Some kinds of matrix models at large N

D=0 Hermitian matrix model ϕ^4

E. Brezin, C. Itzykson, G. Parisi, J. B. Zuber,

Comm. Math. Phys. (1978)

$$S(\Phi) = \frac{1}{2} tr\Phi^2 + \frac{g}{4N} tr\Phi^4$$

Φ is hermitian $N \times N$ matrix.

Statistical integral has form

$$Z = \int d\Phi e^{-S(\Phi)}.$$

The planar vacuum energy

$$E^0(g) = -\lim_{N\to\infty} \frac{1}{N^2} \ln Z.$$

Here convenient measure express via the eigenvalues λ_i of matrix Φ

$$Z \sim \int \prod_i d\lambda_i \prod_{i<j} (\lambda_i - \lambda_j)^2 \exp -(\frac{1}{2} \sum_i \lambda_i^2 + \frac{g}{4N} \sum_i \lambda_i^4).$$

Planar limit $N \to \infty$

$$\lambda(\frac{i}{N}) = \frac{\lambda_i}{\sqrt{N}},$$

equation for density of eigenvalues $u(\lambda)$

$$\frac{1}{2}\lambda + \frac{1}{2} g\lambda^3 = \fint_{-2a}^{2a} d\mu \frac{u(\mu)}{\lambda - \mu}, \qquad |\lambda| \le 2a.$$

Onecut solution at segment $[-2a, 2a]$

$$u_A = \frac{1}{\pi}(\frac{1}{2}g\lambda^2 - \frac{1}{2} + ga^2)\sqrt{4a^2 - \lambda^2},$$

here

$$4ga^4 + a^2 - 1 = 0.$$

The vacuum energy for this model is

$$E^0(g) - E^0(0) = \frac{1}{24}(a^2 - 1)(9 - a^2) - \frac{1}{2}\log a^2.$$

General one-matrix action look as

$$S(\Phi) = \sum_{n=1}^{\infty} t_n tr\Phi^n.$$

Some another methods of planar investigation for one-matrix model

The variational method

A.A. Slavnov, Phys.Lett. B, (1983);

The effective action method

M. Vanderkelen, S. Schelstraete, H. Verschelde. Z.Physik C, (1980);

Planar parquet approximation

I.Ya. Aref'eva, A.P. Zubarev, Phys. Lett. B, (1996);

The quantum collective field method

A. Jevicki and B. Sakita. Nucl. Phys. B, (1980);

Renormalization group approach to matrix models

E. Brezin, J. Zinn-Justin, Phys. Lett. B, (1992).

Goldstone matrix model and multicut solutions

Y. Shimamune, Phys. Lett. B (1982);
I.Ya. Arefeva, A.S. Ilchev, B.K. Mitriushkin, preprint JINR (1984)
G.M. Cicuta, L. Molinari, E. Montaldi, J. Phys. A: Math. Gen. (1987);
J. Jurkiewicz, Phys. Lett. B, (1990);
K. Demeterfi, N. Deo, S. Jain, C.-I. Tan, Phys. Rev., D (1990)

et all

$$S(\Phi) = -\frac{1}{2}tr\Phi^2 + \frac{g}{4N}tr\Phi^4$$

Here solution of quasiclassical equation exits at two segments [a,b] и [c,d]. This twocut solution determines a parameter ξ

$$\xi = \int_a^b u(\lambda)\, d\lambda.$$

When $\xi = \frac{1}{2}$ we have symmetric solution. Exact planar vacuum energy for symmetric phase look as

$$E^0(g) - E^0(0) = -\frac{1}{16g} + \frac{1}{4}\log(4g) - \frac{3}{8}.$$

Multitrace matrix model

S. Das, A. Dhar, A. Sengupta, S. Wadia, Mod. Phys. Lett. A (1990)

$$S(\Phi) = \frac{1}{2} tr\Phi^2 + \frac{g}{4N} tr\Phi^4 + \frac{h}{N^2} \left(tr\Phi^2\right)^2.$$

Goldstone multitrace matrix model

A.O. Shishanin, Theor. Math. Phys. (2007)

$$S(\Phi) = -\frac{1}{2} tr\Phi^2 + \frac{g}{N} tr\Phi^4 + \frac{h}{N^2} (tr\Phi^2)^2.$$

The vacuum energy for symmetric solution

$$E^0(g,h) - E^0(0) = -\frac{g}{16(g+h)^2} + \frac{1}{4}\log(4g) - \frac{3}{8}.$$

Planar two-matrix model

C. Itzykson, J.B. Zuber, J. Math. Phys. (1980);

M.L. Mehta, Comm. Math. Phys. (1981)

Action is

$$S(\Phi_1, \Phi_2) = S(\Phi_1) + S(\Phi_2) - c tr\Phi_1\Phi_2,$$

$$S(\Phi_1) = \frac{1}{2} tr\Phi_1^2 + \frac{g}{4N} tr\Phi_1^4$$

Using formula of Itzykson-Zuber

$$Z \sim \int \prod_i dx_i dy_i e^{-(S(x)+S(y)-c\sum_i x_i y_i)}.$$

Biorthogonal polynomials $P_i(x)$

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} exp(-S(x)-S(y)+cxy)P_i(x)P_j(y)dxdy = h_i\delta_{ij}.$$

Recursion equation

$$xP_i(x) = P_{i+1} + R_iP_{i-1}(x) + T_iP_{i-3}(x).$$

In large N limit ($f_i = \frac{h_i}{h_{i-1}}, x = \frac{i}{N}$) we can obtain closed system algebraic equations for $R(x)$, $T(x)$ and $f(x)$.

Multi-matrix models

S. Chadha, G. Mahoux, M.L. Mehta, J. Phys. A: Math. Gen. (1981)

Open matrix chain has action

$$S_{op}(\Phi_1, ..., \Phi_n) = \frac{1}{2}\sum_{i=1}^{n} tr\Phi_i^2 + \sum_{i=1}^{n}\frac{g_i}{4N}tr\Phi_i^4 +$$

$$-c\sum_{i}^{n-1} tr(\Phi_i)tr(\Phi_{i+1}).$$

Here it is possible to use orthogonal polynomials. Closed matrix chain

$$S_{cl}(\Phi_1, ..., \Phi_n) = S_{op}(\Phi_1, ..., \Phi_n) - ctr(\Phi_1)tr(\Phi_n).$$

The exact method of solving this model is unknown.

Another two-matrix model - multitrace two-matrix model

Simplest model with twotrace term

$$S(\Phi_1,\Phi_2)=\frac{1}{2}tr\Phi_1^2+\frac{g_1}{4N}tr\Phi_1^4+\frac{1}{2}tr\Phi_2^2+ \\ +\frac{g_2}{4N}tr\Phi_2^4-ctr(\Phi_1)^2tr(\Phi_2)^2.$$

When $g_1 = g_2$ this model reduced to one-matrix multitrace model.

Multitrace matrix open chain

$$S(\Phi_1,...,\Phi_n)=\frac{1}{2}\sum_{i=1}^{n}tr\Phi_i^2+\sum_{i=1}^{n}\frac{g_i}{4N}tr\Phi_i^4+ \\ -c\sum_{i}tr(\Phi_i)^2tr(\Phi_{i+1})^2.$$

This models can be applied in two-dimensional gravity, supersymmetric gauge theories and AdS/CFT-correspondes.