Massless Mixed-Symmetry Fields in Minkowski space Unfolded

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Motivation and Program

FIELD THEORIES IN HIGHER-DIMENSIONS

$\Downarrow$

MIXED-SYMMETRY FIELDS

$\Downarrow$

INTERACTIONS???

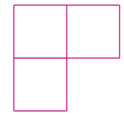
Free higher-spin fields $s = 0, 1, \ldots \infty$	$\oplus$	Unfolded approach	$\Longrightarrow$	Full nonlinear theory **(Vasiliev)**

Massless Fields in Minkowski space

symmetric (Fronsdal:1978)	antisymmetric
$\boxed{\quad s \quad}$	$\boxed{p}$
$\phi_{(\mu_1\mu_2\ldots\mu_s)}$	$\omega_{[\mu_1\mu_2\ldots\mu_p]}$
$\delta\phi_{(\mu_1\mu_2\ldots\mu_s)} = \partial_{(\mu_1}\xi_{\mu_2\ldots\mu_s)}$	$\delta\omega_{[\mu_1\ldots\mu_p]} = \partial_{[\mu_1}\xi_{\mu_2\ldots\mu_p]}$
	$\delta\xi_{[\mu_1\ldots\mu_{p-1}]} = \partial_{[\mu_1}\xi_{\mu_2\ldots\mu_{p-1}]}$
	$\ldots$
	$\delta\xi_\mu = \partial_\mu\xi$
$\Box\phi_{\mu_1\ldots\mu_s} - \partial_{(\mu_1}\partial^\nu\phi_{\nu\mu_2\ldots\mu_s)} + \ldots = 0$	$\Box\omega_{\mu_1\ldots\mu_p} - \partial_{[\mu_1}\partial^\nu\omega_{\nu\mu_2\ldots\mu_p]} = 0$
$\partial^\nu G_{\nu\mu_1\ldots\mu_{s-1}} \equiv 0$	$\partial^\nu G_{\nu\mu_1\ldots\mu_{s-1}} \equiv 0$
	$\ldots$
$\phi^{\nu\rho}{}_{\nu\rho\mu_5\ldots\mu_s} = 0, \quad \xi^\nu{}_{\nu\mu_3\ldots\mu_{s-1}} = 0$	

!gauge symmetry fixes equations

Simplest Mixed-Symmetry Field

The 'Hook' 

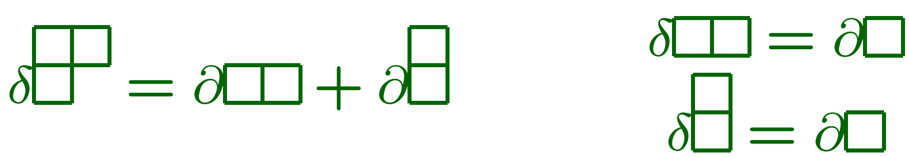
Curtright:1980, Aulakh, Coh, Ouvry:1986

$$\phi_{\mu\nu,\lambda} = -\phi_{\nu\mu,\lambda} \qquad \phi_{\mu\nu,\lambda} + \phi_{\nu\lambda,\mu} + \phi_{\lambda\mu,\nu} = 0$$

$$\Box\phi_{\mu\mu,\lambda} + 2\partial_{[\mu}\partial^{\lambda}\phi_{\mu]\lambda,\nu} - \partial_\nu\partial^\lambda\phi_{\mu\mu,\lambda} - 2\partial_\nu\partial_{[\mu}\phi_{\mu]\lambda,}{}^{\lambda} = 0$$

$$\delta\phi_{\mu\mu,\nu} = \partial_{[\mu}\xi^S_{\mu]\nu} + \partial_{[\mu}\xi^A_{\mu]\nu} - \partial_\nu\xi^A_{\mu\mu}$$

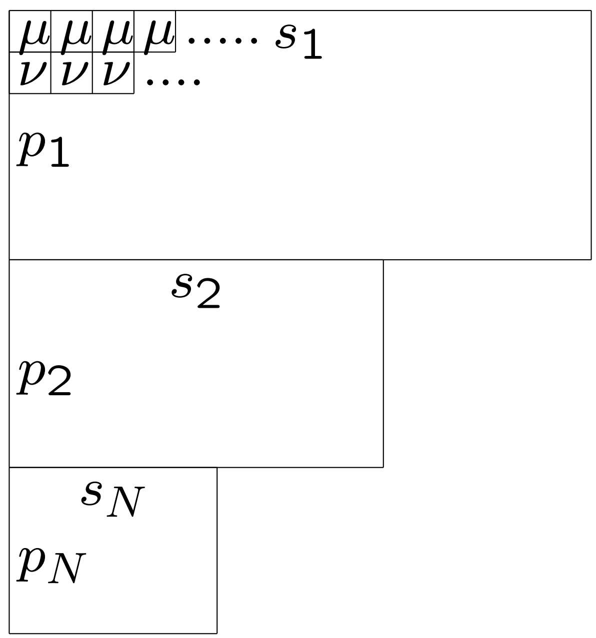
$$\delta\phi_{\mu\mu,\nu} = 0 \qquad \begin{aligned} \delta\xi^A_{\mu\nu} &= \tfrac{2}{3}(\partial_\mu\xi_\nu - \partial_\nu\xi_\mu) \\ \delta\xi^S_{\mu\nu} &= \partial_\mu\xi_\nu + \partial_\nu\xi_\mu \end{aligned}$$

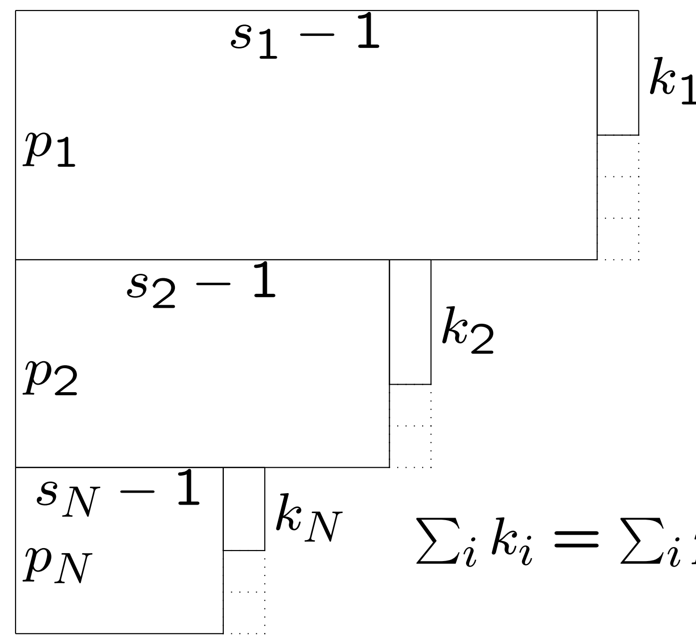


!gauge symmetry fixes equations

General Mixed-Symmetry Field in Minkowski space

Labastida; Bekaert, Boulanger





$\sum_i k_i = \sum_i p_i - r$

$\phi_{\mu(s_1),\nu(s_1),\ldots}$:

$\phi_{\ldots,(\mu\ldots\mu,\mu)\rho\ldots\rho,\ldots} = 0$

$\delta\phi = \sum_i \partial\xi^i$

$\delta\xi^i = \sum_j \partial\xi^{i,j}$

$\ldots$

!offshell symmetry implies unobvious constraints

Unfolding & FDA

Vasiliev, Sullivan, Fre, D'Auria

$$W^A_{\mathbf{q}} \equiv W^A_{\mu_1 \ldots \mu_q} dx^{\mu_1} \wedge \ldots \wedge dx^{\mu_q}$$

$dW^A = F^A(W)$	Equations
$F^B \frac{\delta F^A(W)}{\delta W^B} \equiv 0$	Bianchi/Jacobi identities
$\delta W^A_{\mathbf{q}} = d\xi^A_{\mathbf{q-1}} + \xi^B \frac{\delta F^A(W)}{\delta W^B}$	Gauge symmetry
$\delta \xi^A_{\mathbf{q-1}} = d\eta^A_{\mathbf{q-2}} - \eta^B \frac{\delta F^A(W)}{\delta W^B}$	Reducible gauge symmetry

$$F^A(W) = \sum_n F^A_{B_1 \ldots B_n} W^{B_1} \wedge \ldots \wedge W^{B_n}$$

$$0 = ddW^A = F^B \frac{\delta F^A(W)}{\delta W^B}$$

Lie algebras & FDA

Vasiliev, Sullivan, Fre, D'Auria

Lie algebras $d\Omega^I = -f^I_{JK}\Omega^J\Omega^K$

$$f^I_{J\mathbf{K}} f^J_{\mathbf{LM}} \Omega^{\mathbf{K}}\Omega^{\mathbf{L}}\Omega^{\mathbf{M}} \equiv 0$$ Jacobi identity

Lie algebra modules $dW^{\mathcal{A}}_{\mathbf{q}} = -\Omega^I f_I{}^{\mathcal{A}}{}_{\mathcal{B}} W^{\mathcal{B}}_{\mathbf{q}}$

$$\Omega^{\mathbf{J}}\Omega^{\mathbf{K}}\left(-f^I_{\mathbf{JK}} f_I{}^{\mathcal{A}}{}_{\mathcal{B}} + f_{\mathbf{J}}{}^{\mathcal{A}}{}_{\mathcal{C}} f_{\mathbf{K}}{}^{\mathcal{C}}{}_{\mathcal{B}}\right) W^{\mathcal{B}}{}_{\mathbf{q}} = 0$$

$$D_\Omega W^{\mathcal{A}}_{\mathbf{q}} \equiv dW^{\mathcal{A}}_{\mathbf{q}} + \Omega^I f_I{}^{\mathcal{A}}{}_{\mathcal{B}} W^{\mathcal{B}}_{\mathbf{q}} = 0$$ Covariant derivative

Lie algebra cohomology $D_\Omega W^{\mathcal{A}}_{\mathbf{p}} = f^{\mathcal{A}}_{\mathcal{B}\,IJ...K}\Omega^I\Omega^J...\Omega^K W^{\mathcal{B}}_{\mathbf{q}}$

$$D_\Omega W_{\mathbf{p}} = f_{pq}(\Omega, ..., \Omega) W_{\mathbf{q}},$$

$$D_\Omega W_{\mathbf{q}} = f_{qr}(\Omega, ..., \Omega) W_{\mathbf{r}},$$

$$D_\Omega W_{\mathbf{r}} = ...$$

Free Fields Unfolded

Vasiliev

Lie-algebraic	Field-theoretical
Space-time symmetry Lie algebra $\mathfrak{g} = \mathfrak{iso}(d-1,1)$ $d\Omega + \Omega\Omega = 0$	$\Omega^I = \{\varpi_\mu^{a,b} dx^\mu, h_\mu^a dx^\mu\}$ $dh^a + \varpi^{a,}{}_b h^b = 0$ $d\varpi^{a,b} + \varpi^{a,}{}_c \varpi^{c,b} = 0$ $\varpi_\mu^{a,b} = 0$ and $h_\mu^a = \delta_\mu^a$
$\mathfrak{g}$-modules	a number of Lorentz tensors
$\mathfrak{g}$-cocycles	contractions with h^a

$$\mathcal{D}\omega = (d + f_1(\Omega) + f_2(\Omega,\Omega) + \ldots)\omega = 0 \qquad \omega \in \mathcal{W}_{\mathbf{p}}$$

$$\delta\omega = \mathcal{D}\xi \qquad \xi \in \mathcal{W}_{\mathbf{p-1}}$$

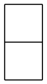
$$\mathcal{D}^2 = 0$$

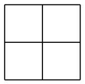
$$D_L \omega^k = f(h,..,h)\omega^{k+1} = \sigma_-(\omega^{k+1})$$

Free Spin-two

$$\delta\phi_{\mu\nu} = \partial_\mu \xi_\nu + \partial_\nu \xi_\mu$$

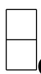
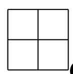
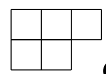
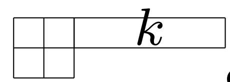
 $e_{\mathbf{1}}^a \equiv e_\mu^a$ $\quad \delta e^a = d\xi^a + h_b \eta^{ab}$ $\quad de^a = h_b \omega^{ab}$ $\quad h_b d\omega^{ab} \equiv 0$

 $\omega_{\mathbf{1}}^{ab} \equiv \omega_\mu^{ab}$ $\quad \delta\omega^{aa} = d\eta^{aa}$ $\quad d\omega^{aa} = h_b h_c C^{aa,bc}$ $\quad h_b h_c dC^{aa,bc} \equiv 0$

 $C_{\mathbf{0}}^{aa,bb} \equiv C^{aa,bb}$ $\quad \delta C^{aa,bb} = 0$ $\quad dC^{aa,bb} = h_c C^{aa,bb,c}$ $\quad \ldots$

$$\phi_{\mu\nu} = h_{a\nu} e_\mu^a + h_{a\mu} e_\nu^a$$

$$\mathcal{W}_{\mathbf{q}} = \{ \quad W_{\mathbf{q}}^a, \quad W_{\mathbf{q}}^{[aa]}, \quad W_{\mathbf{q-2}}^{[aa],[bb]}, \quad W_{\mathbf{q-2}}^{[aa],[bb],c}, \ldots \quad W_{\mathbf{q-2}}^{[aa],[bb],c(k)}, \quad \ldots\}$$

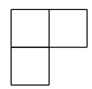
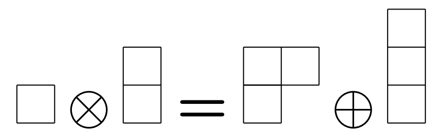
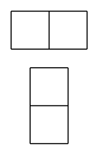
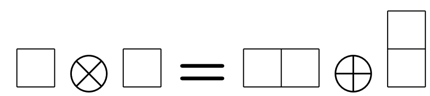
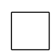
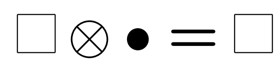
$\mathcal{W}_{\mathbf{q}} = \{$  $_{\mathbf{q}},$  $_{\mathbf{q}},$  $_{\mathbf{q-2}},$  $_{\mathbf{q-2}}, \ldots$  $_{\mathbf{q-2}}, \quad \ldots\}$

$$\mathcal{D} = d + \sigma_-$$

$$\mathcal{D}\omega_{\mathbf{1}} = 0, \qquad \delta\omega_{\mathbf{1}} = \mathcal{D}\xi_{\mathbf{0}}$$

$$\omega_{\mathbf{1}} \in \mathcal{W}_{\mathbf{1}}, \ \xi_{\mathbf{0}} \in \mathcal{W}_{\mathbf{0}}$$

Hook Unfolded, I

$\phi_{\mu\nu,\lambda}$		$e^a_{\mathbf{2}}$	$\delta e^a_{\mathbf{2}} = d\xi^a_{\mathbf{1}}$	
$\xi^S_{\mu\nu}$ $\xi^A_{\mu\nu}$		$\xi^a_{\mathbf{1}}$	$\delta \xi^a_{\mathbf{1}} = d\xi^a_{\mathbf{0}}$	
ξ_μ		$\xi^a_{\mathbf{0}}$	$\delta \xi^a_{\mathbf{0}} = 0$	

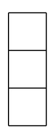
$$h_{a\lambda} e^a_{\mu\nu} = e_{\lambda|\mu\nu} = \phi_{\mu\nu,\lambda} + \psi_{\mu\nu\lambda}$$

$$h_{a\mu} \xi^a_\nu = \xi_{\mu|\nu} = \xi^S_{\mu\nu} + \xi^A_{\mu\nu}$$

$$h_{a\mu} \xi^a = \xi_\mu$$

Hook Unfolded, II

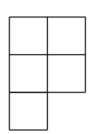
extra field ⇒ algebraic symmetry ⇒ new gauge field ⇒ Jacobi identity ⇒

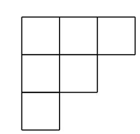


$$\delta e^a_{\mathbf{2}} = h_c h_b \eta^{abc}_{\mathbf{0}} \qquad d e^a_{\mathbf{2}} = h_c h_b \omega^{abc}_{\mathbf{1}} \qquad h_b h_c d\omega^{[abc]}_{\mathbf{1}} \equiv 0$$

new field ⇒ Jacobi identity ⇒ new field

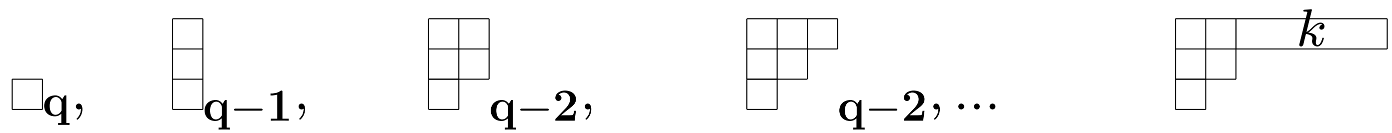
$$d\omega^{[abc]}_{\mathbf{1}} = h_d h_f C^{[abc],[df]}_{\mathbf{0}} \qquad h_d h_f dC^{[abc],[df]}_{\mathbf{0}} \equiv 0 \qquad dC^{[abc],[df]}_{\mathbf{0}} = h_g C^{[abc],[df]}_{\mathbf{0}}$$





Hook, summary

$$\mathcal{W}_{\mathbf{q}} = \{ \quad W^{a}_{\mathbf{q}}, \quad W^{[abc]}_{\mathbf{q-1}}, \quad W^{[abc],[df]}_{\mathbf{q-2}}, \quad W^{[abc],[df],g}_{\mathbf{q-2}}, \dots \quad W^{[abc],[df],g(k)}_{\mathbf{q-2}}, \quad \dots\}$$

$\mathcal{W}_{\mathbf{q}} = \{$ 

$\dots\}$

$$\mathcal{D}\omega_{\mathbf{2}} = 0, \qquad \delta\omega_{\mathbf{2}} = \mathcal{D}\xi_{\mathbf{1}}, \qquad \delta\xi_{\mathbf{1}} = \mathcal{D}\xi_{\mathbf{0}}$$

$\omega_{\mathbf{2}} \in \mathcal{W}_{\mathbf{2}}$, $\xi_{\mathbf{1}} \in \mathcal{W}_{\mathbf{1}}$ and $\xi_{\mathbf{0}} \in \mathcal{W}_{\mathbf{0}}$

General case

p-levels for height-p $\mathbf{Y}$ $\Rightarrow$ $e_{\mathbf{p}}^{\mathbf{Y}_0}$, $\xi_{\mathbf{p-1}}^{\mathbf{Y}_0},\ldots,\xi_{\mathbf{0}}^{\mathbf{Y}_0}$; $\mathbf{Y}_0 = \mathbf{Y}$ without column $\Rightarrow$ a lot of extra fields/parameters

algebraic symmetry

$$\delta e_{\mathbf{p}}^{\mathbf{Y}_0} = \sigma_-(\eta_{\mathbf{q-1}}^{\mathbf{Y}_1})$$
$$\delta \xi_{\mathbf{p-1}}^{\mathbf{Y}_0} = \sigma_-(\eta_{\mathbf{q-2}}^{\mathbf{Y}_1})$$
$$\ldots$$

$\Rightarrow$ new gauge field $de_{\mathbf{p}}^{\mathbf{Y}_0} = \sigma_-(\omega_{\mathbf{q}}^{\mathbf{Y}_1})$ $\Rightarrow$ Jacobi identity $\sigma_-(d\omega_{\mathbf{q}}^{\mathbf{Y}_1}) \equiv 0$ $\Rightarrow$

$\Rightarrow$ new gauge field $d\omega_{\mathbf{q}}^{\mathbf{Y}_1} = \sigma_-(\omega_{\mathbf{r}}^{\mathbf{Y}_2})$ $\Rightarrow$ $\ldots$ $\Rightarrow$ Weyl tensor $\Rightarrow$ $\ldots$

$$\mathcal{W}_{\mathbf{p}} = \{e_{\mathbf{p}}^{\mathbf{Y}_0}, \omega_{\mathbf{q}}^{\mathbf{Y}_1}, \omega_{\mathbf{r}}^{\mathbf{Y}_2}, \ldots\}$$

$$\mathcal{D}\omega_{\mathbf{p}} = 0$$
$$\delta\omega_{\mathbf{p}} = \mathcal{D}\xi_{\mathbf{p-1}}$$
$$\delta\xi_{\mathbf{p-1}} = \mathcal{D}\xi_{\mathbf{p-2}}$$
$$\ldots$$

Local Action

$$S = \langle de_{\mathbf{p}}^{\mathbf{Y}_0} + \tfrac{1}{2}\sigma_-(\omega_{\mathbf{q}}^{\mathbf{Y}_1})|\omega_{\mathbf{q}}^{\mathbf{Y}_1}\rangle$$

$$\delta e_{\mathbf{p}}^{\mathbf{Y}_0} = d\xi_{\mathbf{p-1}}^{\mathbf{Y}_0} + \sigma_-(\xi_{\mathbf{q-1}}^{\mathbf{Y}_1})$$
$$\delta \omega_{\mathbf{q}}^{\mathbf{Y}_1} = d\xi_{\mathbf{q-1}}^{\mathbf{Y}_1} + \sigma_-(\xi_{\mathbf{r-1}}^{\mathbf{Y}_2})$$

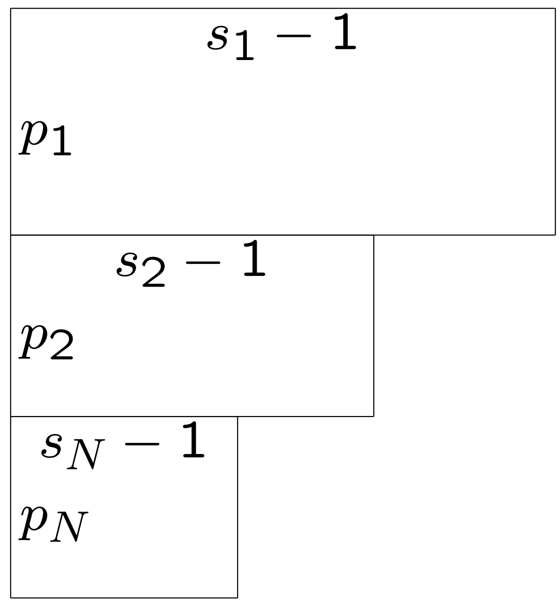
$$S = \int \left[de_{\mathbf{2}}^{a} + \tfrac{1}{2}h_b h_c \omega_{\mathbf{1}}^{abc}\right]\omega_{\mathbf{1}}^{aaa}\epsilon_{aaaab(d-4)}h^b ... h^b$$

!two-term action for any spin

Conclusions

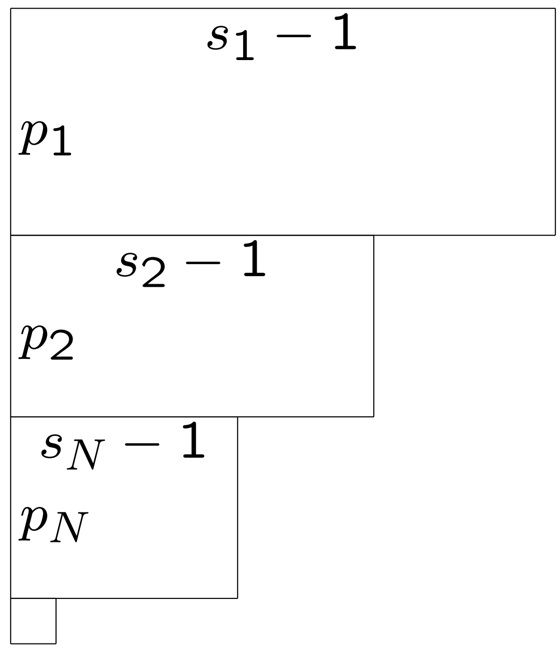
- $\mathcal{D}\omega_{\mathbf{p}} = 0$, $\delta\omega_{\mathbf{p}} = \mathcal{D}\xi_{\mathbf{p-1}}$, $\mathcal{D}^2 = 0$
- $d\omega^k = \sigma_-(\omega^{k+1})$, $\mathcal{D} = d + \sigma_-$, $(\sigma_-)^2 = 0$
- **Lie modules/cohomology interpretation**
- **Manifest gauge invariance**
- **Manifest general covariance**
- **Simple field content, tensor-valued forms**
- **Two-term action for any spin**
- **nonlinear deformations???**

General Case



form degree $= p_1 + p_2 + \ldots + p_N$

General Case



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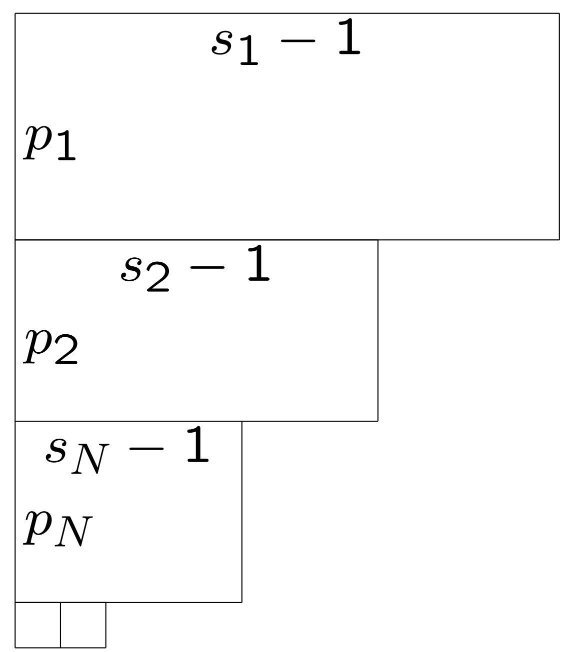

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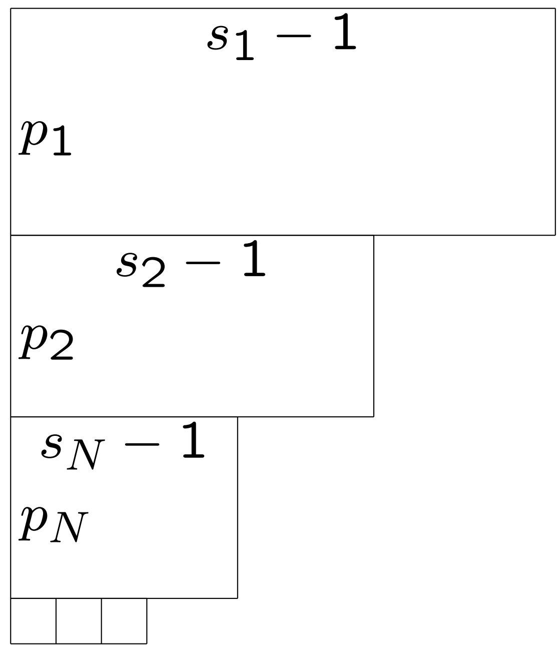

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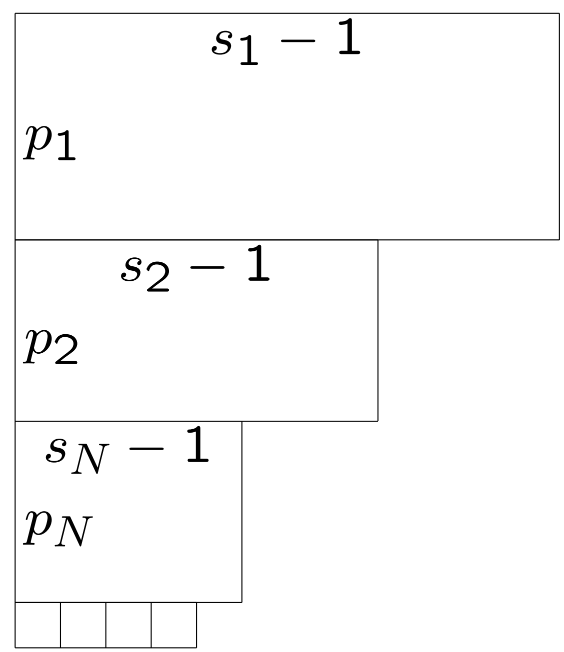

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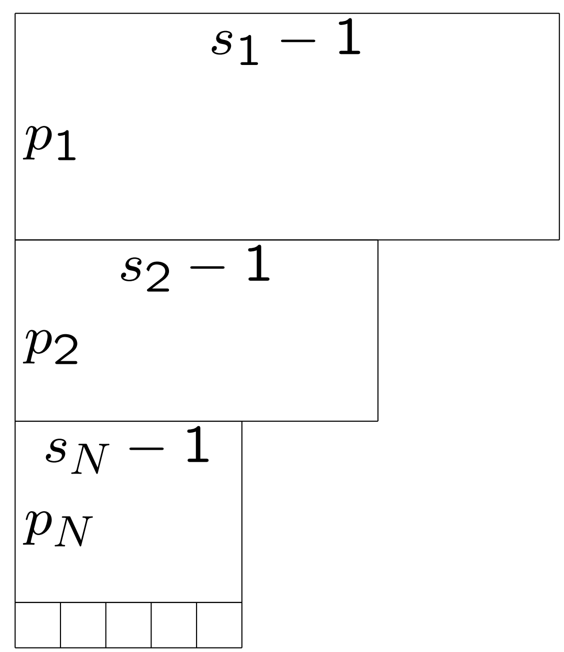
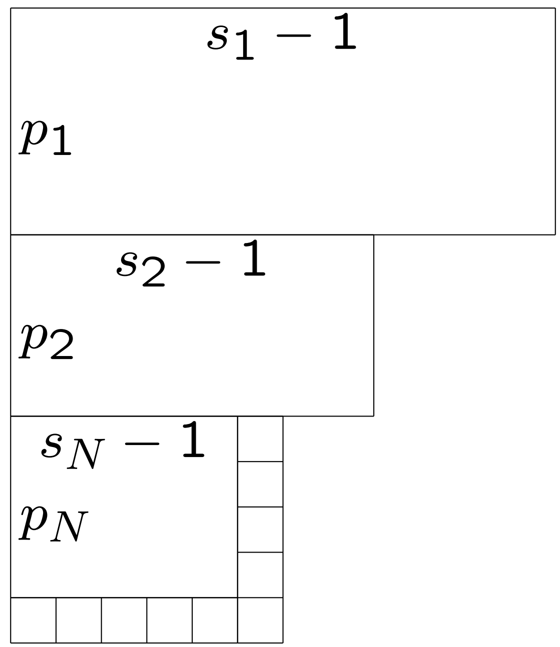
General Case



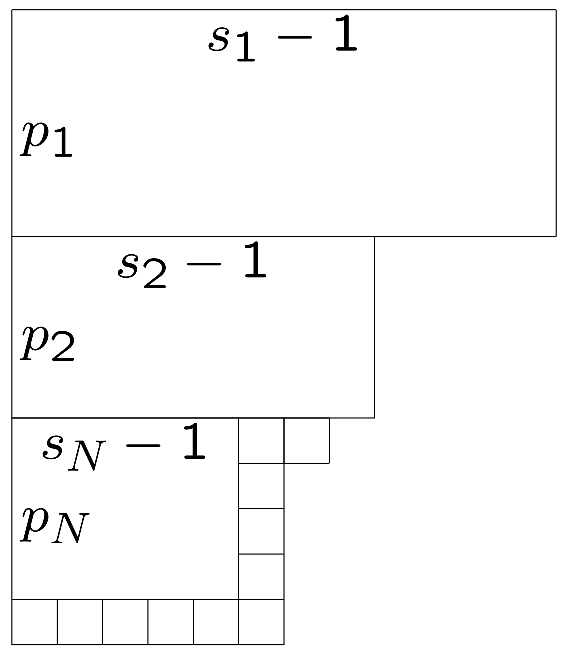
form degree $= p_1 + p_2 + \ldots + p_N$

General Case



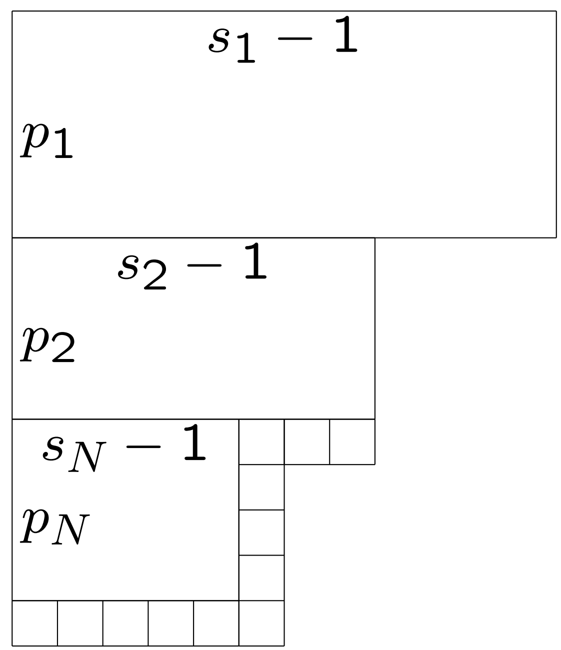
form degree $= p_1 + \ldots + p_{N-1}$

General Case



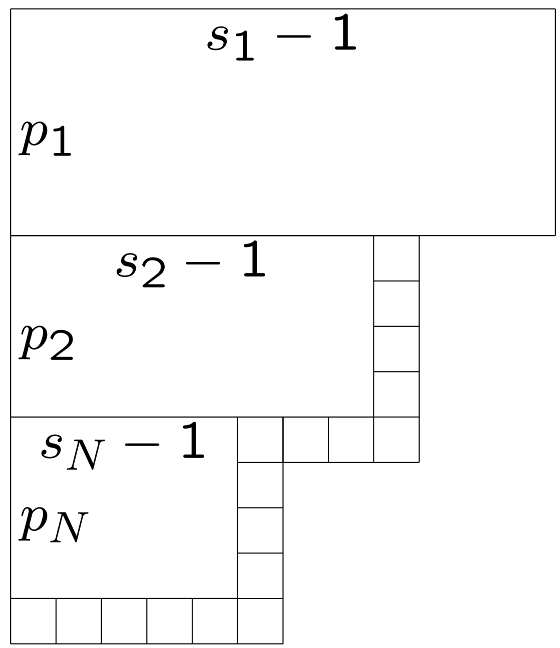
form degree $= p_1 + \ldots + p_{N-1}$

General Case



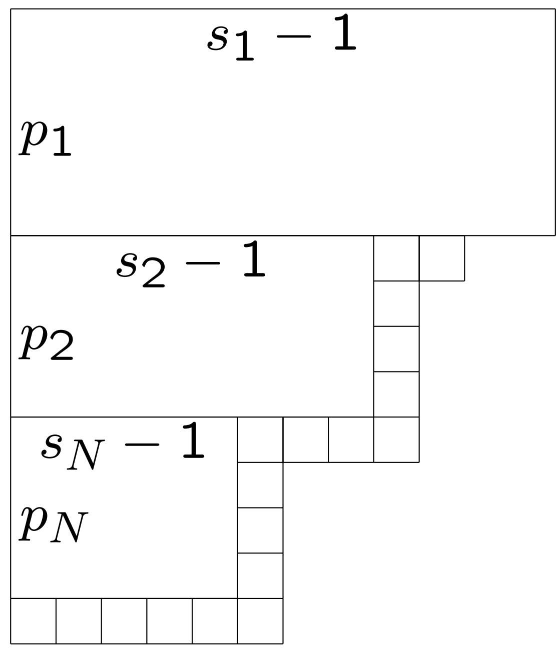
form degree $= p_1 + ... + p_{N-1}$

General Case



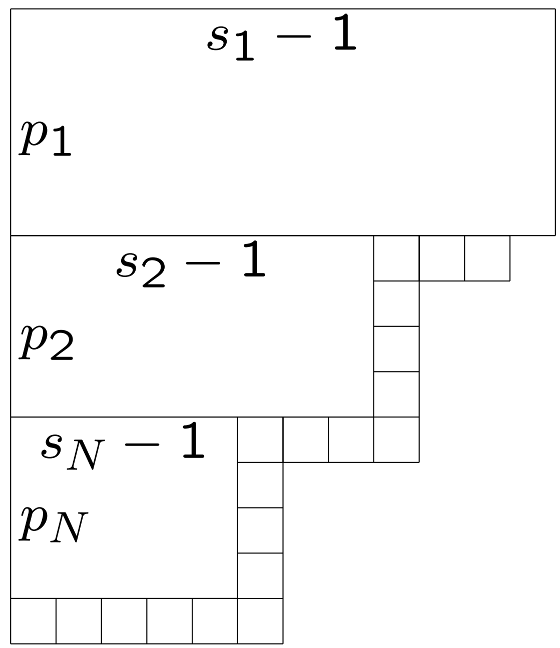
form degree $= p_1$

General Case



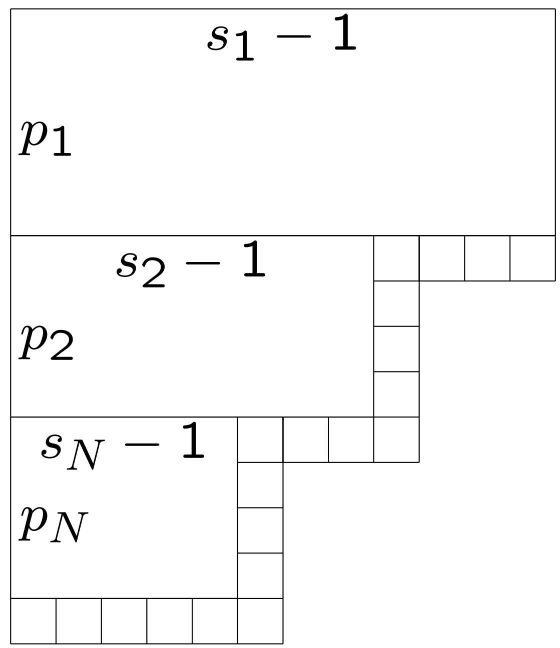
form degree $= p_1$

General Case



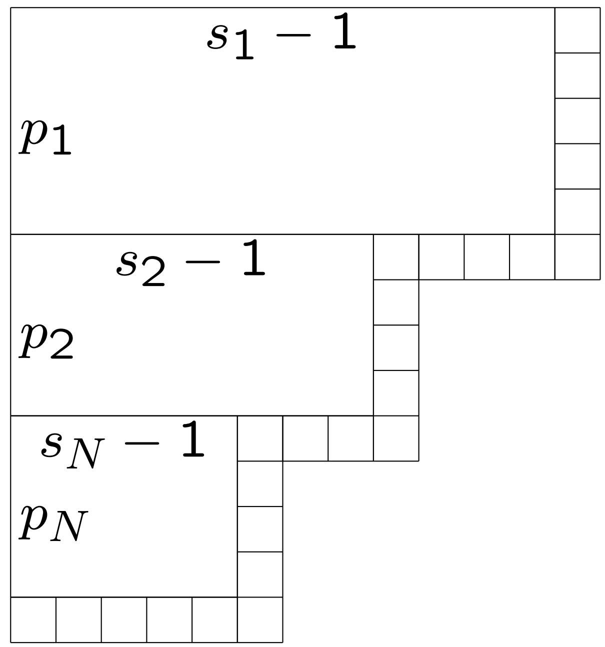
form degree $= p_1$

General Case



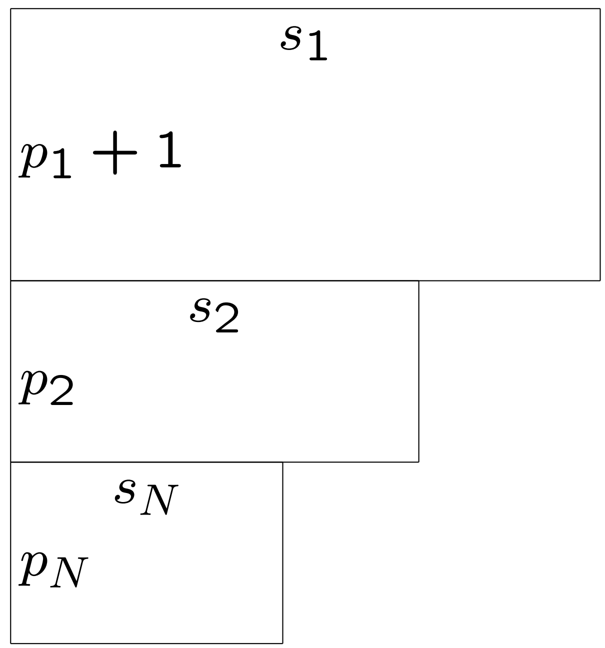
form degree $= p_1$

General Case



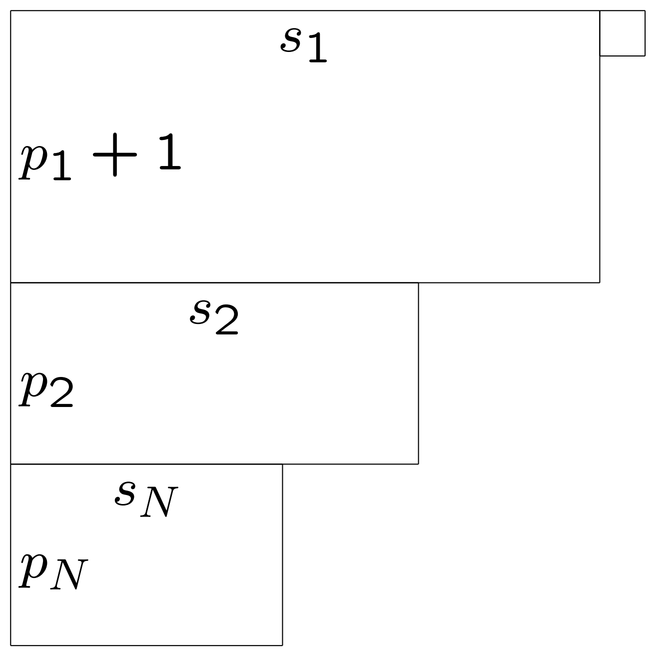
form degree =0

General Case



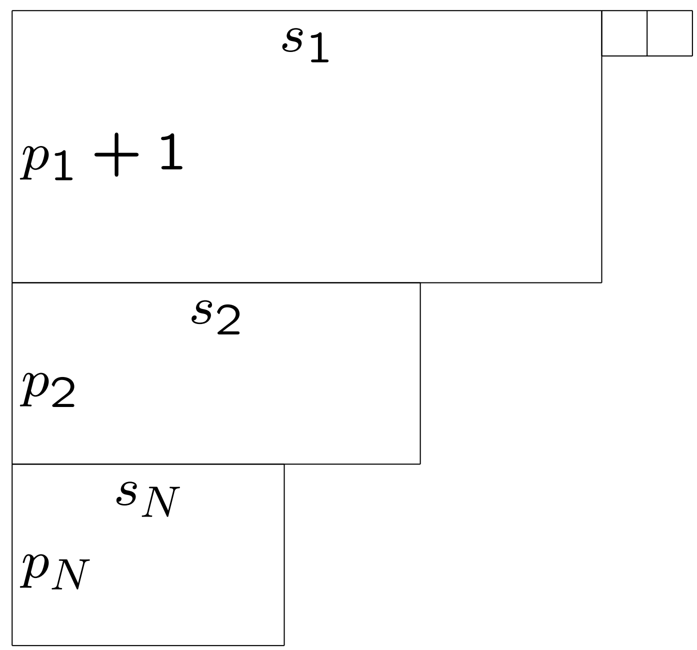
form degree =0

General Case



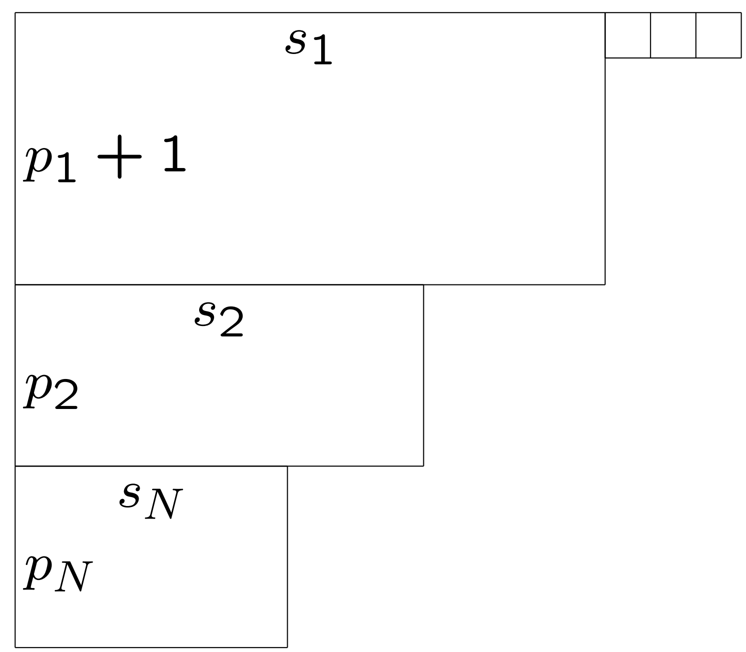
form degree $=0$

General Case



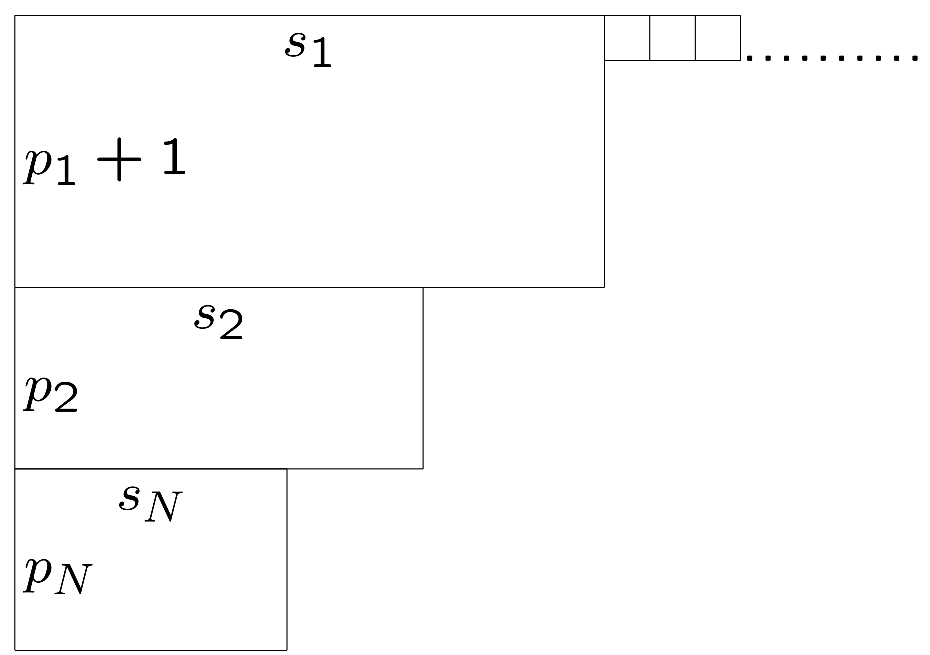
form degree =0

General Case



form degree =0

General Case



form degree =0