New results for the kaon wave function

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K. G. Chetyrkin and A. Khodjamirian, AAP, Phys. Lett. B **661**, 661 (2008).

- Light-cone distribution amplitudes
- Correlation function at NNLO
- Numeric analysis with QCD sum rules
- Conclusion

1. Light-cone distribution amplitudes

LCDA enter factorization formulae used for description of exclusive processes in QCD:

- Pion EM form factor at large Q^2
- Light cone sum rules for form factors of heavy hadrons
- QCD factorization in B-meson decays
- Soft-collinear effective theory

Twist-2 LCDA of the kaon $\varphi_K(u,\mu)$

$$\langle K^-(q)|\bar{s}(z)\gamma_\mu\gamma_5\,[z,-z]\,u(-z)|0\rangle_{z^2=0}$$

$$= -iq_\mu f_K \int_0^1 du\; e^{iuq\cdot z - i\bar{u}q\cdot z}\varphi_K(u,\mu)$$

s- and $\bar{u}$ carry the momentum fractions u and $\bar{u} = 1-u$;

$$[x_1, x_2] = P\exp(i\int_0^1 dv(x_1-x_2)_\rho A^\rho(vx_1+\bar{v}x_2))$$

μ is the normalization scale.

Gegenbauer polynomials expansion

$$\varphi_K(u,\mu) = 6u\bar{u}\left(1+\sum_{n=1}^{\infty} a_n^K(\mu)C_n^{3/2}(u-\bar{u})\right)$$

$a_n^K(\mu)$ - Gegenbauer moments.

a_1^K is related to the difference between the longitudinal momenta of the strange and nonstrange quarks in the kaon.

We determine a numerical value of this asymmetry parameter $a_1^K(\mu)$ at a low scale $\mu \sim 1\ \mathrm{GeV}$.

The method is based on QCD sum rules. One relates a_1^K to the vacuum-to-kaon matrix element of a local operator with one derivative

$$\langle K^-(q)|\bar{s}\gamma_\nu\gamma_5 i \overleftrightarrow{D}_\lambda u|0\rangle = -iq_\nu q_\lambda f_K \frac{3}{5} a_1^K$$

$$\overleftrightarrow{D}_\lambda = \overrightarrow{D}_\lambda - \overleftarrow{D}_\lambda$$

Previous result (average)

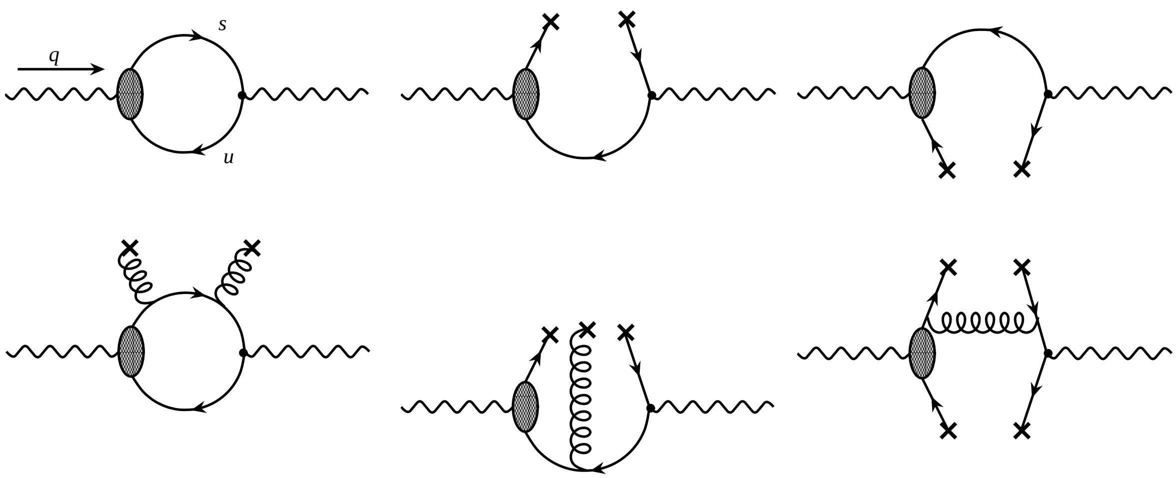
$$a_1^K(1\ \text{GeV}) = 0.06 \pm 0.03$$

2. Correlation function

Correlation function for a_1^K reads

$$\Pi_{\mu\nu\lambda}(q) = q_\mu q_\nu q_\lambda \Pi(q^2) + \dots$$

$$= i \int d^4x \; e^{iq\cdot x} \langle T \left\{ \bar{u}(x)\gamma_\mu\gamma_5 s(x), \bar{s}(0)\gamma_\nu\gamma_5 i \overleftrightarrow{D}_\lambda u(0) \right\} \rangle$$



Diagrams for OPE at LO:
PT loop and quark-condensate, gluon-, quark-gluon- and four-quark condensate diagrams.

OPE gives an expansion for $\Pi(q^2)$

$$\Pi(Q^2,\mu) = \frac{\mathcal{A}_2(Q^2,\mu)}{Q^2} + \frac{\mathcal{A}_4(Q^2,\mu)}{Q^4} + \frac{\mathcal{A}_6(Q^2,\mu)}{Q^6} + \ldots$$

$\mathcal{A}_j$ has a double expansion in α_s and m_s^2
(u, d-quark masses are neglected)

$$\mathcal{A}_d = a_d^{(0,0)} + \left(\frac{\alpha_s}{\pi}\right) a_d^{(1,0)} + \left(\frac{\alpha_s}{\pi}\right)^2 a_d^{(2,0)} + \left(\frac{m_s^2}{Q^2}\right) a_d^{(0,1)}$$

$$+ \left(\frac{m_s^2}{Q^2}\right)^2 a_d^{(0,2)} + \left(\frac{\alpha_s}{\pi}\right) \left(\frac{m_s^2}{Q^2}\right) a_d^{(1,1)} +$$

Numerical role of small parameters at $Q^2 \simeq 1\ \mathrm{GeV}^2$:
for $\alpha_s(1\ \mathrm{GeV}) = 0.47$ and $m_s(1\ \mathrm{GeV}) < 150\ \mathrm{MeV}$, one has $m_s^2/Q^2 \leq 0.02 \ll \alpha_s/\pi \simeq 0.15$.

So far only the $O(\alpha_s)$ correction to the quark-condensate contribution $\mathcal{A}_4$ was calculated

For the largest $d = 2, 4$ terms of the OPE the NNLO accuracy in α_s is achieved

The techniques of loop calculations are employed (programs QGRAF, FORM and MINCER).

$$d=2: \quad \mathcal{A}_2(Q^2,\mu) = \frac{m_s^2}{4\pi^2}\Bigg(1+\frac{\alpha_s}{\pi}\left[\frac{26}{9}+\frac{10}{9}l_Q\right]$$

$$+\left(\frac{\alpha_s}{\pi}\right)^2\left[\frac{366659}{11664}-\frac{29}{9}\zeta(3)+\frac{14449}{972}l_Q+\frac{605}{324}l_Q^2\right]$$

$$+3\frac{m_s^2}{Q^2}\left(\frac{5}{2}+l_Q\right)\Bigg);$$

$$d=4: \quad \mathcal{A}_4(Q^2,\mu) = -m_s\langle\bar{s}s\rangle\Bigg(1-\frac{\alpha_s}{\pi}\left[\frac{112}{27}+\frac{8}{9}l_Q\right]$$

$$-\left(\frac{\alpha_s}{\pi}\right)^2\left[\frac{28135}{1458}-4\zeta(3)+\frac{218}{27}l_Q+\frac{49}{81}l_Q^2\right]+2\frac{m_s^2}{Q^2}\Bigg)$$

$$-m_s\langle\bar{u}u\rangle\left(\frac{4\alpha_s}{9\pi}+\left(\frac{\alpha_s}{\pi}\right)^2\left[\frac{59}{54}+\frac{49}{81}l_Q\right]\right);$$

$$d=6: \quad \mathcal{A}_6(Q^2,\mu) = \frac{2}{3}m_s\langle\bar{s}Gs\rangle+\frac{1}{3}m_s^2\langle G^2\rangle\left(1+l_Q\right)$$

$$-\frac{32}{27}\pi\alpha_s\left(\langle\bar{s}s\rangle^2-\langle\bar{u}u\rangle^2\right),$$

The hadronic dispersion relation

$$\Pi(q^2) = \frac{\frac{3}{5}a_1^K f_K^2}{m_K^2 - q^2} + \int_{s_h}^{\infty} ds \frac{\rho^h(s)}{s - q^2}\,.$$

The $\rho^h(s)$ includes contributions of $K\pi\pi$, $K^*\pi$, $K\rho$, $K_1(1270)$, $K_1(1400)$,... The lower limit of integration is $s_h = (m_K + 2m_\pi)^2$. To approximate $\rho^h(s)$, we employ the quark-hadron duality

$$\rho^h(s)\Theta(s - s_0^h) = \rho^{OPE}(s)\Theta(s - s_0^K)\,,$$

where s_0^K is the effective threshold

Finally, the sum rule for a_1^K reads

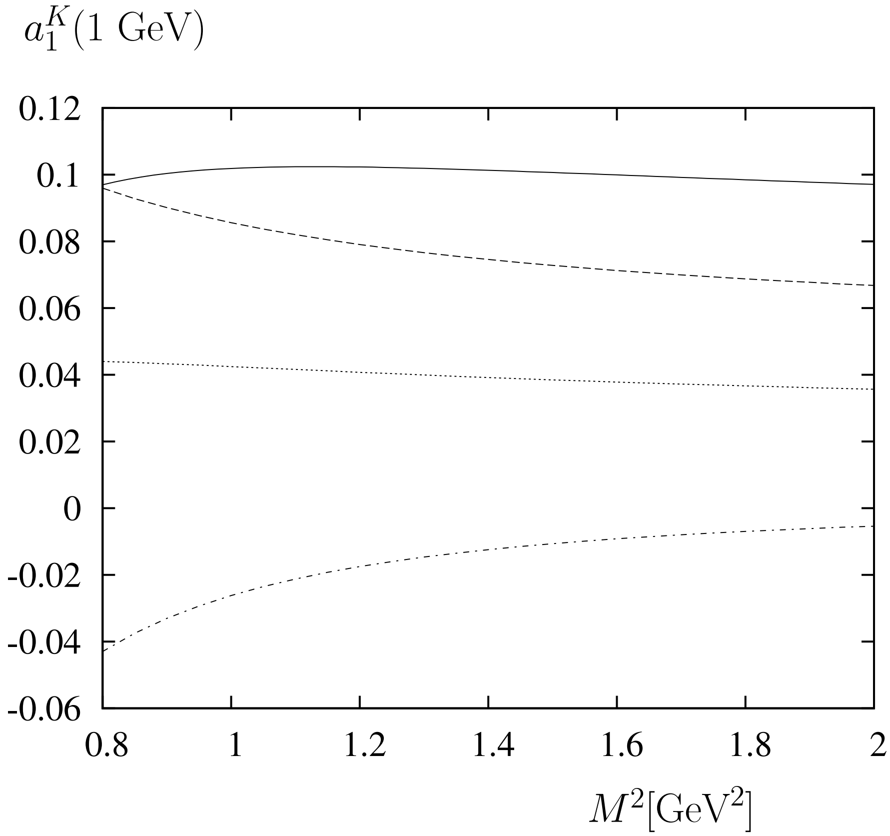
$$a_1^K = \frac{5}{3f_K^2} e^{m_K^2/M^2}\left(\Pi(M^2) - \int_{s_0^K}^{\infty} ds \rho^{OPE}(s) e^{-s/M^2}\right)$$

3. Numerical analysis

Input parameters:

- kaon mass $m_K^\pm = 493.58$ MeV
- kaon decay constant $f_K = 159.8 \pm 1.4 \pm 0.44$ MeV
- strange quark mass $m_s(2\ \mathrm{GeV}) = 98 \pm 16$ MeV ($m_s(1\ \mathrm{GeV}) = 128 \pm 21$ MeV)
- $\alpha_s(m_Z) = 0.1176 \pm 0.002$ ($\alpha_s(1\ \mathrm{GeV})/\pi = 0.15 \pm 0.01$)
- $\langle \bar{q}q(2\ \mathrm{GeV})\rangle = -(0.264^{+0.031}_{-0.020}\ \mathrm{GeV})^3$
- $\langle \bar{s}s\rangle / \langle \bar{q}q\rangle = 0.8 \pm 0.3$
- $\langle \bar{s}Gs\rangle = m_0^2 \langle \bar{s}s\rangle(1\ \mathrm{GeV})$ with $m_0^2 = 0.8 \pm 0.2\ \mathrm{GeV}^2$
- $\langle G^2\rangle = 0.012 \pm 0.012\ \mathrm{GeV}^4$

Borel parameter interval $M^2 = 1.0 - 2.0 \text{ GeV}^2$
Threshold parameter $s_0^K = 1.05 \text{ GeV}^2$ (sum rule for f_K).
s_0^K-dependence is weak



$a_1^K(1 \text{ GeV})$ as a function of the Borel parameter (solid); $d = 2$, $d = 4$ and $d = 6$ terms are shown with dashed, dotted and dash-dotted lines

Numerical prediction of the sum rule is

$$a_1^K(1\ \mathrm{GeV}) = 0.100$$

$$\pm 0.003|_{\mathrm{SR}} \pm 0.003|_{\alpha_s} \pm 0.035|_{m_s} \pm 0.022|_{m_q} \pm 0.013|_{cond}$$

Adding the individual uncertainties in quadrature we obtain the interval

$$a_1^K(1\ \mathrm{GeV}) = 0.10 \pm 0.04$$

Let us now discuss the structure of the perturbative series as it follows from the numerical analysis.

PT corrections enhance $d = 2$, $O(m_s^2)$ term

$$\Pi^{(m_s^2)} = \frac{m_s^2}{4\pi^2}\left[1 + 3.53\left(\frac{\alpha_s}{\pi}\right) + 33.7\left(\frac{\alpha_s}{\pi}\right)^2\right]$$

For quark condensate contribution ($d = 4$) corrections are smaller

$$\Pi^{(m_s\langle\bar{s}s\rangle)} = m_s\langle\bar{s}s\rangle\left(1 - 3.77\left(\frac{\alpha_s}{\pi}\right) - 10.8\left(\frac{\alpha_s}{\pi}\right)^2\right),$$

Thus, at NNLO the numerical pattern of the sum rule for a_1^K changes: relative weight of $d = 2$ term becomes larger

Bad convergence of m_s^2-part and determiation of m_s:
the Borel-transformed pseudoscalar correlation function is

$$\Pi^{(5)''(m_s^2)} = \frac{3m_s^2}{8\pi^2}(1 + 4.82\left(\frac{\alpha_s}{\pi}\right) + 22.0\left(\frac{\alpha_s}{\pi}\right)^2)$$

4. Conclusion

The NNLO PT corrections to the QCD sum rule for a_1^K are numerically important, they change the relative magnitude of the $d = 2$ (loop diagrams) and $d = 4, 6$ (condensate) terms in the OPE and give

$$a_1^K(1 \text{ GeV}) = 0.10 \pm 0.04$$

while previous result (average)

$$a_1^K(1 \text{ GeV}) = 0.06 \pm 0.03$$

The uncertainty of a_1^K is still large due mainly to the poor precision of the light quark masses: m_s directly entering the sum rule and $m_{u,d}$ determining the quark-condensate densities via Gell-Mann-Oakes-Renner relation.

Our result for a_1^K is larger than two recent lattice determinations:

$$a_1^K(2\ \mathrm{GeV}) = 0.0453 \pm 0.0009 \pm 0.0029$$

V. M. Braun *et al.*, [QCDSF/UKQCD Collaboration] Phys. Rev. D **74**, 074501 (2006)

and

$$a_1^K(2\ \mathrm{GeV}) = 0.048 \pm 0.003$$

M. A. Donnellan *et al.*, “Lattice Results for Vector Meson Couplings and Parton Distribution Amplitudes,” arXiv:0710.0869 [hep-lat];
P. A. Boyle, M. A. Donnellan, J. M. Flynn, A. Juttner, J. Noaki, C. T. Sachrajda and R. J. Tweedie [UKQCD Collaboration], “A lattice computation of the first moment of the kaon’s distribution amplitude,” Phys. Lett. B **641**, 67 (2006).

By evolving our result to the scale $2\ \mathrm{GeV}$ we find

$$a_1^K(2\ \mathrm{GeV}) = 0.08 \pm 0.04$$