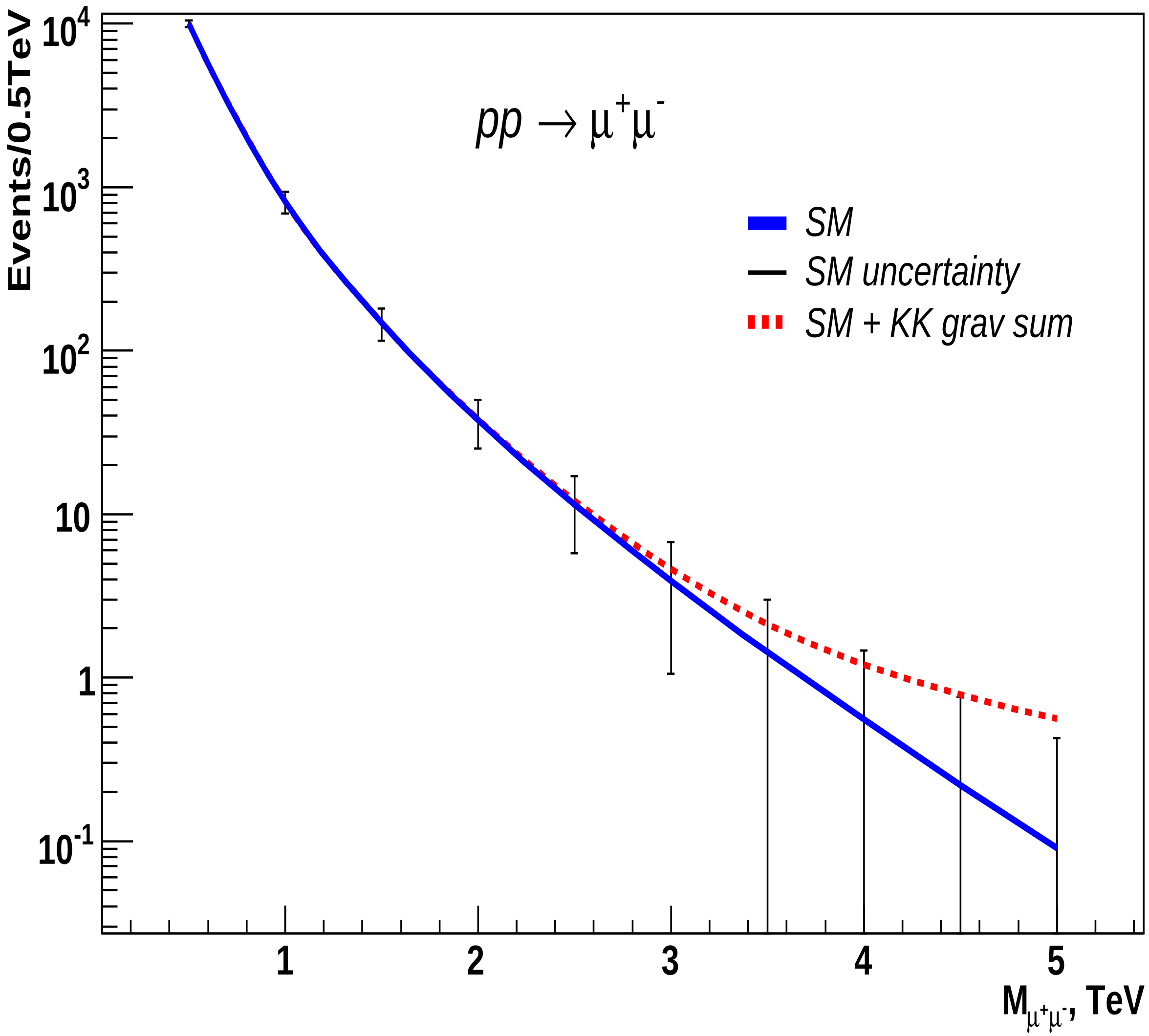


Figure 6: Dilepton invariant mass distribution for 95% CL parameter $\frac{1}{\Lambda_\pi m_1} \times (1TeV)^2$=0.055 for the LHC ($L = 100 fb^{-1}$)

$$\Gamma_1 \approx \frac{m_1^3}{\Lambda_\pi^2 \cdot 4\pi} \frac{97}{80}$$

Estimate

$$\Gamma_1 < \xi m_1, \qquad \xi < 1$$

Thus

$$\frac{m_1}{\Lambda_\pi} < 3.2\sqrt{\xi}$$

Tevatron:

$$\frac{1}{\Lambda_\pi m_1} \leq 1.2\, TeV^{-2} \quad \rightarrow \quad \Lambda_\pi > \frac{0.51}{\xi^{\frac{1}{4}}}\, TeV$$

LHC:

$$\frac{1}{\Lambda_\pi m_1} \leq 0.055\, TeV^{-2} \quad \rightarrow \quad \Lambda_\pi > \frac{2.38}{\xi^{\frac{1}{4}}}\, TeV$$