Lattice calculation of the strange quark contribution to the nucleon's form factors

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work done in collaboration with

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Nucleon form factors

$$\langle \bar{N}(p')|j_\mu|N(p)\rangle = \bar{u}(p')\big(\gamma_\mu F_1(q^2) + \imath\sigma_{\mu\nu}q_\nu F_2(q^2)/(2M)\big)u(p)$$

$$\begin{aligned} G_E(Q^2 \equiv -q^2) &= F_1(Q^2) - Q^2 F_2(Q^2)/(4M^2) \\ G_M(Q^2) &= F_1(Q^2) + F_2(Q^2) \end{aligned}$$

$$\langle \bar{N}(p')|j^5_\mu|N(p)\rangle = \bar{u}(p')\big(\gamma_5\gamma_\mu G_A(Q^2) + \gamma_5 q_\mu G_P(Q^2)/M\big)u(p)$$

G_E, G_M, G_A, G_P determined from $e - p$ and $\nu - p$ scattering experiments at Jlab, Bates, BNL

see e.g. J. Arrington, C.D. Roberts, J.M. Zanotti nucl-th/0611050

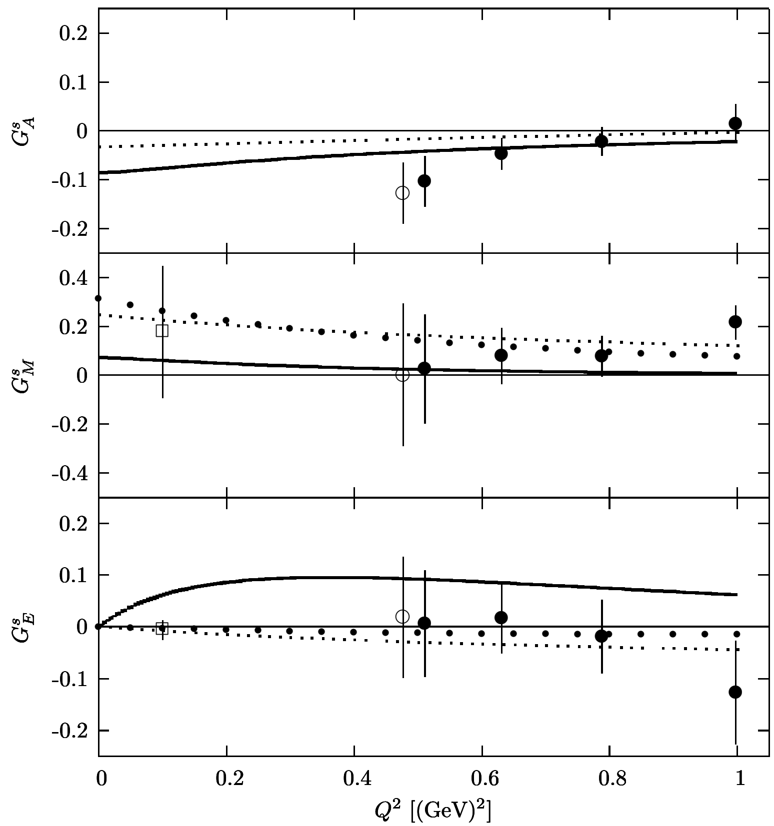
Strange quark contribution to the form factors

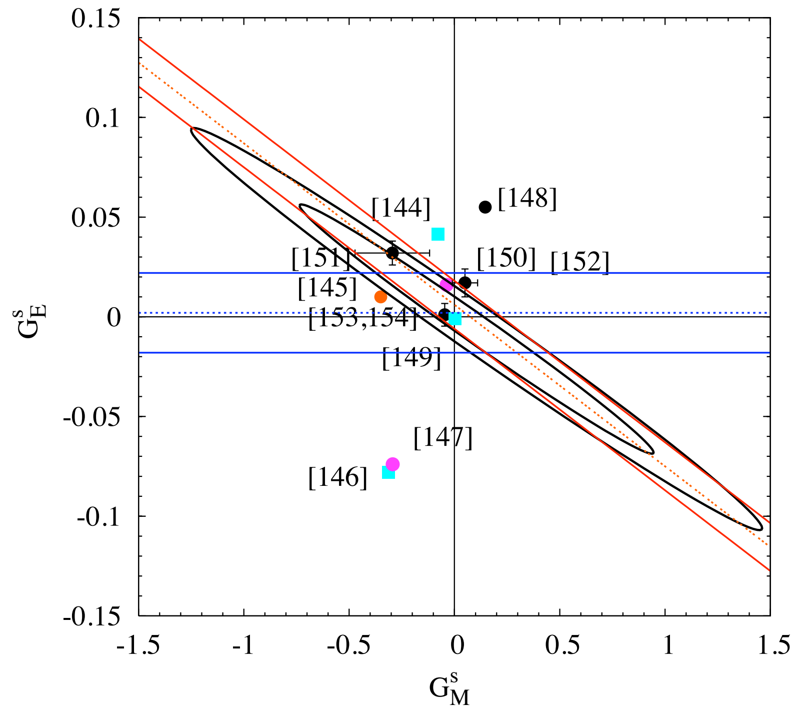
$$
\begin{aligned}
G_E^{\gamma,p} &= \frac{2}{3}G_E^u - \frac{1}{3}G_E^d - \frac{1}{3}G_E^s \\
G_E^{\gamma,n} &= \frac{1}{3}G_E^u - \frac{2}{3}G_E^d - \frac{1}{3}G_E^s \\
G_E^{Z,p} &= \Big(1 - \frac{8}{3}\sin^2\theta_W\Big)G_E^u + \Big(-1 + \frac{4}{3}\sin^2\theta_W\Big)G_E^d \\
&\quad + \Big(-1 + \frac{4}{3}\sin^2\theta_W\Big)G_E^s \\
G_A^{Z,p} &= \frac{1}{2}(-G_A^u + G_A^d + G_A^s)
\end{aligned}
$$

see e.g. Stephen Pate arXiv:0704.111

$G_S^s = \langle \bar{N}(p')|\bar{s}s|N(p)\rangle$: relevant for nucleon structure and for BSM implications (see J. Ellis, K. Olive, C. Savage arXiv:0801.365)

Challenging for experiment and theory





Left: from Pate 2007, points are data, lines are model fits. Right: from Arrington, Roberts and Zanotti 2006, curves are experimental bounds, points are various model or lattice calculations.

Lattice calculation of the strange quark contribution

One should calculate

$$\langle N|\bar{\psi}_s \Gamma \psi_s|N\rangle =$$
$$\lim_{t\to\infty} \frac{\sum_{\vec{x},\vec{y}} e^{\imath(\vec{p}\cdot\vec{x}+\vec{q}\cdot\vec{y})} \langle \psi\psi\psi(\vec{x},t)\ \bar{\psi}_s\Gamma\psi_s(\vec{y},0)\ \bar{\psi}\bar{\psi}\bar{\psi}(\vec{0},-t)\rangle}{\sum_{\vec{x}} \langle \psi\psi\psi(\vec{x},t)\ \bar{\psi}\bar{\psi}\bar{\psi}(\vec{0},-t)\rangle} - \langle \bar{\psi}_s \Gamma \psi_s\rangle$$

Disconnected diagram calculation

For each gauge field configuration one must calculate separately the propagators $P_{cs,c's'}(U;x,y)$ which solve

$$[D(U)P(U)]_{cs}(x) = \delta(x,y)\delta_{c,c'}\delta_{s,s'}$$

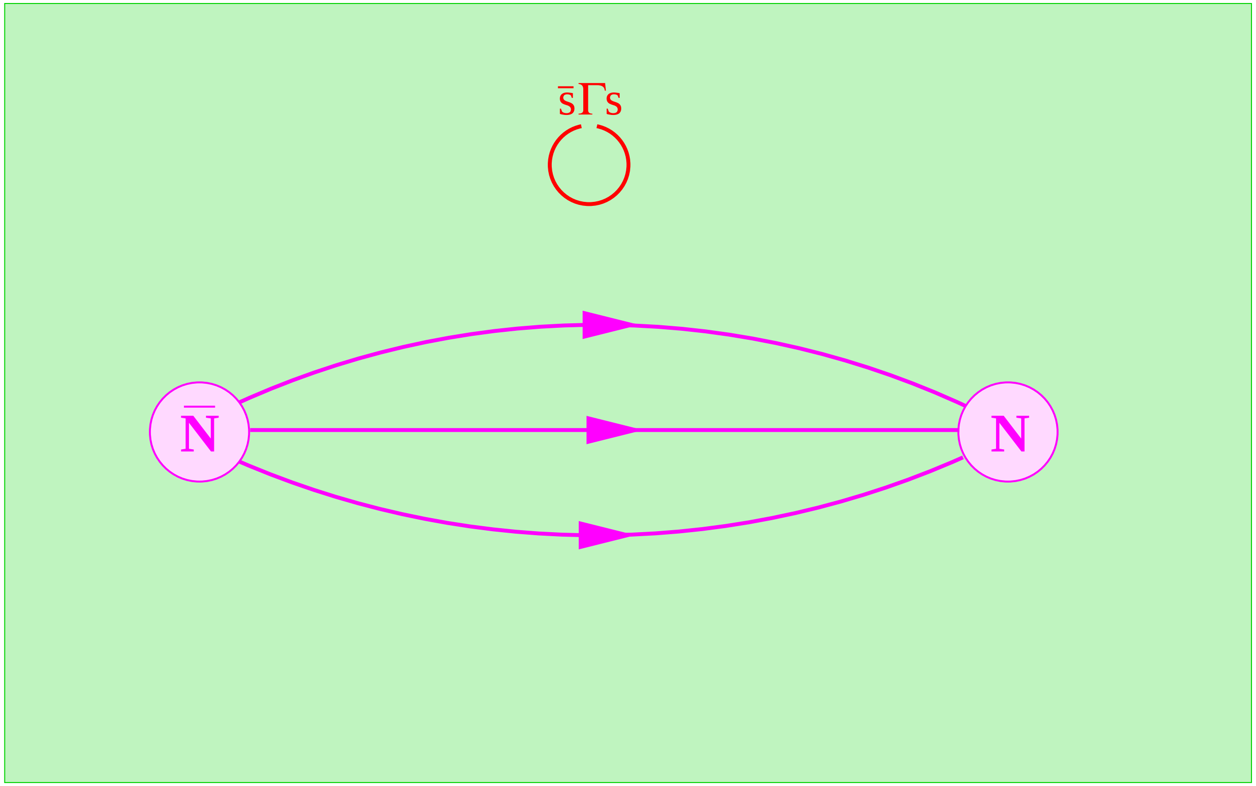
for the light and strange quarks. From the light quark propagators one forms nucleon propagators (with implicit sums over color and spin indices)

$$\sum_{\vec{x}} e^{\imath\vec{p}\cdot\vec{x}}\epsilon_{c_1,c_2,c_3}\epsilon_{c'_1,c'_2,c'_3}\Psi^*_{s_1,s_2,s_3}\Psi_{s'_1,s'_2,s'_3}P_{c_1s_1,c'_1s'_1}(U;\vec{x},t,\vec{0},-t)$$
$$P_{c_2s_2,c'_2s'_2}(U;\vec{x},t,\vec{0},-t)P_{c_3s_3,c'_3s'_3}(U;\vec{x},t,\vec{0},-t)$$

and from the strange quark propagators one calculates

$$\sum_{\vec{y}} e^{\imath\vec{q}\cdot\vec{y}}\mathrm{Tr}[P(U;\vec{y},0,\vec{y},0)\Gamma]$$

Their product is then averaged over the gauge field configurations.



Challenges: a) The strange quark propagator insertion should be calculated for every point of a time slice.

b) The “signal” is the result of a minute correlation between the nucleon propagators and the $\bar{s}s$ insertions.

Stochastic source methods

Calculate $\chi(x)$ which solves

$$(D\chi)(x) = \sum_y \eta(y)\delta(x,y); \quad \chi(x) = \sum_y P(x,y)\eta(y)$$

Average over a large number of η with

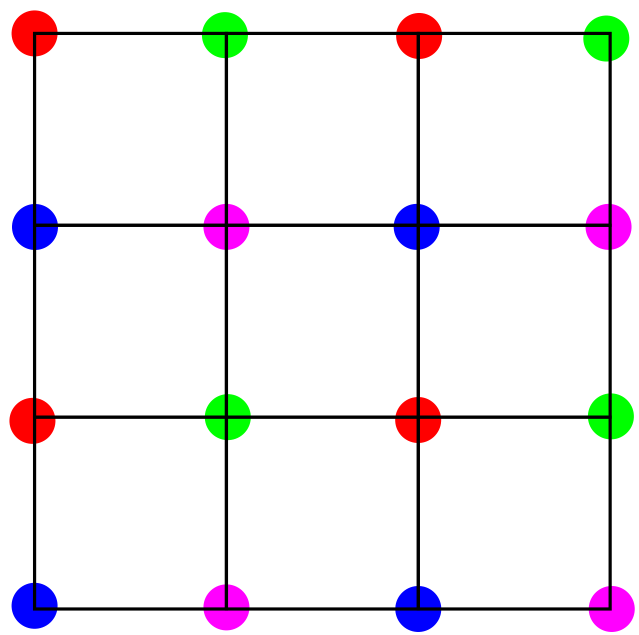
$$\overline{\eta^*(x)\eta(y)} = \delta(x,y)$$

Then

$$\overline{\sum_x \eta^*(x)\chi(x)} = \sum_x P(x,x)$$

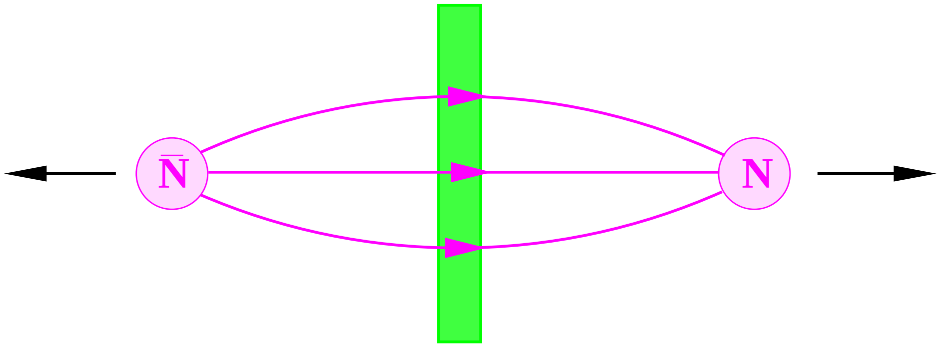
Variance reduction: dilution

The stochastic source methods avoids the need of calculating a separate propagator for each point of the time-slice, but can lead to a high variance and therefore to the need of repeating the calculation with vary many sources.

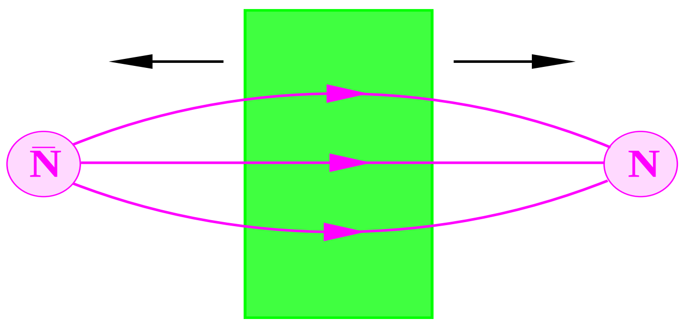


Dilution: One does not populate all of the time slice with sources, but only a subset of points. The increased separation reduces the variance, but one must repeat the calculation for all the subsets.

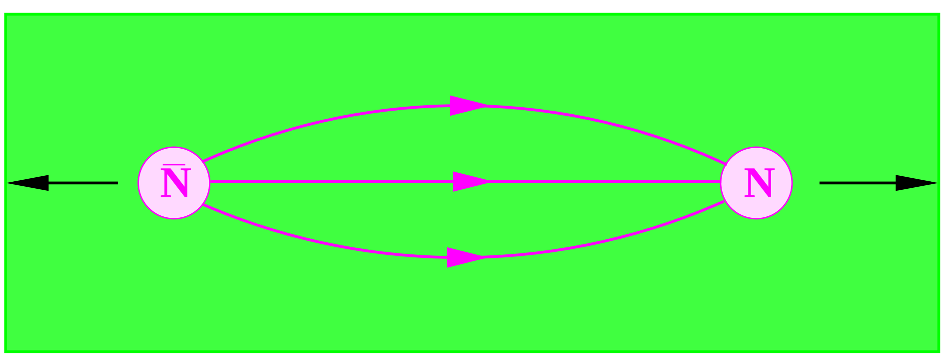
Computational methodologies



Evaluate $\bar{s}\Gamma s$ on a single time slice. Move the nucleon sources in time. (our approach)



Evaluate $\bar{s}\Gamma s$ on a region expanding in time, fit linearly in t. cfr. S.J. Dong, K.F. Liu, A.G. Williams, 1998



Evaluate $\bar{s}\Gamma s$ on all of space-time, move the nucleon sources and take finite differences. cfr. R. Lewis, W. Wilcox, R.M. Woloshyn, 2002

Preliminary results

From research in progress with R. Babich, R. Brower, M. Clark, G. Fleming, J. Osborn + D. Schaich

Results based on 863 gauge field configurations, with two flavors of dynamical quarks (Wilson discretization), on anisotropic $24^3 \times 64$ lattices ($a_s = 0.108(7)\mathrm{fm}$, $a_t = 0.036(2)\mathrm{fm}$), with $m_\pi \approx 400\mathrm{MeV}$, generated by the LHPC collaboration.

$\beta = 5.5;\ \chi_0 = 2.38;\ \kappa \equiv 1(2 + 6/\chi_0 + 2m);$
$m = -0.4125 \to m_\pi = 400(36)\mathrm{MeV};\ m = -0.4086 \to m_\pi = 572(29)\mathrm{MeV};$

we use $m_s = -0.38922$

Dilution

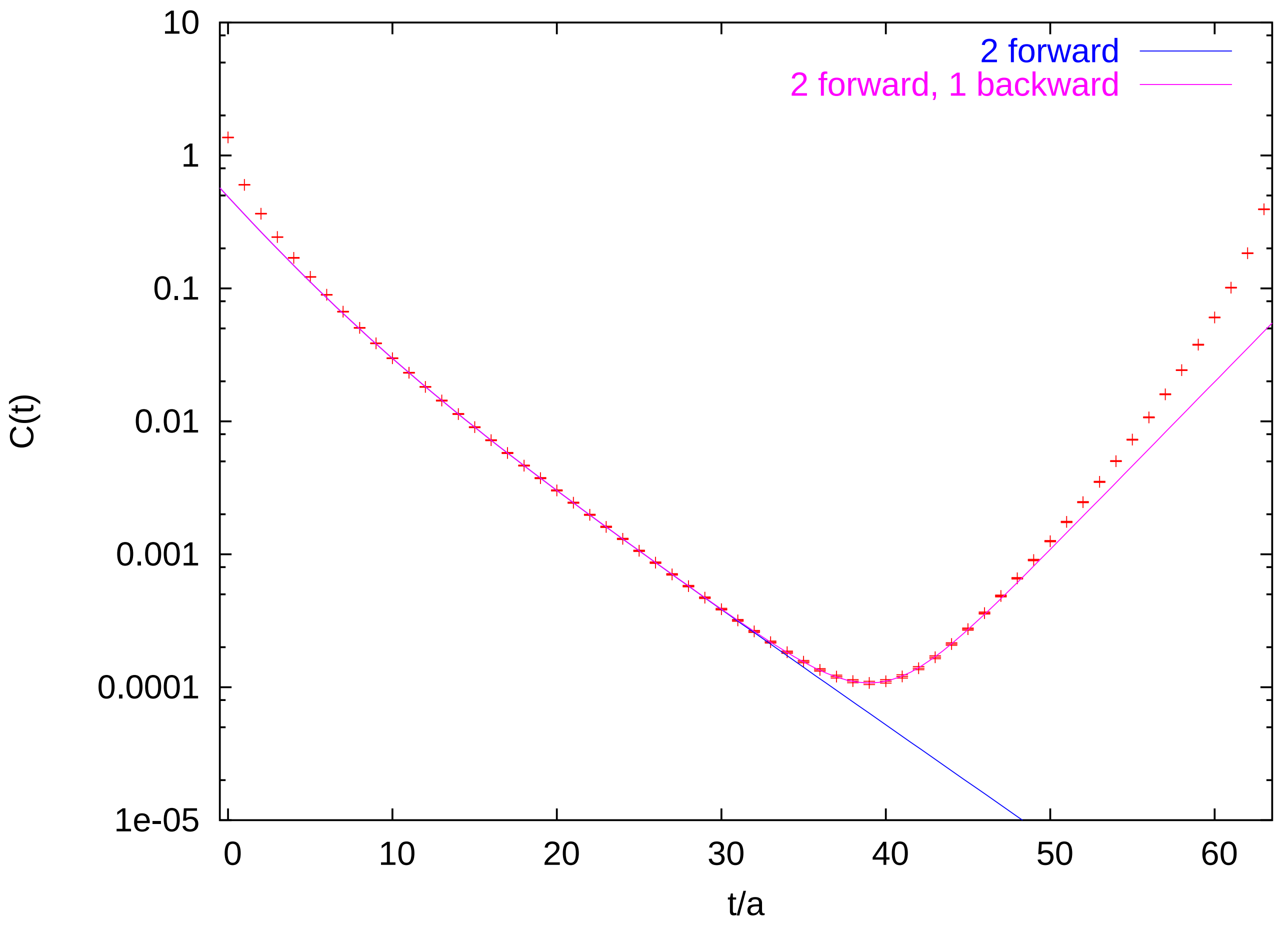
On each lattice configuration we evaluate the strange quark propagators for all 12 color and spin values of sources placed within the time slices at $t = 7, 23, 39, 65$ and set on the diagonally staggered vertices of a 4^3 spatial sublattice, with separation $6 \times \sqrt{3}$.

We invert $4 \times 4^3/4 = 64$ sources simultaneously, and perform $4 \times 6^3 = 864(\times 12)$ inversions per configuration.

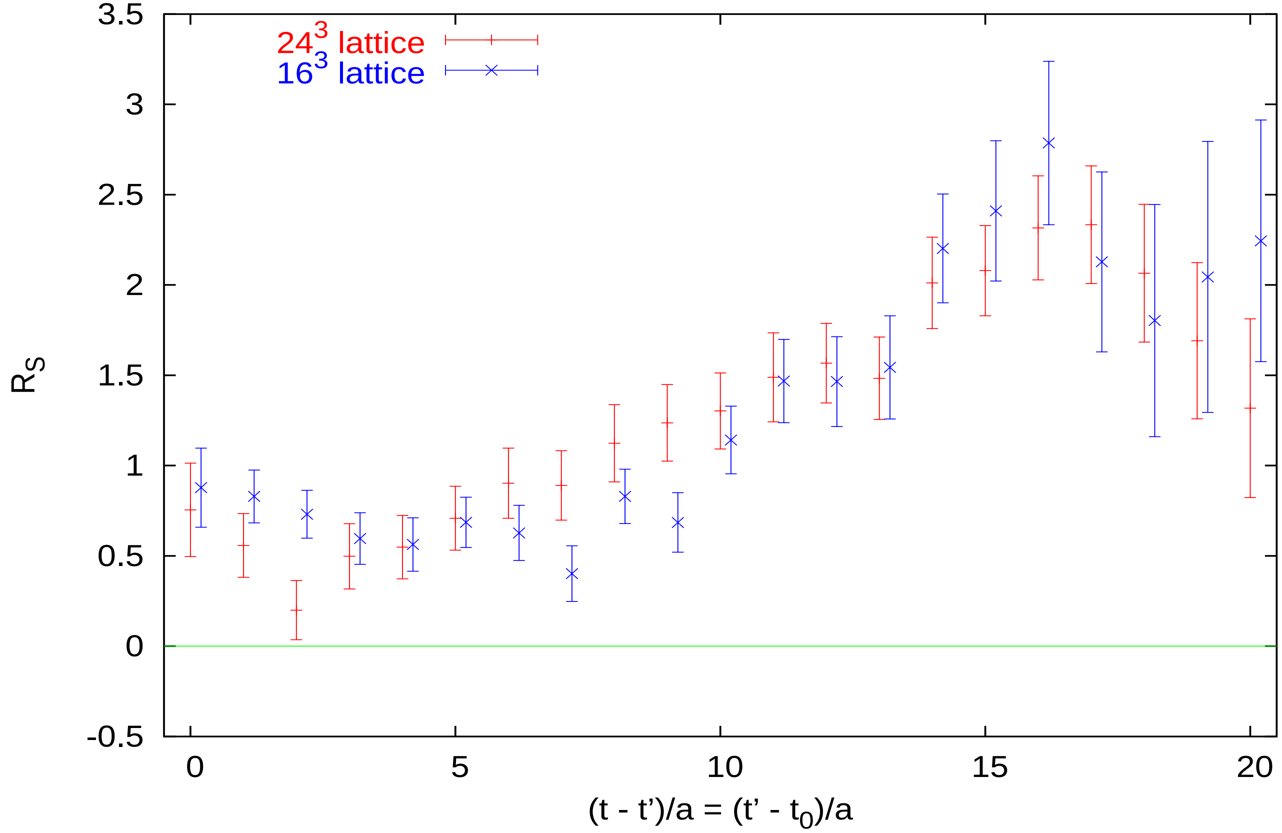
We do not use stochastic noise and rely on separation and gauge variance to avoid non-diagonal contributions.

We calculate nucleon propagators with spread out quark sources at a fixed space point and at all values of t.

The nucleon propagator

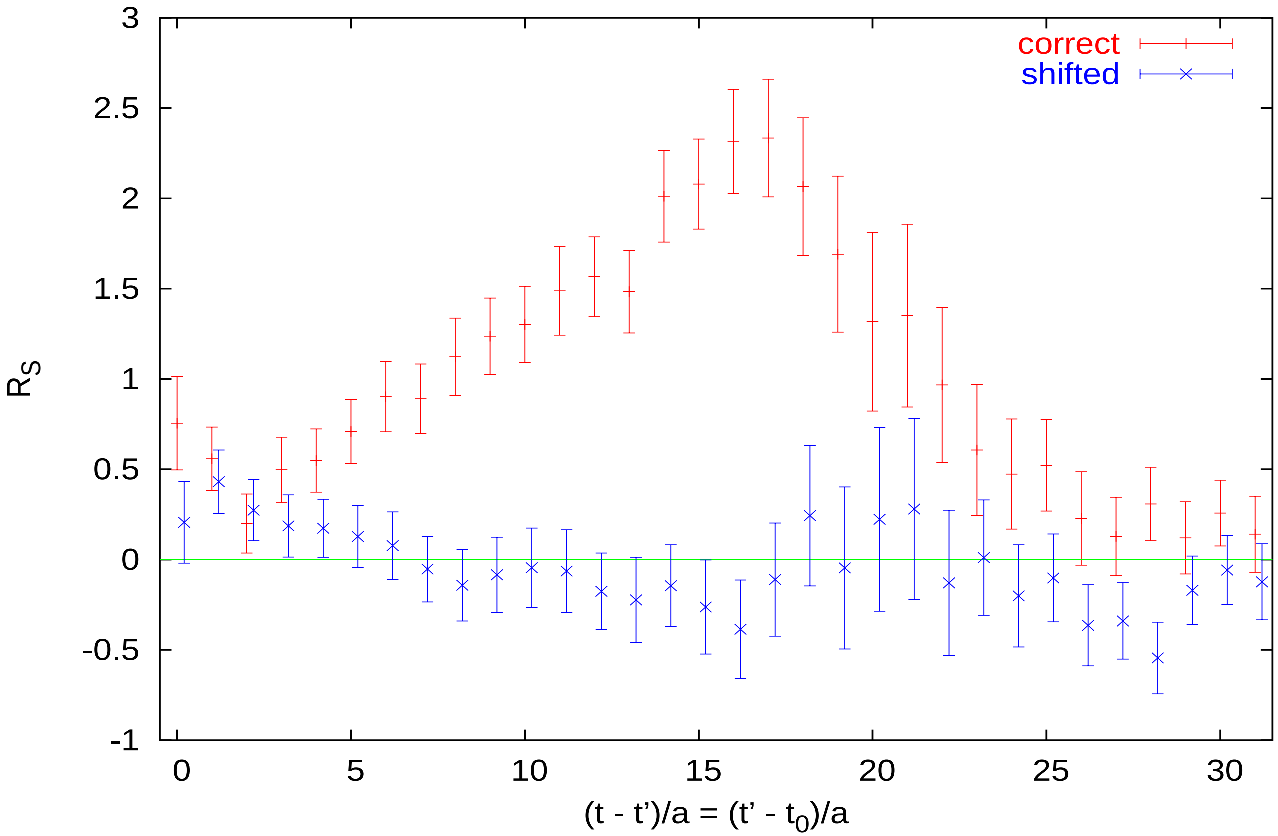


Scalar density



Raw lattice results for the zero-momentum scalar density $\sum_{\vec{x}} \bar{\psi}_s \psi_s(\vec{x}, 0)$.
Previous results on a $16^3 \times 64$ lattice are also shown.

Check of statistical significance



The data points in blue have been obtained averaging the product of propagators and scalar density on decorrelated configurations.

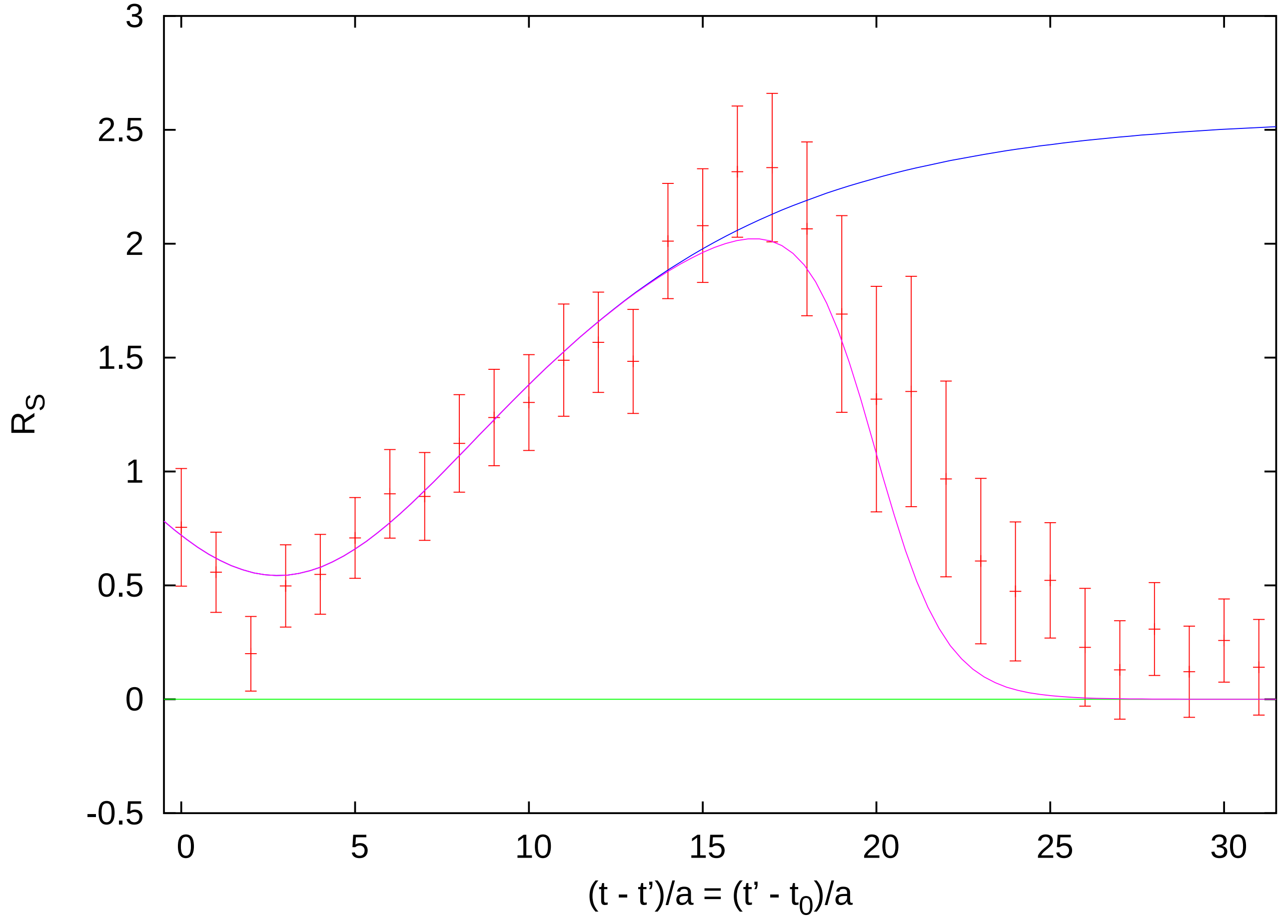
A physically motivated fit

Assume a two state contribution to the $\bar{s}\Gamma s$ matrix elements and a two state plus backward propagating state to the nucleon propagator. Then one would expect the data to exhibit a behavior

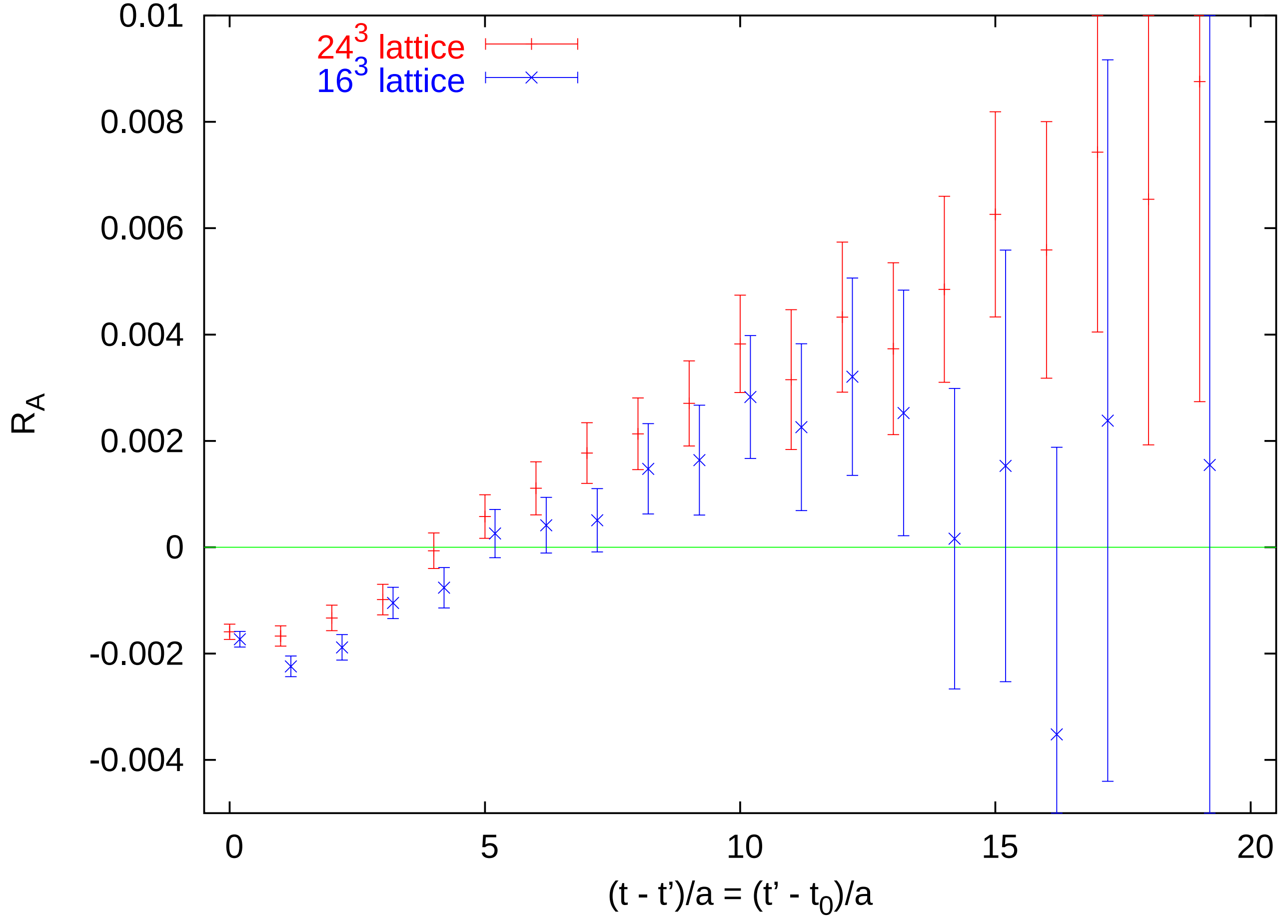
$$R_\Gamma \approx \frac{c_1^2 j_1 e^{-2m_1 t} + c_1 c_2 (j_{1,2} + j_{1,2}^*) e^{-(m_1+m_2)t} + c_2^2 j_2 e^{-2m_2 t}}{c_1^2 e^{-2m_1 t} + c_2^2 e^{-2m_2 t} + c_B^2 e^{-2m_B(L/2-t)}}$$

where c_1, c_2, c_B can be extracted from the nucleon propagator, $j_1, j_2, j_{1,2}$ are unknown matrix elements.

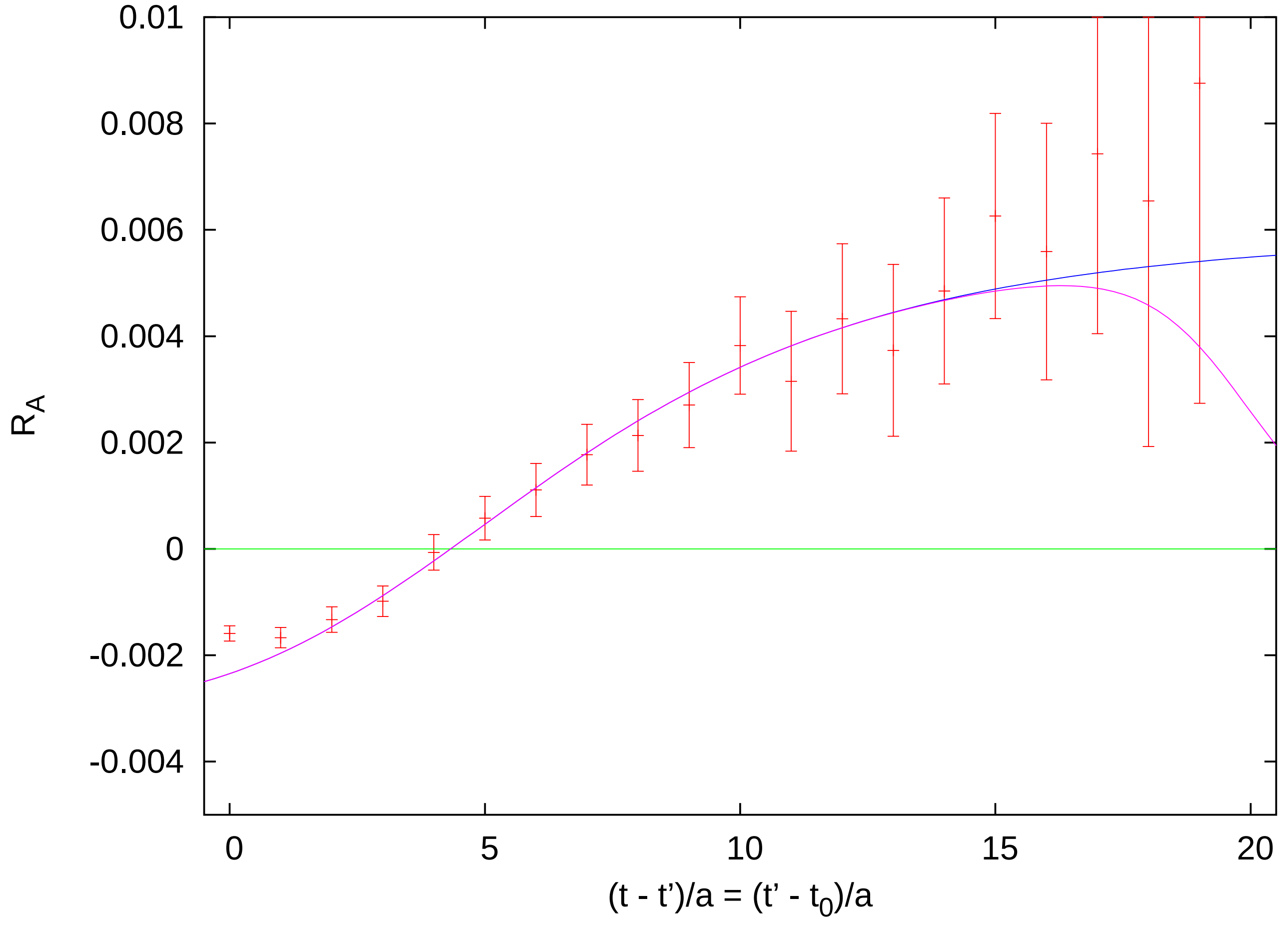
The fit (over $3 \leq (t - t')/a \leq 16$)



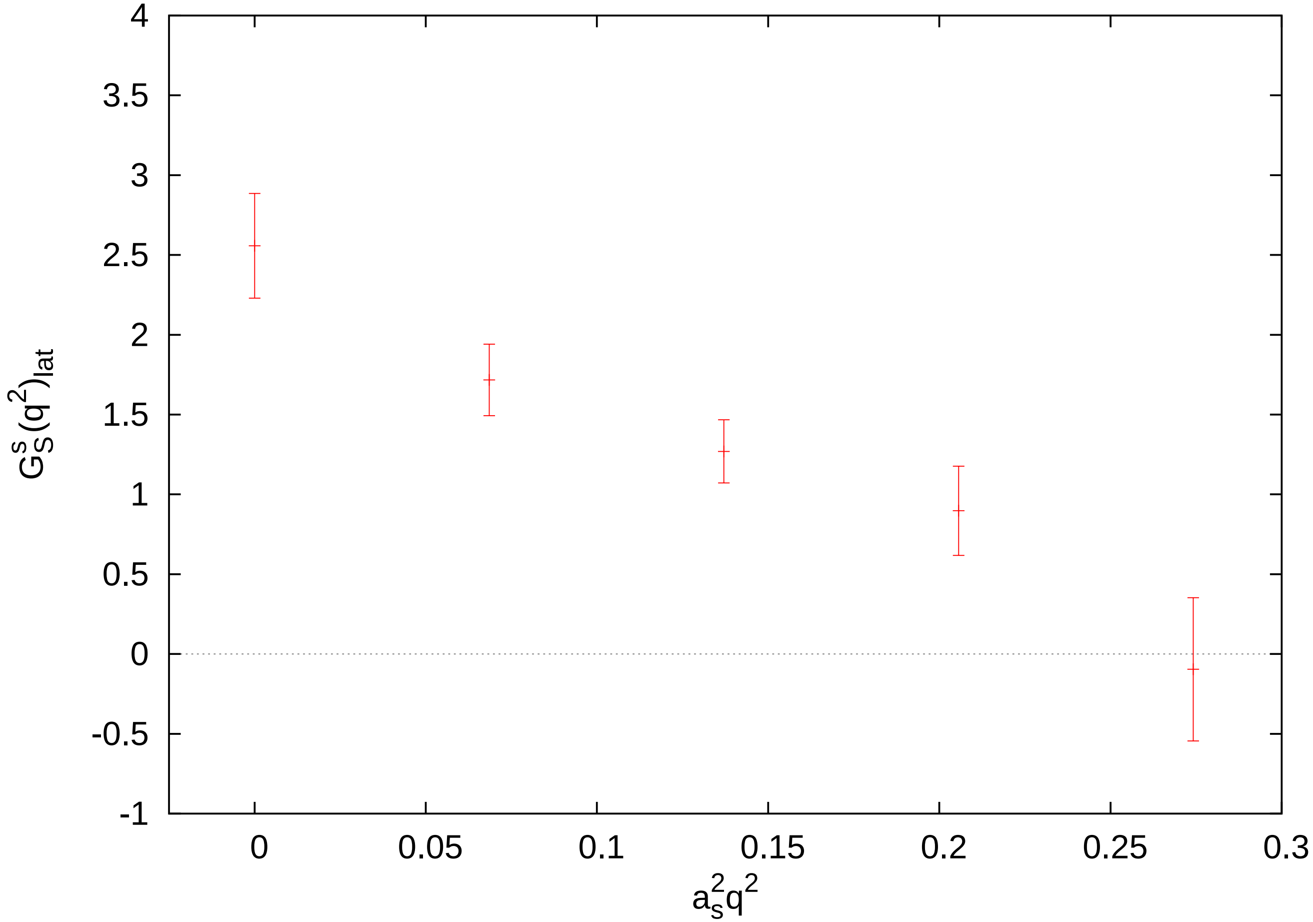
Results for the zero-momentum axial density



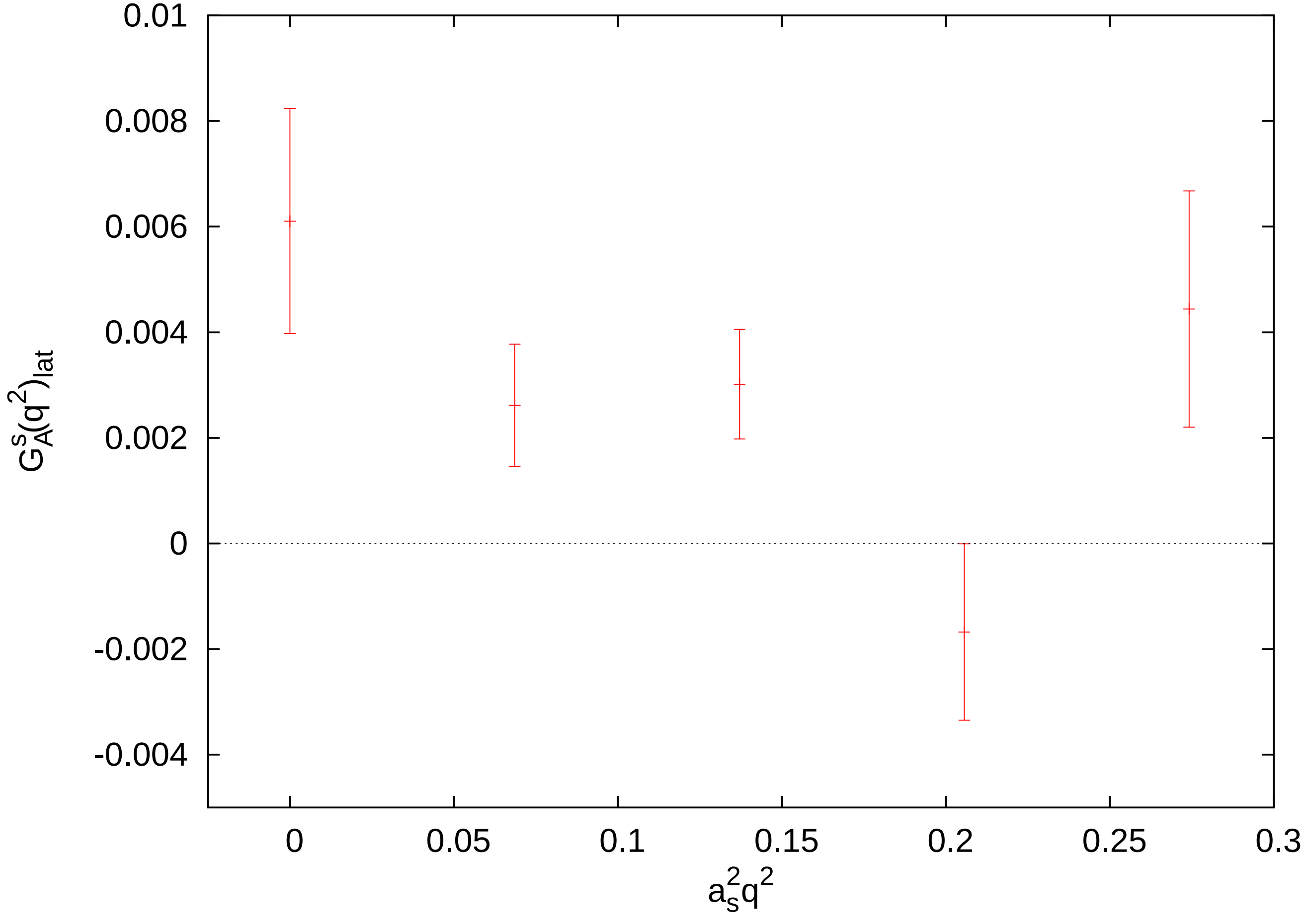
A two state fit



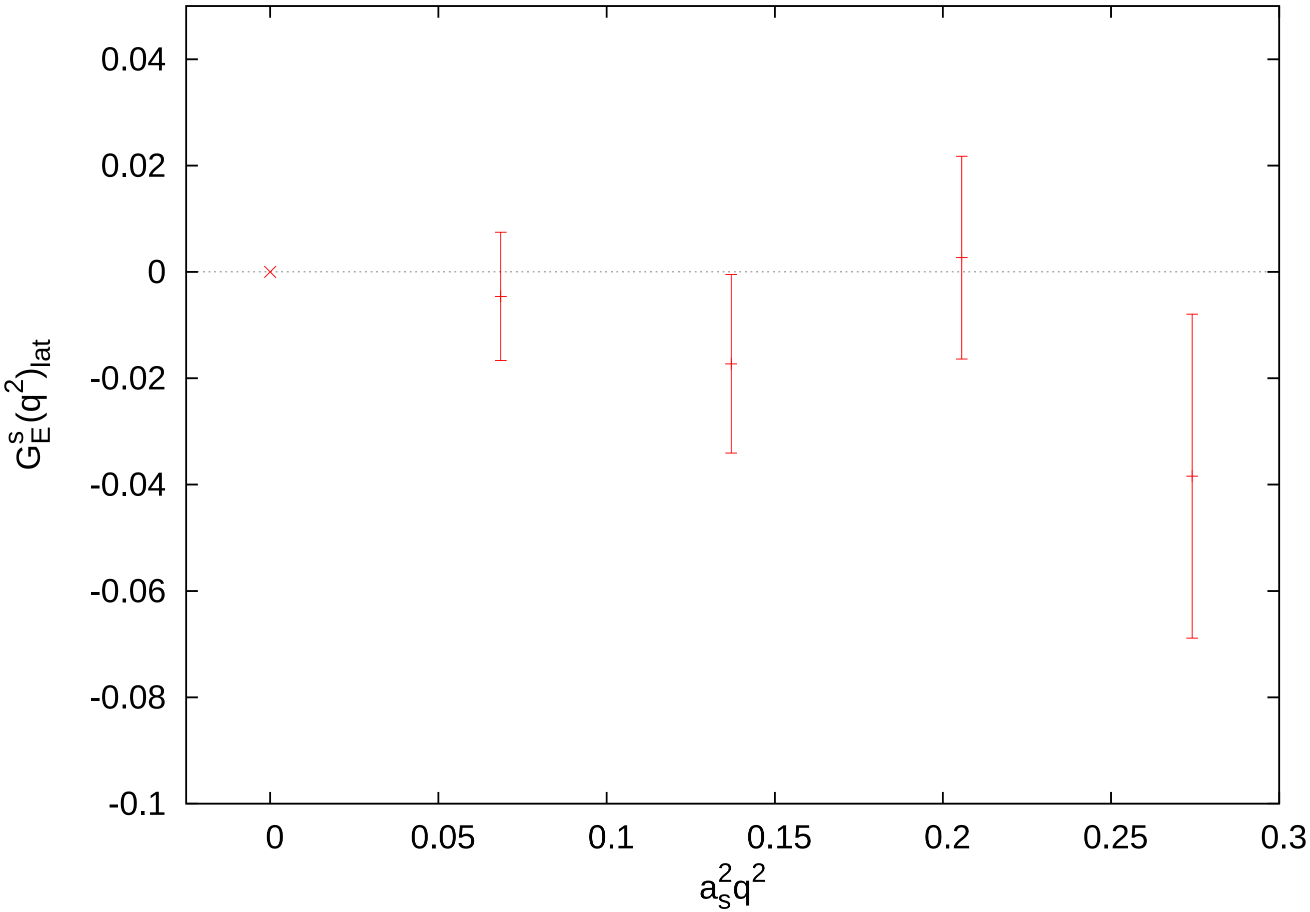
q^2 dependence, scalar density



q^2 dependence, axial density



q^2 dependence, electric form factor



Multigrid variance reduction

We need $\mathrm{Tr} A^{-1} = \mathrm{Avg}_i \left[\eta_i^\dagger A^{-1} \eta_i\right]$

Introduce a restriction over a coarse lattice P and the prolongator $P^\dagger$. Let $\hat{A}$ be the projection of A over the coarse lattice. Using $\mathrm{Tr} P^\dagger \hat{A}^{-1} P = \mathrm{Tr} \hat{A}^{-1} P P^\dagger = \mathrm{Tr} \hat{A}^{-1}$:

$$\mathrm{Tr} A^{-1} = \mathrm{Avg}_i \left[\eta_i^\dagger (A^{-1} - P^\dagger \hat{A}^{-1} P) \eta_i\right] + \mathrm{Tr} \hat{A}^{-1}$$

If $\hat{A}$ reproduces well the long range behavior of A, the variance of the subtracted term can be substantially reduced, while the calculation of $\mathrm{Tr} \hat{A}^{-1}$ will be less computationally demanding.

Multigrid variance reduction

With a free $(A = \sum_\mu \nabla_\mu^\dagger \nabla_\mu + m^2)$ on a 32^4 lattice:

m	s	$\sqrt{\sigma}$	s'	$\sqrt{\sigma}'$
0.1	1.619×10^5	1.932×10^2	4.012×10^4	1.266×10^2
0.01	1.723×10^5	1.000×10^4	5.047×10^4	1.281×10^2
0.001	1.162×10^6	1.000×10^6	1.040×10^6	1.281×10^2

where $s = \mathrm{Avg}_i\,[\eta_i^\dagger A^{-1} \eta_i]$ and σ is its variance, $s' = \mathrm{Tr}\hat{A}^{-1}$ and σ' is the variance of $\mathrm{Avg}_i\,[\eta_i^\dagger (A^{-1} - P^\dagger \hat{A}^{-1} P)\eta_i]$.

Conclusions and future work

Our results indicate that progress in computer power and algorithms have brought disconnected matrix elements within the realm of calculable quantities.

We plan:

- to repeat the calculation on $24^3 \times 128$ anisotropic lattices with clover (S-W) fermions;
- use more elaborated nucleon wave-functions;
- implement multigrid variance reduction, if practical.