Lattice chirality and the decoupling of mirror fermions

Erich Poppitz

U of T

with Tanmoy Bhattacharya, Matthew Martin — Los Alamos
hep-lat/0605003 PRD74(2006)08528

with Joel Giedt — Minnessota (now at Rensselaer)
hep-lat/0701004 JHEP10(2007)076

with Yanwen Shang — Toronto
arXiv:0706.1043 [hep-th] JHEP08(2007)081
arXiv:0801.0587 [hep-lat]: explanatory/defense from misguided criticism

& work in progress with Yanwen Shang

why should one care? (at least a little bit)

currently most popular scenarios for LHC-scale physics involve weakly coupled models of electroweak symmetry breaking (S,T...)

however, a survey of weakly coupled models reveals that they are all fine-tuned

- we have not found the right weak coupling model
- we shouldn't worry about fine tuning
- we should look at strong coupling theories

while QCD-type technicolor models are out by electroweak precision tests, there may (?) exist other kinds of dynamics that work just fine

e.g., Bagger, Falk, Swartz, 1999; Chanowitz, 2004

the kinds of strong-coupling gauge dynamics we reasonably understand - from experiment, theory, or simulations - are only a few

the real theory could involve strong chiral gauge dynamics, "tumbling," and possibly other strange things...

e.g., Holdom, 1996 + ...

all good questions: do we know such/which gauge theories "work"?
do we understand the strong dynamics involved?
can we calculate and make predictions?

with not-so-good answers: not really

we do not understand strong - chiral, in particular - gauge dynamics well

- how well do we understand strong chiral gauge dynamics?

- can we make progress?

- what tools do we currently have to study strong chiral gauge dynamics?

tools one trusts	tools you don't really know whether to trust unless confirmed by other means - experiment, numerics, or the tools on the left
't Hooft anomaly matching	"MAC"
in SUSY aided by "power of holomorphy"	truncated Schwinger-Dyson equations

- there's not much there...

the recently popular AdS/QCD type duals - I'd put them solidly on the r.h.s. -
are not straightforwardly applied to chiral gauge theories

in large-N limit - taking "quintessential" chiral theory - "tumbling" etc...

e.g., Dimopoulos, Raby, Susskind '80s

SU(5) with 5* and 10 becomes SU(N) with (N-4) N* and an N(N-1)/2

have different symmetries and symmetry realizations, as easily made evident, say, from the study of the supersymmetric case

e.g., Trivedi, EP, 1995

- at large-N "quark" loops are not suppressed [N(N-1)/2-dim representation],
- so "mesons" are not free at infinite N
- one doesn't expect a nice classical "super" gravity description in a slice of AdS or a deformation thereof

while I motivated the desire to study chiral dynamics via topical "beyond the Standard Model"/LHC physics, recall that the SM itself is a chiral gauge theory albeit weakly coupled at energies < O(TeV)

do we have a nonperturbative formulation of the SM?

SOME PURIST MIGHT EVEN ASK: "DOES THE SM EXIST?"

here comes our interest in the lattice:

- recall that the lattice is a controlled way to precisely calculate many things in QCD - though not all! - notably, spectra and various matrix elements

- in addition, gives a nonpertubative definition of the theory
  and provides for some important rigorous results - e.g., about continuity between Higgs/ confining phases

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SUPERSYMMETRIC? - some recent developments (I gave a talk at "Quarks-2004") are especially promising (working!) in lower dimensional, nonchiral, cases

"phenomenologically interesting" N=1 + matter, 4d case still open

see, Joel Giedt's review hep-lat/0602007

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NOT about LHC physics via strong chiral gauge dynamics; I will not discuss a potential theory of the real world: won't tell you which chiral gauge theory naturally breaks EW symmetry with small S-parameter...

I'd like to tell you ("in pictures") where the lattice chiral gauge theory problem is at, and what attempts are being made at improvement and progress

I think that it is a theoretically appealing problem, fun to think about ...and that doing this may even turn out to be useful (in the longer run, of course)
many tools come together - both theoretical and "experimental"

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CHIRAL?

THIS TALK's GOALS and rough **OUTLINE:**

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the approach I'll describe today is a combination of "old" and "new"

will put in larger perspective shortly, motivating the necessity to pursue this line of thought

the idea goes like this:

formulating vectorlike gauge theories (like QCD) on the lattice is not too much of a problem - there are doublers, of course, but we've learned...

so, one can ask a natural question -

can one start with a vectorlike theory, for example:

$SU(5)$ with 5^* 5 10 10^* } all Weyl, L

names: "light" (5^*, 10)   "mirror" (5, 10^*)

and then, deform the theory in such a way that

- mirrors decouple from the low-energy spectrum
- the gauge symmetry remains unbroken

?

before attempting to answer - WHY DO WE DO THIS?

a lightning review of current situation with chiral lattice gauge theories

based on seminal works of

Ginsparg, Wilson (1982); *Callan, Harvey (1985);* D.B. Kaplan (1992); Narayanan, Neuberger (1994); Neuberger (1997); P. Hasenfratz, Laliena, Niedermaier (1997); Luescher (1998); Neuberger (1998),

Luescher has proven (1999-2000) that an exactly gauge invariant lattice action and measure exist **for an* anomaly free chiral gauge theory** based on the Neuberger-Dirac (or "Ginsparg-Wilson") operator D[A]

... roughly:

$$Z_{chiral}[A] = e^{if[A]} \int \Pi\, dc\, d\bar{c}\;\; e^{\,\bar{c}^k \cdot D[A]_{kp} \cdot c^p}$$

- f[A] must be there, for gauge invariance, locality, smoothness wrt A

* - appropriate f[A] proven to exist for an anomaly free U(1) in finite V; SU(2)xU(1) in infinite V

- outside of perturbation theory, for a general gauge group, there is no explicit formulation of f[A]

fascinating theoretical achievement, but not good for practical use, e.g. numerical simulations

before attempting to answer - WHY DO WE DO THIS?

because the measure is explicitly known only perturbatively, one must nonperturbatively tune higher-dimensional gauge field operators to restore gauge invariance - nonperturbative tuning -

- usually considered an anathema

[(?) - but see Golterman and Shamir, 1998+: proposal with argued only finite number of tunings]

- it appears that a formal solution might be very hard to construct;
we may as well look for a dynamical one:

attempting to construct a chiral lattice gauge theory by decoupling the mirrors from a vectorlike theory - where the measure is known explicitly - is worthwhile* and of possible practical importance

a further moral support to a dynamical approach is provided by the fact that, while we don't know how chiral gauge theories arise from the real world UV theory, this is (very roughly) how it happens in string theory (or higher-dim) models that lead to chiral 4d gauge theories

---

* a message for enthusiasts: there may be other, not thought of ways to do this!

now, back to our question -

- can one start with a vectorlike theory, for example:

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	10	10^*	
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- a normal continuum field theorist would say: no!
- a string theorist might say: may be
  e.g., if one allows the liberty to think of orbifolding as decoupling of states
- lattice may afford new possibilities:

everybody knows that four-fermi interactions, if taken strong enough, break chiral symmetries

$$\frac{g}{\Lambda^2} (\bar{\Psi}\Psi)(\bar{\Psi}\Psi) \quad , \quad gN > 8\pi^2$$

as per the NJL "gap equation" *made "believable" via large-N, gN=const, limit, aka "mean field"*

- few continuum people know, however, that if one takes coupling even stronger, the theory enters a "**strong-coupling symmetric phase**," with only massive excitations and unbroken chiral symmetry

- why haven't most people heard about these phases?

because these phases are a "lattice artifact" - the physics is that of "lattice particles" with small hopping probability

thus, these "lattice particles" are "heavier than the UV cutoff"
think of an almost-insulator

I’m not sure who discovered them first

Eichten, Preskill (1986 paper on “Chiral gauge theories on the lattice”)

- 4-fermi interactions ... [E-P]

A. Hasenfratz, Neuhaus (1988)

- strong Yukawa case - similar

E-P story “retold”

SU(5)	“light”	“mirror”
	5^*	5
	10	10^*
	1	1

g_1: $10^* - 5 - 5 - 1$

g_2: $10^* - 10^* - 10^* - 5$

a toy example with SU(4) “chiral” symmetry (the one to be gauged)

$$H_{4\psi} = \sum_x g\left(\psi_a \psi_b \psi_c \psi_d \, \epsilon^{abcd} + hc\right)$$

space lattice only (any dimension); canonical anticommutation relations:

$$\{\psi_a(x), \psi_b^+(y)\} = \delta_{ab}\,\delta_{xy}$$

at g>>1 in lattice units, hopping is negligible:

$$H = \sum_x H_{0\,x} + H_1$$

$H_{0\,x}$ → 4-fermi; H_1 → hopping $\sim \psi^+ \vec{\alpha}\cdot\vec{\nabla}\,\psi$

to leading order, at every site the same simple 4-fermion QM problem, rename: $\psi_{a\,x} \to a_a$, $\bar{\psi}_{b\,x} \to a_b^+$

$$H_0 = g\left(a_a a_b a_c a_d + a_a^+ a_b^+ a_c^+ a_d^+\right)\epsilon^{abcd}$$

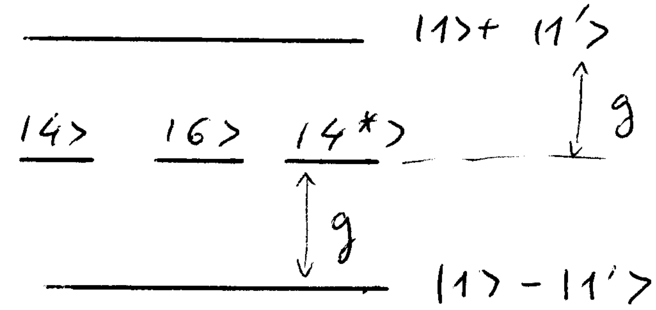
H_0 conserves F(mod 4); 16 states = 1 + 1’ + 4 + 4* + 6 under SU(4)

H_0 connects only 1 (= all fermions empty) and 1’ (= all fermions occupied)

so:

(1-1’) has energy -g; (1+1’) has energy +g, 4,4*,6 have energy 0.

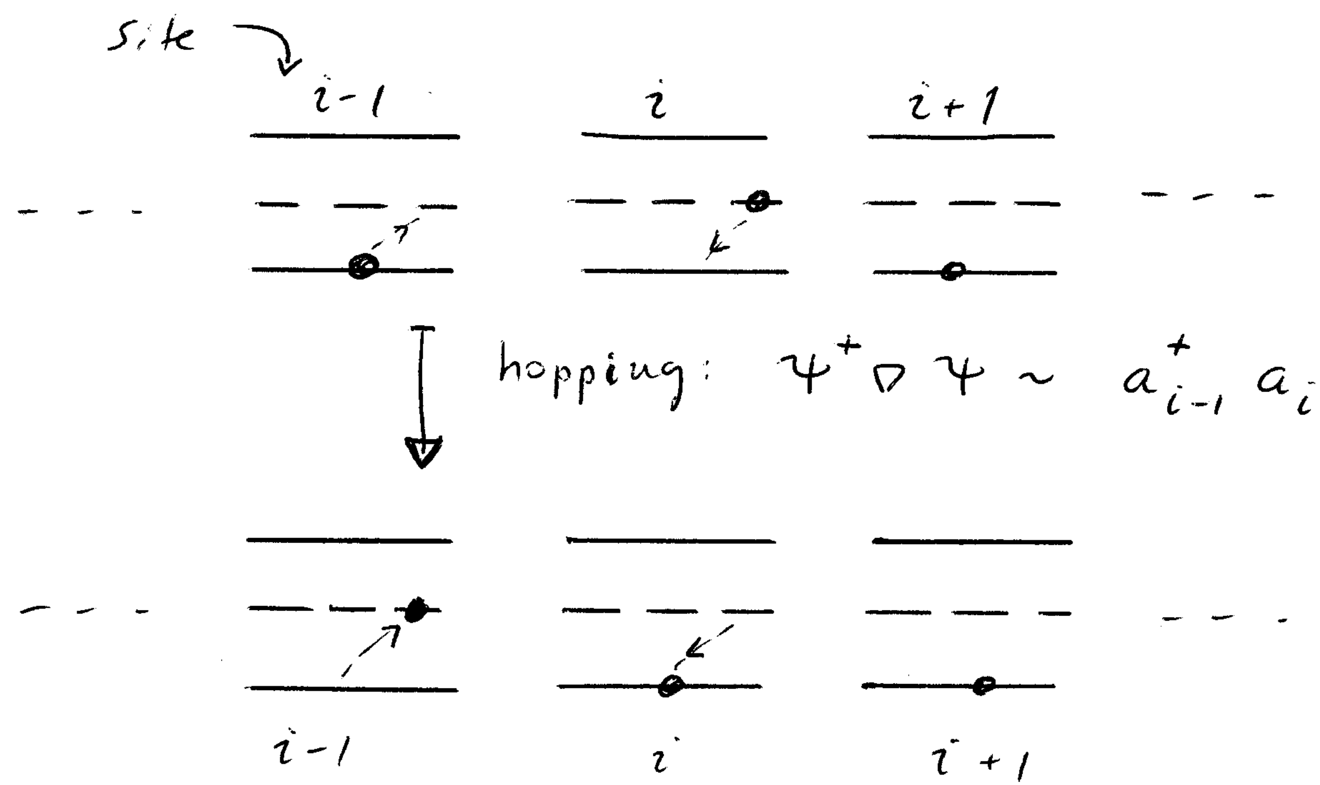
in the infinite-g limit, the lattice theory ground state is unique (hence, SU(4) - “chiral” - singlet) with gap = g



at first order in 1/g, hopping turns on, site-localized states form bands and propagate

propagating states heavy, mass ~ g/a >> 1/a,
a - the UV-cutoff

the 1/g (strong-coupling) expansion has finite radius of convergence, hence this story represents the true ground state of theory, for g sufficiently large



- very much like “static limit” of lattice QCD, but infinite mass limit replaced by infinite four-fermi
- large-g phase same in any dimension, like high-T statmech where disorder always wins

simple SU(4) exercise, with a bit more group theory, can be repeated for SU(5) of E-P

(btw, singlet needed by E-P to have sensible "static limit" of Euclidean fermion path integral)

$g_1 \quad 10^* - 5 - 5 - 1$

$g_2 \quad 10^* - 10^* - 10^* - 5$

two 4-fermi terms;
mirror global symmeries, including anomalous ones, must be broken, or else get extra zero modes in instanton (indeed, extra light particles)

showing that at infinite g SU(5) ground state unique and singlet

the "E-P dream" was, essentially, to use this* phase to decouple the mirrors

*I am simplifying E-P story

- no continuum limit of this mirror theory - "everything mirror" is cutoff scale and heavier and decoupled from IR physics... ideally

- gauge field appears only in hopping terms and so contributions of mirror sector to gauge field action should be ~ 1/g

- did the "E-P dream" come true?

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no! - the reason was that, in 1986, there was no way to define chiral components of a spinor field on the lattice - even a two-component Weyl field on the lattice (such as E-P used) has opposite chirality massless excitations in it, because of the fermion doubling

because of the lack of L/R separation on lattice - notice that L/R separation requires the notion of chiral symmetry - the strong 4-fermi was "felt" by both "mirror" and "light" fermions

hence, both "mirror" and "light" fermions became heavy at strong-4 fermi, while at weak 4-fermi, both "mirror" and "light" were massless, i.e. the theory was vectorlike

- study of E-P model by Golterman, Petcher, Rivas (1993)
- also, study of related "waveguide" models proposed by Kaplan (1992), by Golterman, Shamir (1992)

- so what has changed?

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after a series of seminal papers in the 90's (Kaplan, Narayanan/Neuberger, Neuberger, P. Hasenfratz/Laliena/Niedermayer, Luescher, Neuberger) it was realized that there is an exact definition of chirality at any nonzero lattice spacing without doublers

...rediscovering, in 1997, Ginsparg&Wilson's work of 1982!

definition of L and R components of Dirac fermions without doublers

- somewhat complicated, but exact at any (a, N)
- exact chirality transforms, anomaly, Ward identity, index theorem...

so, naturally, one can ask, can the "E-P dream" be resurrected as well?

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important note: Creutz, Rebbi, Tytgat, Xue, 1996, similar proposal using E-P + domain wall - before GW operator and exact chirality - symmetries become exact only as size becomes infinite, so less "pretty," hence more difficult to study theoretically - there was no follow-up work whatsoever

to explain our proposal and later/current work in more detail requires technicalities; can ask me in person later

- the structure is like this:

$$Z_{vector}[A] = Z_{light}[A] \times Z_{mirror}[A]$$

$$Z_{light}[A] = \int \Pi\, d\bar{c}\, dc\, e^{\bar{c}^k D_{kp}[A] c^p}$$

$$Z_{mirror}[A] = \int \Pi\, d\bar{b}\, db\, e^{\bar{b}^k D_{kp}[A] b^p + S_{4fermi}[\bar{b}, b, A]}$$

- light and mirror Z separate explicitly; light fields do not "feel" strong mirror interaction
- measure is explicitly defined
- global symmetries, incl. anomalous, are exactly the ones of the target continuum theory
- Z_mirror/Z_light separation singular in A if anomalous mirror rep
- Z_mirror is a smooth function of A iff anomaly free

due to "splitting theorem" of Yanwen Shang, EP, 0706.1043 [hep-th]

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I didn't tell all: c, b not usual local fermion variables, slight nonlocality having to do with implementing exact lattice chirality

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4. with gauge fields included, is the long-distance theory unitary?
   we have defined a complex Euclidean partition function: different treatment of conjugate mirror fermion variables through the different chiral projectors

but we are getting there:

- with the slightly nonlocal Yukawa/4-fermi mirror interactions, is it still true that a "strong coupling symmetric phase" exists?
  are the mirrors heavy?

yes, on both questions in the 2d models studied

Joel Giedt, EP, hep-lat/0701004

yes, on 1st question in the 4d model studied

P. Gerhold, K. Jansen, arXiv:0707.3849[hep-lat]

(no Majorana type coupling due to different motivation;
unlifted "mirror" zero modes quite easy to predict and spot)

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2 in typical models, there is more than one strong Yukawa/4-fermi interaction - needed to break all classical mirror global symmetries, less there will be extra unlifted instanton mirror zero modes - and there can be a nontrivial phase structure as their ratios change

there is a nontrivial phase structure in the 2d model (v-like Schwinger model at strong chirally invariant Yukawa) studied

reaching symmetric phase at strong coupling does not appear to require tuning (a large region in coupling space)

Joel Giedt, EP, hep-lat/0701004

but we are getting there:

3 what happens if one tries to decouple an anomalous mirror representation?

we think that Z_mirror/Z_light singular split has something to do with it;
important to differentiate between options
(massless mirror fermion, Green-Schwarz field, nonunitarity)
work still in progress

Yanwen Shang, EP, arXiv:0706.1043[hep-th] + in progress

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are the mirrors heavy? **YES** Joel Giedt, EP, hep-lat/0701004
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2 in typical models, there is more than one strong Yukawa/4-fermi interaction - needed to break all classical mirror global symmetries, less there will be extra unlifted instanton mirror zero modes - and there can be a nontrivial phase structure as their ratios change **YES, seems OK**
Joel Giedt, EP, hep-lat/0701004

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not obvious, future work

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of the continuum target

but

d.) requires (more) numerical work to study *and, most importantly,*

e.) ***we have not seen reasons to give up -***

we don’t know if we have succeeded or “not failed”, yet!

I have not failed. I've just found 10,000 ways that won't work.

Thomas A. Edison

backup slides

backup slides

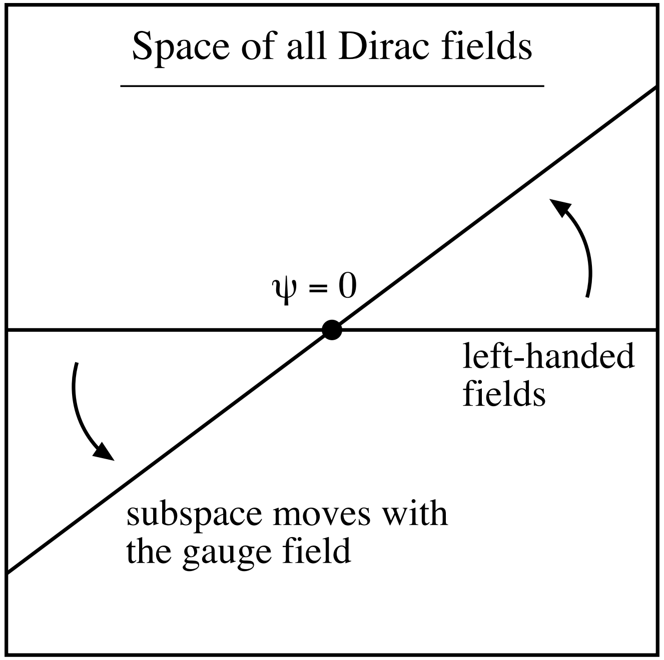
why we care?

because most of the "action" regarding anomalies happens in the $\hat{\gamma}_5$ -eigenvectors and in their A-dependence!

- change in direction perpendicular to eigenspace completely determined by solving perturbatively for change of eigenvector due to small changes of "parameter" A:

$$\hat{\gamma}_5 \, t_i = -t_i$$

- change in parallel direction - as usual in perturbation theory - not determined (phase of Z); but not always completely arbitrary (Berry phase!)



since chiral Z defined via t,v - to what extent is chiral Z arbitrary? (its phase, that is)

Neuberger, 1998, showed that, in anomalous case, sum of "Berry curvatures" for the "eigenstates" t_i of the "Hamiltonian" $\hat{\gamma}_5$ (as a function of gauge background) integrates to an integer over some closed 2-manifolds in gauge field space

hence, no smooth, wrt gauge field, choice of t_i exists (or of the corresponding Berry connection)

- chiral partition function is not smooth wrt A for an anomalous representation
- fermion expectation values are not smooth functions of A and there are no Schwinger-Dyson equations...

Luescher proved that no other obstructions exist (...) shortly thereafter (1999-00)

$$\delta \log Z[U] = \sum_i (\delta t_i^\dagger \cdot t_i) + \left\langle \frac{\delta S}{\delta O} \, \delta O \right\rangle$$

since change of ImLog Z due to change of A largely controlled by eigenvectors, our "splitting theorem" encodes, on the lattice, the fact that anomalies do not depend on the action

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similarity to continuum topological classification - Alvarez-Gaume, Ginsparg, 1985:
consider gauge loop in gauge connection space

$$A^\theta = g^{-1}(\theta) A g(\theta) + g^{-1}(\theta) \mathrm{d}_x g(\theta), \qquad g(\theta, x): \qquad \mathbf{S}^1 \times \mathbf{S}^{2n} \to \mathbf{G} \quad (\pi_5(G))$$

change of phase w of chiral determinant along loop = winding number $n = \frac{1}{2\pi}\int_0^{2\pi} \mathrm{d}\theta \, \frac{\partial w}{\partial \theta}(\theta, A)$

such loops with nonzero n exist iff a non-contractible two-sphere in gauge orbit space

$$\pi_5(G) = \pi_2(\mathcal{A}/\mathcal{G}) \quad \text{if } \pi_1(G) = 0$$

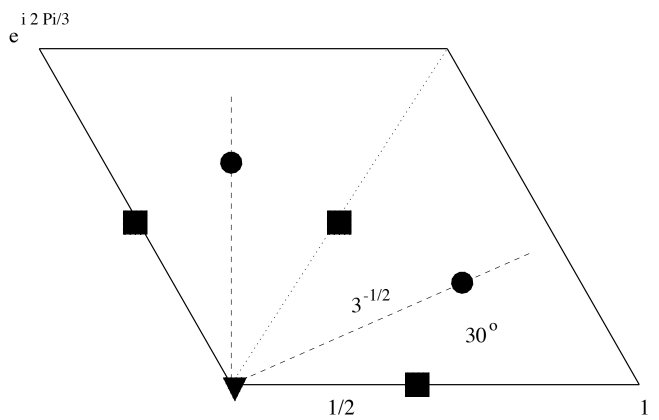
$$\delta \log Z[U] = \sum_i (\delta t_i^\dagger \cdot t_i) + \left\langle \frac{\delta S}{\delta O} \, \delta O \right\rangle$$

since change of ImLog Z due to change of A largely controlled by eigenvectors, our "splitting theorem" encodes, on the lattice, the fact that anomalies do not depend on the action

for example, consider

T_6 / Z_3 orientifold of type-I theory

(one T_2 shown)



11 D3 branes at origin
9 D3 branes "stuck" at Z_2 f.p.s
12 D3 branes removed from origin

Lykken, Trivedi, EP, 1998

vectorlike SO(32) N=4 theory

at a disconnected branch of moduli space

vectorlike SO(11) N=2 theory

orbifold to chiral N=1:

	$SU(5)$	$U(1)$
$A_{i=1,2,3}$	$\begin{smallmatrix}\Box\\\Box\end{smallmatrix}$	2
$\bar{Q}_{i=1,2,3}$	$\overline{\Box}$	-1

now this is complicated, is not proven to be nonperturbatively consistent (but some indications), and is even further from being nonperturbatively useful... but example can inspire us to look further along the lines of decoupling states from vectorlike theories!

Ginsparg-Wilson relation, its solution, and consequences:

[GW] $\{D, \gamma_5\} = a D \gamma_5 D$ *Ginsparg-Wilson, 1982*
"A remnant of chiral symmetry on the lattice"

but what is D? - ressurection by *Neuberger* **(explicit solution** *D is "local" w/ exponential tail***)**
and by *Hasenfratz, Laliena, Niedermayer, 1997;*

given D, define: $\hat{\gamma}_5 = (1 - aD)\gamma_5$

then GW is equivalent to:

$\hat{\gamma}_5^2 = 1$ or $\hat{\gamma}_5 D = -D \gamma_5$

and there is an exact chiral symmetry (GW, 1982; formulation of Luescher, 1999)

$\psi \to e^{i\alpha\gamma_5}\psi \qquad \bar{\psi} \to \bar{\psi} e^{i\alpha\hat{\gamma}_5}$

of lattice action $S = \sum_{xy} \bar{\psi}_x D_{xy} \psi_y$

note that, really, we have $\bar{\psi}_x \longrightarrow \sum_{x'} \bar{\psi}_{x'} \left(e^{i\alpha\hat{\gamma}_5}\right)_{x'x}$ with $\hat{\gamma}_{5\,x'x} \sim e^{-\frac{|x'-x|}{a}}$

$$\psi \to e^{i\alpha_\pm P_\pm} \psi$$

$$\bar{\psi} \to \bar{\psi} e^{-i\alpha_\pm \hat{P}_\mp}$$

global L and R symmetries of action $U(1)_+ \times U(1)_-$

field dependence of transformation leads to nontrivial Jacobian

$$1 \mp \frac{\alpha_\pm}{2} \, tr(\gamma_5 D)$$

Jacobian vanishes for vector U(1), where both + and - done with same parameter

then properties of D are useful to (easily, really!) to show that "index theorem in QCD with finite cutoff" holds

$$-\frac{1}{2} tr(\gamma_5 D) = n_+^0 - n_-^0$$

moral:

exact lattice chiral symmetry (not usual one for all modes!), exact, incl. anomalous, Ward identities, axial charge violation, ... in vectorlike theories - **big success!**

biggest drawback - no explicit Hamiltonian formulation! only evidence for unitarity (and some plausible words...)

is it still true that a "strong coupling symmetric phase" exists?

Joel Giedt, EP (2007)

toy 2d Yukawa-unitary Higgs model with Ginsparg-Wilson fermions

exact chiral symmetry, zero gauge fields in simulation:

$$
\begin{aligned}
S &= S_{light} + S_{mirror} \\
S_{light} &= \left(\bar{\psi}_+, D_1 \psi_+\right) + (\bar{\chi}_-, D_0 \chi_-) \\
S_{mirror} &= \left(\bar{\psi}_-, D_1 \psi_-\right) + (\bar{\chi}_+, D_0 \chi_+) \\
&+ \; y \left\{ \left(\bar{\psi}_-, \phi^* \chi_+\right) + (\bar{\chi}_+, \phi \psi_-) + h \left[\left(\psi_-^T, \phi \gamma_2 \chi_+\right) - \left(\bar{\chi}_+, \gamma_2 \phi^* \bar{\psi}_-^T\right) \right] \right\}
\end{aligned}
$$

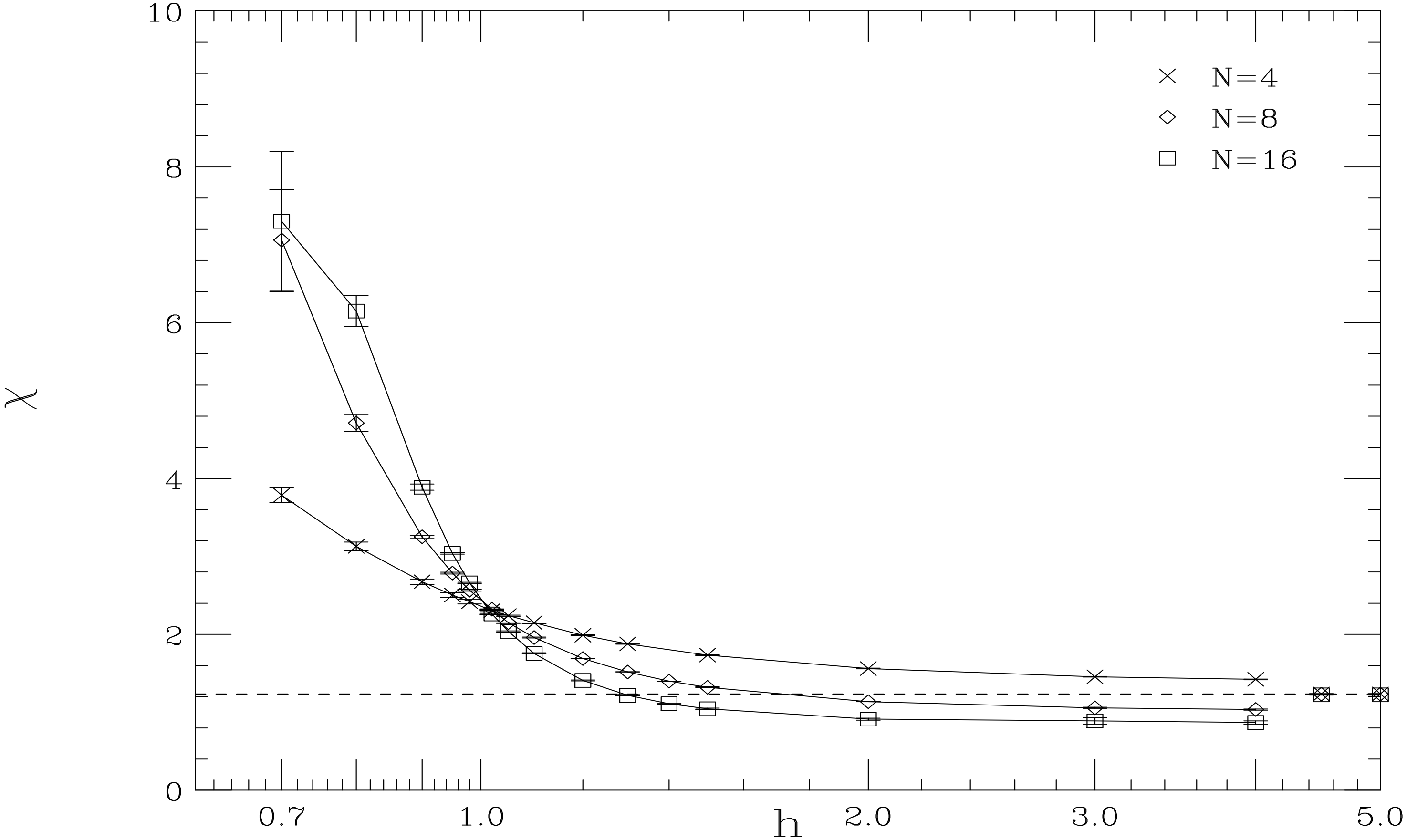
$$
S_\kappa = \frac{\kappa}{2} \sum_x \sum_{\hat{\mu}} [2 - (\; \phi_x^* \, U_{x,x+\hat{\mu}} \, \phi_{x+\hat{\mu}} + \text{h.c.} \;)]
$$

all simulations at infinite y: economic reasons! i.e. by dropping mirror kinetic terms

using only mirror partition function work out split of measure and action at U=1...

2d Yukawa-unitary Higgs model with Ginsparg-Wilson fermions,

show one plot only: scalar susceptibility at infinite y, as function of h, kappa=0.1
(~ inverse "mass squared" of scalar in lattice units)



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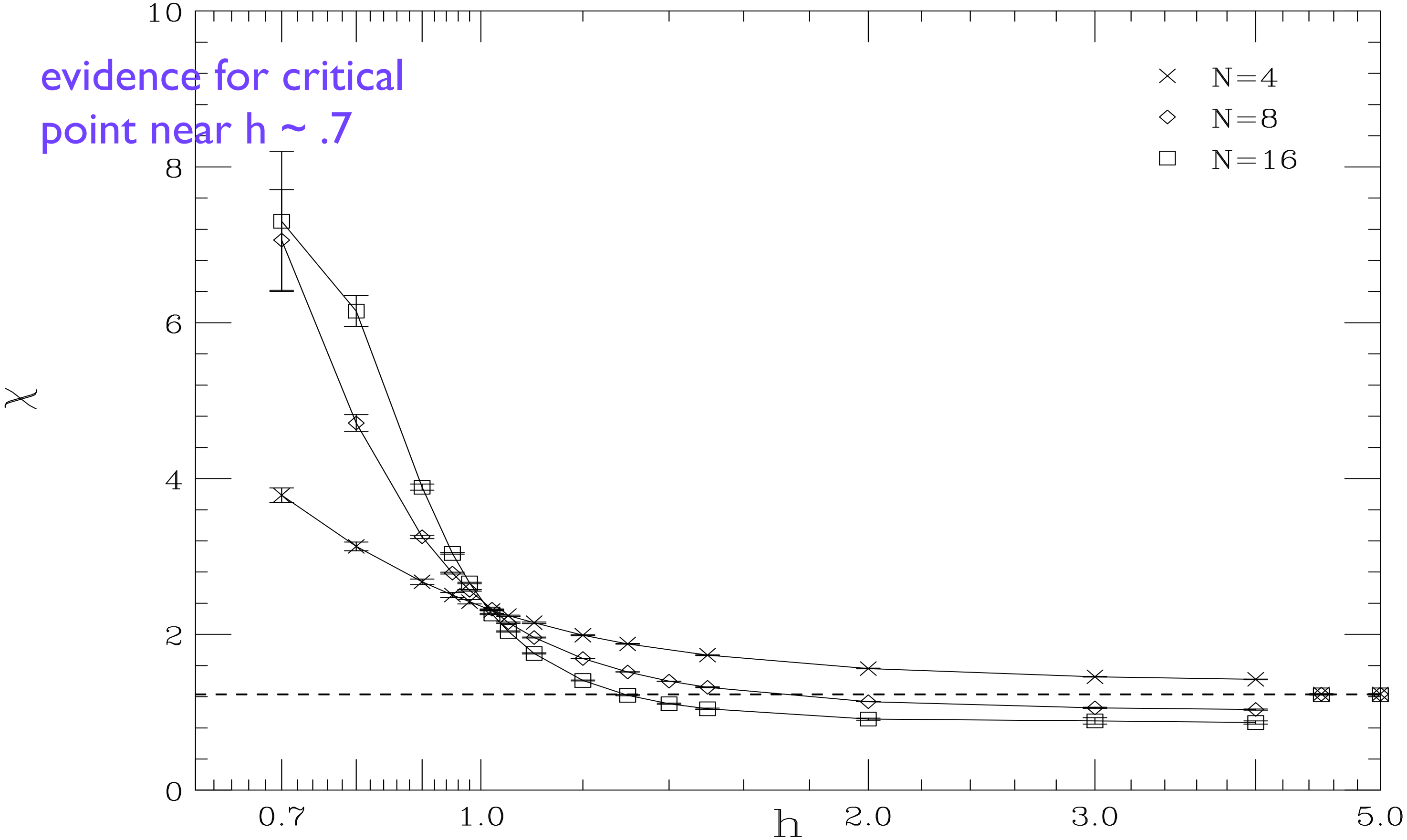

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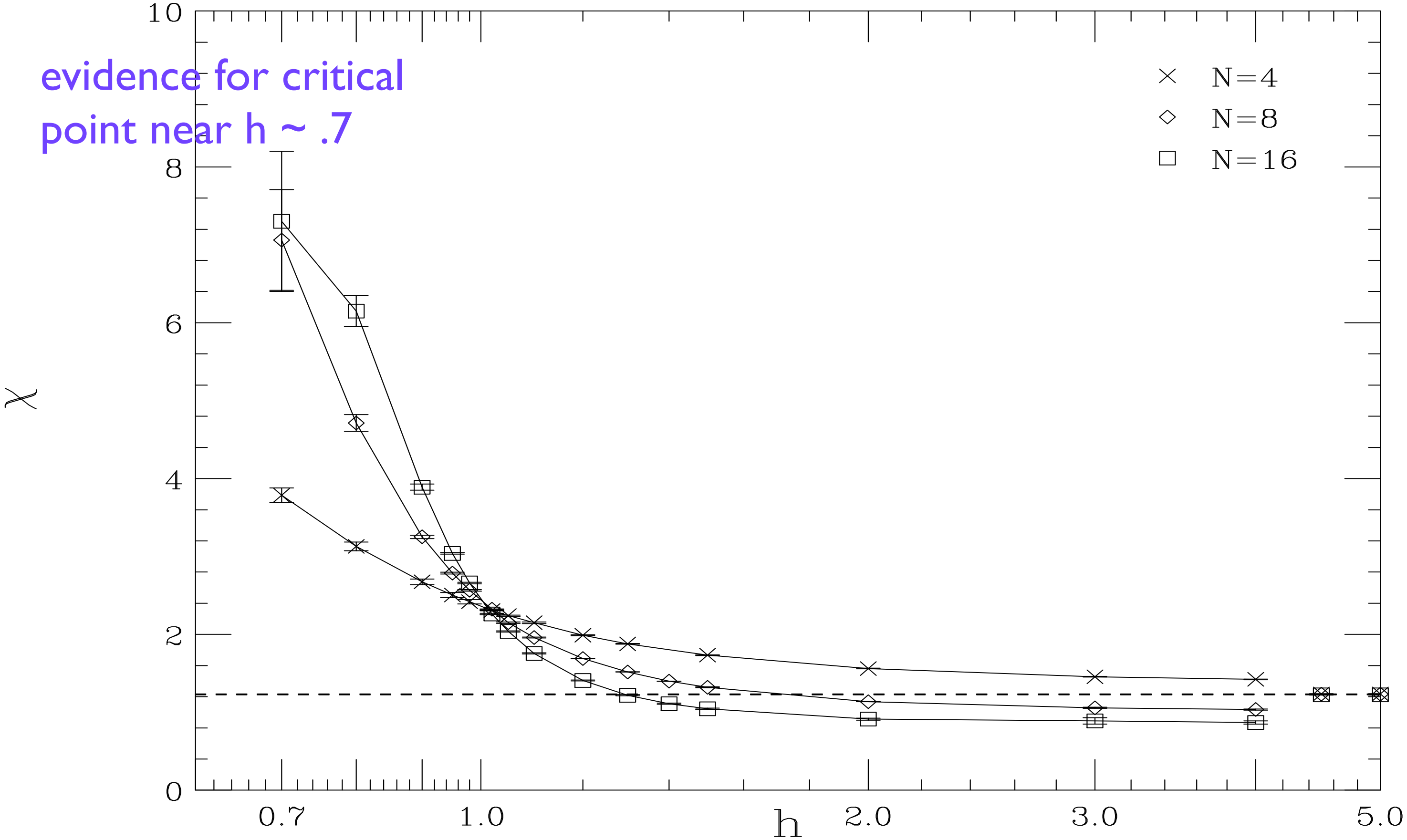

Joel Giedt, EP (2007)

- also measured other order parameters:
Binder cumulant, fermion-composite susceptibilities, and vortex density -
all show similar behavior as a function of h, no indication of long-range correlations for h > 1

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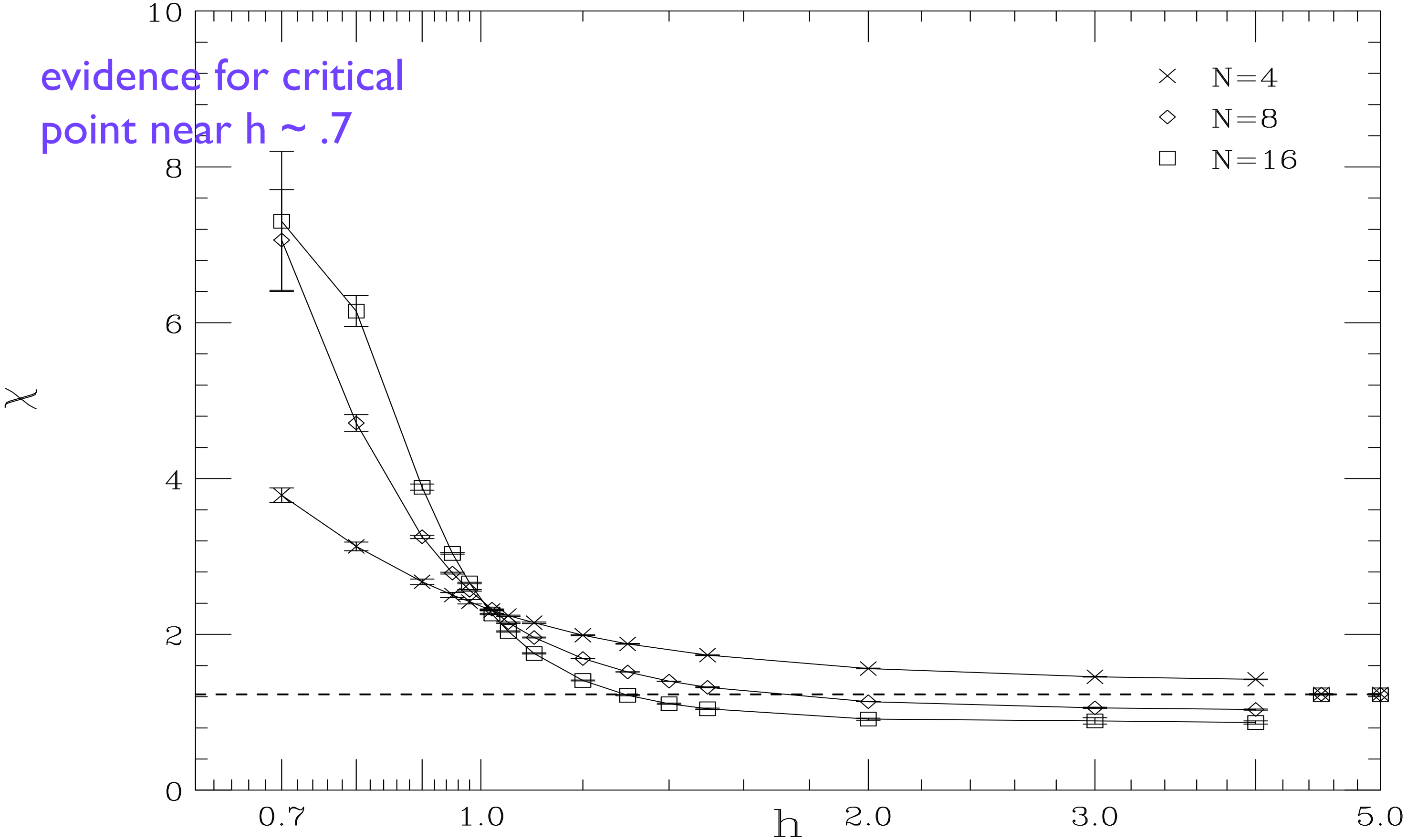

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- strong coupling symmetric phase exists

strong coupling symmetric phase exists also in at least one 4d model with exactly chiral fermions:

$SU(2)_L \times SU(2)_R$ chirally invariant Yukawa-Higgs model with GW fermions

P. Gerhold, K. Jansen (2007) --- starting from different motivation, but looked at large Yukawa...
single Yukawa coupling-only Dirac, no Majorana

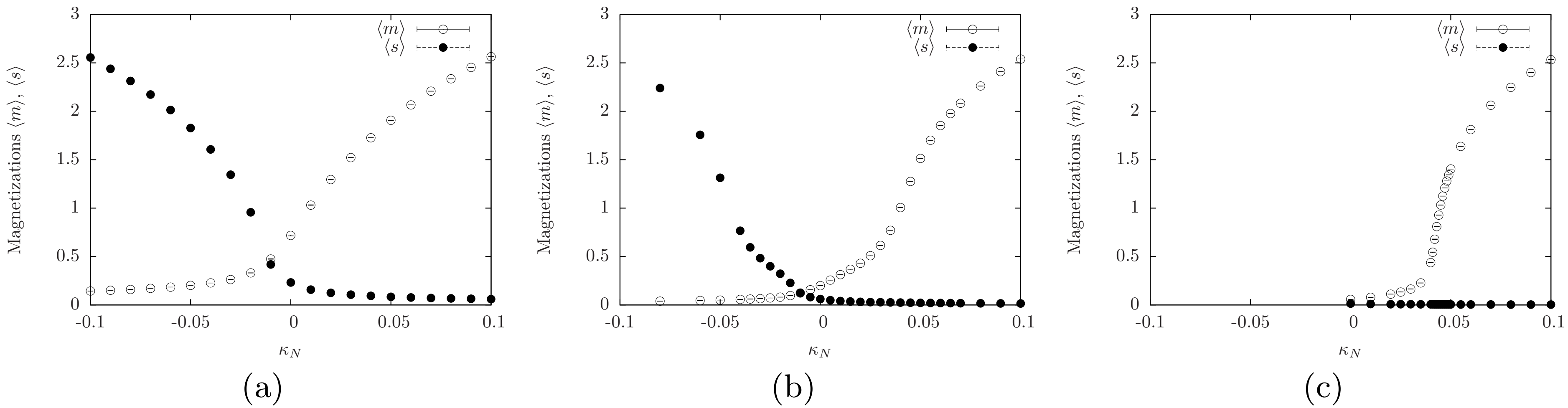


FIG. 8: The behaviour of the average magnetization $\langle m \rangle$ and staggered magnetization $\langle s \rangle$ as a function of κ_N on a 4^4- (a), 8^4- (b) and 16^4-lattice (c). In the plots we have chosen $\tilde{y}_N = 30$, $\tilde{\lambda}_N = 0.1$ and $N_f = 2$.

- strong coupling symmetric phase exists

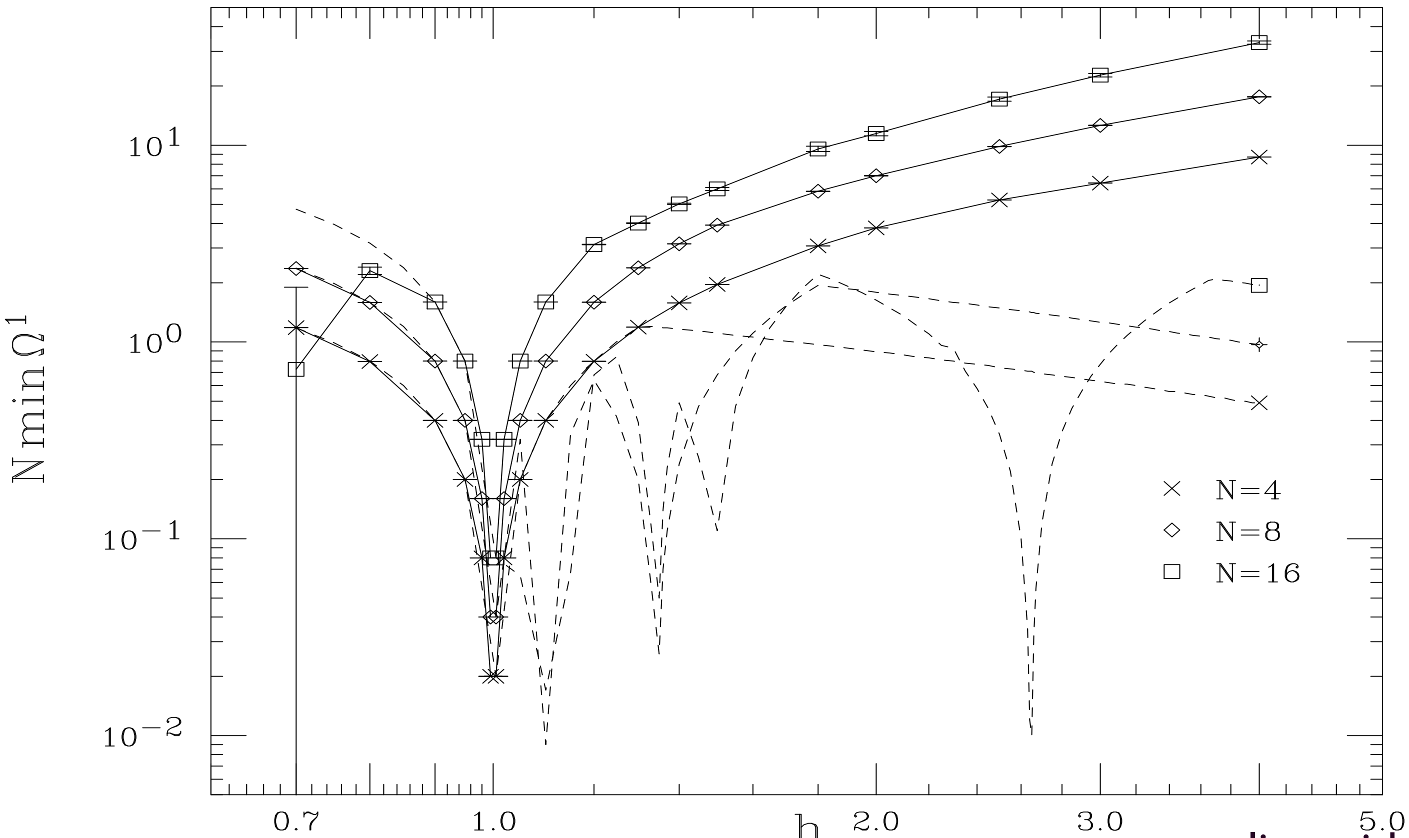
are mirrors heavy?

Joel Giedt, EP (2007)

2d Yukawa-unitary Higgs model with Ginsparg-Wilson fermions

lowest, as function of momentum, inverse eigenvalue of the L-R components of charged mirror fermion Green's function - zero means massless pole

dotted lines: "broken" (spin-wave) phase values, where perturbation theory good, check that agrees with MC



in units of y times inverse size of system, 1/L

scaling with y, N, L: $\sim y\, N^{x}\ L^{-1}$

values of exponent x depend weakly on kappa, h, but x is usually about 1

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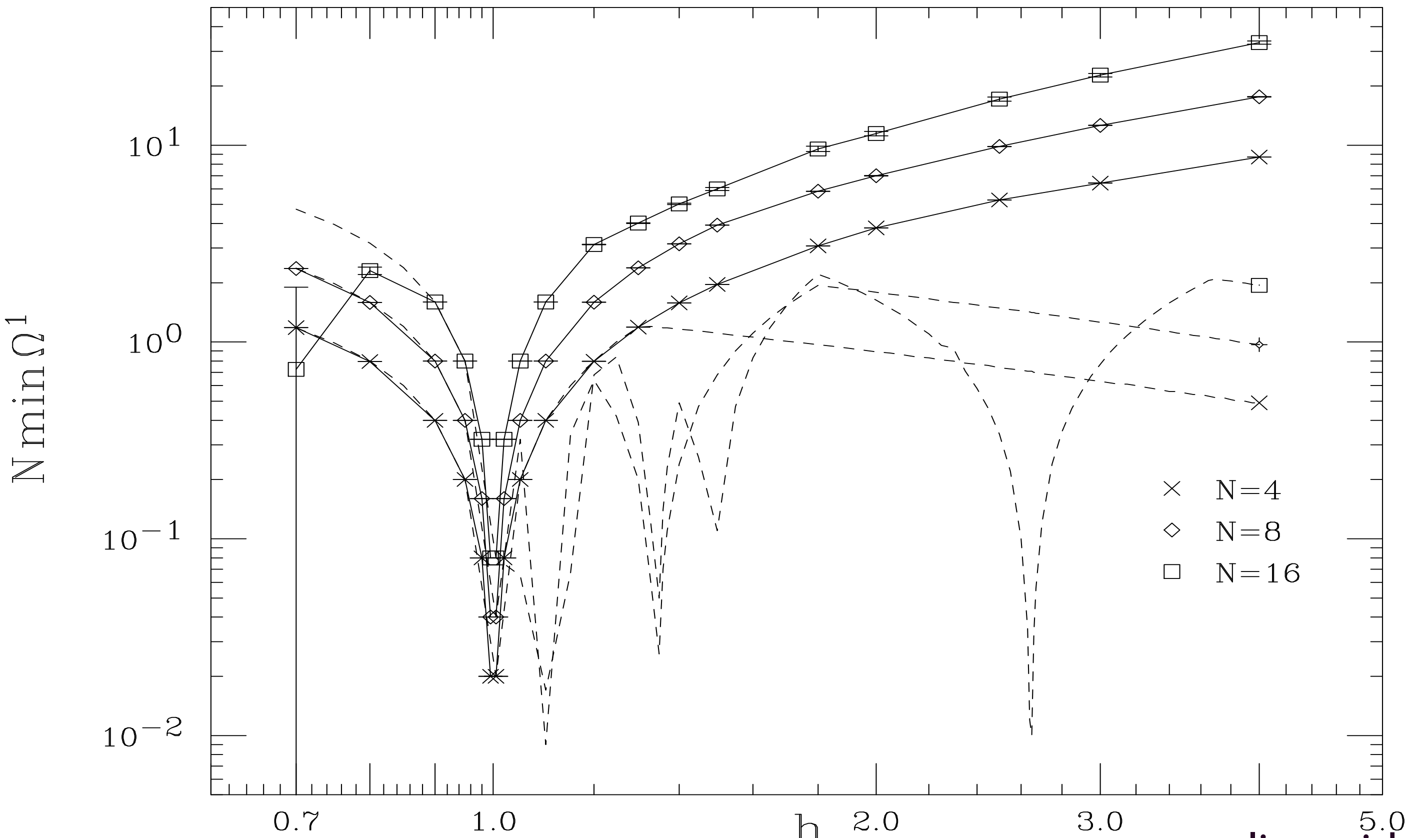

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- mirrors look heavy -

but more to come - have we decoupled an anomalous representation without a trace?

what happens if one tries to decouple an anomalous mirror representation?

Yanwen Shang, EP, arXiv:0706.1043[hep-th] + in progress

I discussed split of action into "light" and "mirror" components,
using GW to split kinetic term and defining Yukawa/4-fermi only in "mirror" terms:

$$\bar{\Psi}_q D_q \Psi_q = \bar{\Psi}_{q,+} D_q \Psi_{q,+} + \bar{\Psi}_{q,-} D_q \Psi_{q,-}$$

how does measure split?

$$\Psi_x = \sum_i c_i^+ w_i(x) + c_i^- u_i(x) \longleftarrow \gamma_5$$

eigenvectors of γ_5

$$\bar{\Psi}_x = \sum_i \bar{c}_i^+ t_i^\dagger(x) + \bar{c}_i^- v_i^\dagger(x) \;. \longleftarrow \hat{\gamma}_5$$

complete set of eigenvectors of modified $\hat{\gamma}_5$ - t, v depend on gauge field

split measure as:

$$\prod_x d\Psi_x \, d\bar{\Psi}_x = \frac{1}{J} \prod_i dc_i^- d\bar{c}_i^- \, dc_i^+ \, d\bar{c}_i^+$$

$$J = \det\| w_i^+(x) \; u_j(x) \| \; \det \| v_j^+(x) \; t_j^+(x) \|$$

light action is "free" (D[A])

mirror action has Yukawa, multi-fermion ...

$$S = S_{light}(\psi_+, \bar{\psi}_+) + S_{mirror}(\psi_-, \bar{\psi}_-, \phi, \dots)$$

$$Z_{mirror} = \int \prod_{\substack{i \\ x}} dc_i^- \, d\bar{c}_i^- [d\phi_x] \, e^{S_{mirror}(c^-, \bar{c}^-, t, w, \phi, \dots)}$$

depend on A_μ (for t)

operators do, too (for $\phi, \dots$)

mirror partition function - integral over c^- , $\bar{c}^-$ - now depends on gauge field through:

- eigenvectors t depending on gauge field (mirror action depends on t)
- operators depending on gauge field

How does mirror Z depend on gauge field - as the gauge background changes, how does Z_{mirror} change?

remarkably, the change of the basis vectors factorizes in the change of Z - no matter how complicated the mirror action: Yanwen Shang, EP, 2007 - really, our most useful result - proved a

"splitting theorem" for a background variation of a general chiral theory

(e.g., Z_mirror)

$$\delta \log Z[U] = \sum_i (\delta t_i^\dagger \cdot t_i) + \left\langle \frac{\delta S}{\delta O} \, \delta O \right\rangle$$

Neuberger, 1998, showed that in anomalous case the eigenvectors t can not be chosen to be smooth functions of the gauge background (topological obstruction similar to continuum)

- in anomalous case, Z_mirror is not a smooth function of A - no SD eqns...

- our "splitting theorem" encodes, on the lattice, the idea that anomaly is independent on the action (is also invaluable for computing gauge field correlators in the mirror)

our simulations of anomalous mirror theory used precisely this singular mirror partition function, defined via D chiral eigenvectors, discontinuous at A=0 (U=1)

... possibilities:

- expect mirror spectrum different from A=0 once path integral over A done
- perhaps new massless states to cancel light anomaly ... what are they?
- or somehow unitarity is violated (Jackiw/Rajaraman...) - after all we have a complex Euclidean partition function, not a Hamiltonian formulation.

to figure out what really happens, in this “would-be-anomalous” model,
work in two directions:

- ... future: simulations with gauge fields - need code, Joel Giedt, Blue Gene...

- one can still learn a lot (e.g. about unitarity!) by studying mirror polarization operator; in progress with Yanwen Shang, no need of new code

  ...only note couldn’t do this before “splitting theorem”

- clearly, anomaly-free models a lot more interesting
  (but expensive to study!)
  singular “light”-”mirror” splits do not afflict them

- but it is still good (faster and cheaper!) to understand precisely what happens in anomalous case, at least for purely theoretical reasons
  (and how/if problems are resolved in anomaly-free case)

finally, one could ask:

suppose indeed of practical use (e.g. sign/phase problem manageable)

when simulating an arbitrary chiral theory more than half of the resources go on simulating the mirror sector,
which you really don't care about...

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but what is the alternative?

- short of solving string theory

- or nonperturbative fine tuning a la Golterman-Shamir