Tunneling via unstable semiclassical solutions.

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Classically allowed and forbidden processes

Recently a new mechanism for tunneling has been discovered.

F. Bezrukov and D. Levkov, quant-ph/0301022

K. Takahashi and K.S. Ikeda, J. Phys. A **36**, 7953 (2003);

Classically allowed transitions

- All the trajectories corresponding to a given classically allowed process contribute almost equally into the quantum amplitude, $\mathrm{e}^{iS/\hbar}$.
- The unstable trajectories constitute a set of zero measure in the phase space of any regular system, accordingly, their contribution is vanishingly small.

Classically forbidden transitions

- The contributions of different semiclassical trajectories into the amplitude are of order $\mathrm{e}^{-\mathrm{Im}\, S/\hbar}$; they all are *exponentially different*.
- Clearly, only one distinguished complex trajectory (or, rather, its small vicinity) saturates the tunneling amplitude. This distinguished trajectory may well be unstable.

What are the physical reasons of the new mechanism?

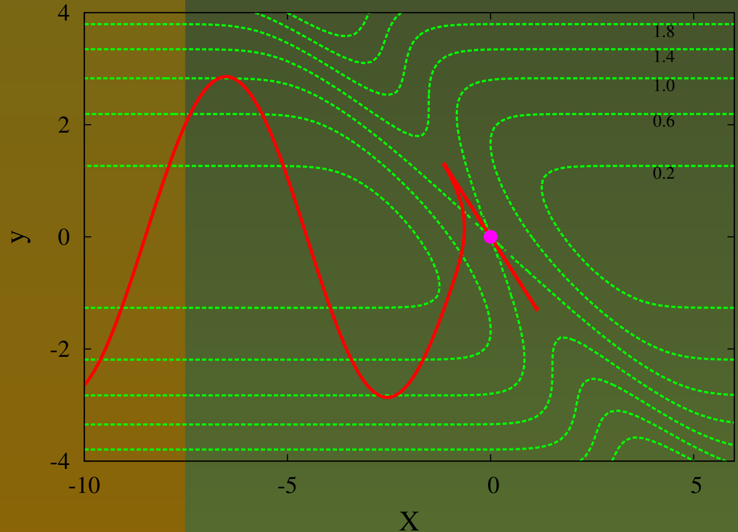
Necessary conditions for the new mechanism:

- High energy
- Multiple degrees of freedom
- Classical transitions are dynamically forbidden
- Nonlinear interaction between the degrees of freedom

Consider a system with the classical action in dimensionless units

$$S = \int dt \left[\frac{\dot{X}^2}{2} + \frac{\dot{y}^2}{2} - V(X,y) \right] \,, \quad V(X,y) = \frac{\omega^2 y^2}{2} + \mathrm{e}^{-(X+y)^2/2} \,, \quad \omega = \frac{1}{2} \,.$$

We are interested in the transition from the left in a state with wave function $|E, E_y\rangle$ to the right.



At some conditions it is preferable for a particle to transfer some part of the energy into translatory degree of freedom.

Difficulties.

Difficulties of the semiclassical description of the sphaleron-driven process.

- It's not easy to find unstable solution numerically
- There is a problem to describe semiclassically the subsequent decay of an unstable intermediate state
- The standard semiclassical expression leads to zero value for pre-exponential factor

ε–regularization technique

One inserts into the path integral for the amplitude of the process the unity factor

$$1 = \int_0^{+\infty} d\tau\, \delta(T_{\mathrm{int}}[\vec{x}] - \tau) = \int_0^{+\infty} d\tau \int_{-i\infty}^{i\infty} \frac{id\varepsilon}{2\pi\hbar}\, \mathrm{e}^{-\varepsilon T_{\mathrm{int}}[\vec{x}]/\hbar + \varepsilon\tau/\hbar}\,, \qquad T_{\mathrm{int}}[\vec{x}] = \int dt V(\vec{x})$$

It leads to an additional purely imaginary term in the equations of motion:

$$\ddot{\vec{x}} + (1 - i\varepsilon)\vec{V}'(\vec{x}) = 0\,.$$

For the probability we obtain

$$\mathscr{P} = \int_0^{+\infty} \frac{d\tau_+}{\sqrt{\pi}} \sqrt{-\frac{d\varepsilon}{d\tau_+}} A_\varepsilon \mathrm{e}^{-F_\varepsilon/\hbar} \, , \qquad \tau_+ = \mathrm{Re}\,\tau = \mathrm{Re}\, T_{\mathrm{int}}[\vec{x}]$$

Integrating over τ_+, we obtain

$$\mathscr{P} = \hbar^{\gamma/2} A(E, E_y) \mathrm{e}^{-F(E,E_y)/\hbar}$$

Direct tunneling:

$$\boxed{\gamma = 1}$$

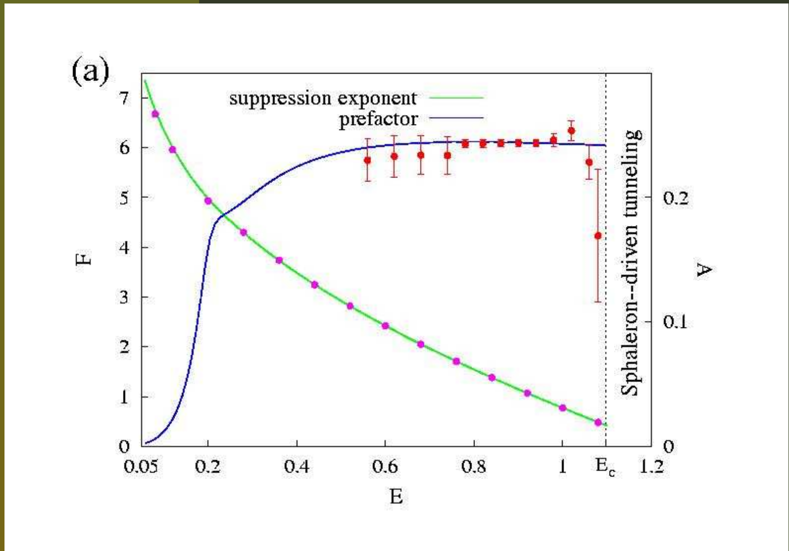
$$\mathscr{P} = \hbar^{\frac{1}{2}} A e^{-F/\hbar}$$

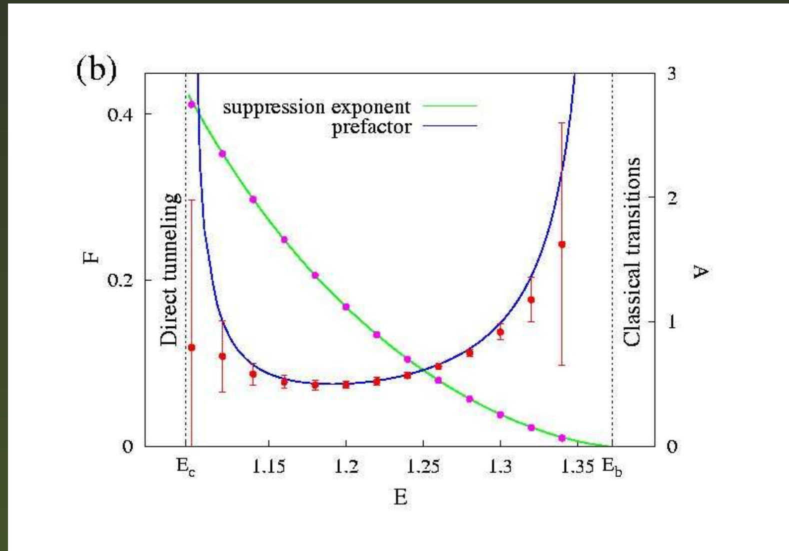
Sphaleron-driven tunneling:

$$\boxed{\gamma = 2}$$

$$\mathscr{P} = \lim_{\varepsilon \to 0} \frac{\hbar A_\varepsilon \mathrm{e}^{-F_\varepsilon/\hbar}}{\varepsilon \sqrt{-4\pi \frac{d\tau_+}{d\varepsilon}}}$$

Results





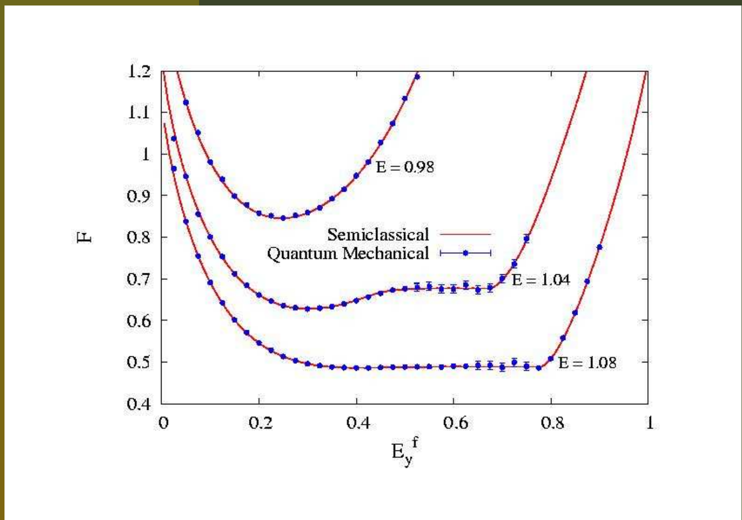
The semiclassical (line) and quantum–mechanical (points) results for the suppression exponent F and the pre-exponential factor A plotted for $E_y = 0.05$. Quantum–mechanical data we extrapolate to $\hbar = 0$ by fitting them with the function

$$\hbar \log \mathscr{P}_{\text{ex}}(\hbar) = -F_{\text{ex}} + \frac{\gamma}{2}\hbar \log \hbar + \hbar \log A_{\text{ex}} + \hbar^2 C_{\text{ex}}$$

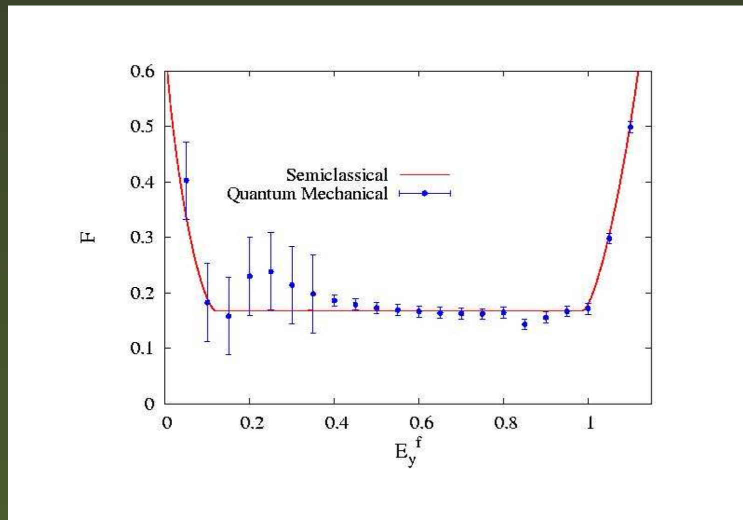
Discussion

The method of ε–regularization allow us to study a different experimental signatures of the effect of tunneling via unstable intermediate state

- Increase in tunneling time as compared to the direct tunneling transition
- The effect of spreading for the spectra with respect to final quantum numbers of a particles



(a)



(b)

(a) The suppression exponent of tunneling into the exclusive final states with fixed oscillator energy E_y^f. The graphs are plotted for $E = 0.98$, $E = 1.04$ and $E = 1.08$; $E_y = 0.05$.
(b) For the initial quantum numbers $E = 1.2$, $E_y = 0.05$.