Naturalness of Scalar Fields and the Standard Model

Partial Rehabilitation of Scalar Fields

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History—General

K. G. Wilson, Phys. Rev. D **3**, 1818 (1971). ’Mass terms ... must break a symmetry...’

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$$Q < \Lambda \sim m/\sqrt{\lambda}$$

Composite scalars

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L. Susskind, Phys. Rev. D **20**, 2619 (1979)

$$m^2 = m_0^2 + \Lambda^2 P(\lambda_0)$$

Fine tuning

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Fine tuning

L. Susskind, Phys. Rept. **104**, 181 (1984)...Supersymmetry

History—Practical

M. J. G. Veltman, Acta Phys. Polon. B 12, 437 (1981)...Veltman condition:

$$|m^2 - m_0^2| < m_0^2$$

$\Lambda \approx 1.2$ TeV. (Dimensional regularization)

History—Practical

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R. Barbieri and G. F. Giudice, Nucl. Phys. B 306, 63 (1988)... BG condition:

$$\left|\frac{\lambda_0}{m^2}\frac{\partial m^2}{\partial \lambda_0}\right| < q$$

q is a measure of fine tuning

Corrections

Leading order in λ_0:

$$m^2 = m_0^2 + \Lambda^2 P(\lambda_0)$$

Higer orders (previously considered):

$$P(\lambda_0) \to P(\lambda_0, \log(\Lambda/m_0))$$

More important corrections involve higer powers of Λ^2. For example, there appears a contribution

$$\lambda_0^3 \Lambda^4 / m_0^2$$

The higer the order of perturbation theory in λ_0, the higher the powers of cutoff appearing in the expansion. Resummation is needed.
Victor Kim & GP, arXiv:0712.0402 [hep-ph], PRD

All Order Evergreen Classics

In a different context, dependence on cutoff has been studied to all orders of the perturbation theory in λ. General theory of renormalization:

$$m_0^2 = m^2 - \Lambda^2 P(\lambda) + \Big(\gamma(\lambda) \log(\Lambda^2/m^2)\Big)$$

$$\lambda_0 = \lambda + \log(\Lambda^2/m^2) \frac{\beta(\lambda)}{2}$$

The fact that higher powers of cutoff are not appearing in the expressions for bare parameters in terms of physical ones is a basic result of renormalization theory. Solving for m^2, λ returns series in λ_0 involving higher powers of Λ^2. Resummation of higher powers of cutoff is effectively performed within standard renoramlization programm.

Are logs of Λ important?

Fine tuning argument neglects logs of Λ in the relation between bare and physical parameters. First, are logs important for Veltman condition

$$|m^2 - m_0^2| < m_0^2?$$

Log of Λ makes the difference between λ and λ_0. The correct relation between bare and physical masses is

$$m^2 = m_0^2 + \Lambda^2 P(\lambda)$$

The leading behaviour of m^2 in Λ is not affected by neglecting the log. So, Veltman's estimate for the scale of new physics ($\Lambda \approx 1.2$ TeV) is not qualitatively changed by higher order corrections.

Barbieri-Giudice Condition

$$\left|\frac{\lambda_0}{m^2}\frac{\partial m^2}{\partial \lambda_0}\right| < q$$

We need to express the derivative

$$\frac{\partial m^2}{\partial \lambda_0}$$

in terms of physical parameters and cutoff. Neglecting the log yields

$$\frac{\partial m^2}{\partial \lambda_0} = \Lambda^2 P'(\lambda)$$

So, neglecting the logs results in the derivative growing as Λ^2.

Jacobian matrix

More generally, we need to compute the Jacobian matrix

$$J = \begin{pmatrix} \frac{\partial \lambda}{\partial \lambda_0} & \frac{\partial \lambda}{\partial m_0^2} \\ \frac{\partial m^2}{\partial \lambda_0} & \frac{\partial m^2}{\partial m_0^2} \end{pmatrix}$$

Because we have expressions of bare parameters in terms of physical, we have the inverse matrix:

$$J^{-1} = \begin{pmatrix} \frac{\partial \lambda_0}{\partial \lambda} & \frac{\partial \lambda_0}{\partial m^2} \\ \frac{\partial m_0^2}{\partial \lambda} & \frac{\partial m_0^2}{\partial m^2} \end{pmatrix}$$

It is

$$\begin{pmatrix} 1 + \log(\frac{\Lambda^2}{m^2})\frac{\beta'(\lambda,g)}{2} & -\frac{\beta(\lambda,g)}{2m^2} \\ -\Lambda^2 P'(\lambda, g) & 1 \end{pmatrix}$$

Determinant

So, the desired Jacobian is

$$J = det(J) \begin{pmatrix} 1 & \frac{\beta(\lambda,g)}{2m^2} \\ \Lambda^2 P'(\lambda,g) & 1 + \log(\frac{\Lambda^2}{m^2}) \frac{\beta'(\lambda,g)}{2} \end{pmatrix}$$

The determinant is

$$det(J) = \frac{1}{-\frac{\Lambda^2}{m^2} P'(\lambda,g) \frac{\beta(\lambda,g)}{2} + \log(\frac{\Lambda^2}{m^2}) \frac{\beta'(\lambda,g)}{2} + 1}$$

We see that neglecting logs is equivalent to replacement

$$det(J) \to 1$$

Thus, it leads to a qualitative mistake in esimating the behaviour of the derivatives in bare parameters at large Λ

Result

Finally, we obtain the Jacobian at large cutoff:

$$\begin{pmatrix} \frac{\partial \lambda}{\partial \lambda_0} & \frac{\partial \lambda}{\partial m_0^2} \\ \frac{\partial m^2}{\partial \lambda_0} & \frac{\partial m^2}{\partial m_0^2} \end{pmatrix} \xrightarrow[\Lambda \to \infty]{} \begin{pmatrix} 0 & 0 \\ -\frac{2m^2}{\beta(\lambda)} & 0 \end{pmatrix}$$

This means that physical coupling exhibits universality, i.e. it becomes independent of bare parameters in the limit of infinite cutoff. The physical mass of a scalar does depend on bare coupling, and the derivative has a finite continuum limit.

We conclude that fine tuning problem is a problem of leading order perturbation theory. Resummation of cutoff powers removes the fine tuning problem.

Application to Standard Model

BG condition takes now the form

$$\left|\frac{2\lambda_0}{\beta(\lambda, g)}\right| < q$$

Using the scalar coupling beta-function of the Standard Model, the relation between couplings, masses and Higgs vev, and neglecting the difference between bare and physical coupling, we obtain the inequality

$$\frac{4m_H^2 v^2}{|p(m_H, m_Z, m_W, m_t)|} < \frac{3q}{4\pi^2},$$

where

$$\begin{aligned} p(m_H, m_Z, m_W, m_t) &= m_H^4 + m_H^2(2m_t^2 - m_Z^2 - 2m_W^2) \\ &- 4m_t^4 + m_Z^4 + 2m_W^4 \end{aligned}$$

...Application to Standard Model

This polynomial of Higgs mass vanishes at mass value about 200 GeV. Thus, at this value Higgs mass is very sensitive to the value of bare Higgs selfcoupling. If we forbid such sensitivity, moderate values of Higgs mass are forbidden. For example,

$$\begin{aligned} q &= 10 \text{ forbids interval } [96 \text{ GeV} \ldots 540 \text{ GeV}] \\ q &= 15 \text{ forbids interval } [113 \text{ GeV} \ldots 438 \text{ GeV}] \\ q &= 20 \text{ forbids interval } [126 \text{ GeV} \ldots 380 \text{ GeV}] \end{aligned}$$

Conclusions

1. There is no fine tuning problem in the theory of scalar field. The physical selfcoupling of a scalar is independent of bare parameters, while its physical mass depends on the bare selfcoupling in the following way:

$$m^2 = \Lambda^2 \exp\Big(-\frac{2(\bar{\lambda}_0 - \lambda_0)}{\beta(\lambda)}\Big),$$

where $\bar{\lambda}_0$ is the value of bare coupling at which physical mass equals Λ^2.
2. At $\Lambda \sim 1.2$ TeV about half of the Higgs mass squared comes from radiative corrections. So, at TeV energies, perturbation theory is not reliable.