ON EXISTENCE OF NONSINGULAR SOLUTIONS IN STATIC BRANEWORLDS

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Motivation

- Large extra dimensions — one of the most popular extensions of the SM

- Randall-Sundrum type II setup Randall, Sundrum, 1999

$$ds^2 = \mathrm{e}^{-2k|z|}\left(dt^2 - d\vec{x}^{\,2}\right) - dz^2$$

  - $4d$-brane at $z = 0$
  - $\Lambda < 0$ in the bulk
  - any slice $z = const$ is Lorentz invariant
  - static gravitational potential on the brane is Newtonian

$$V(r) = G_N \frac{m_1 m_2}{r}\left(1 + \frac{1}{r^2 k^2}\right)$$

- Matter in the bulk may lead to the metric

$$ds^2 = A(t,z,\vec{x})dt^2 - B(t,z,\vec{x})d\vec{x}^{\,2} - C(t,z,\vec{x})dz^2$$

$A(t,z,\vec{x}) \neq B(t,z,\vec{x}) \neq C(t,z,\vec{x}) \Longrightarrow$ Lorentz violation in the bulk $\Longrightarrow$

- Lorentz violation on the brane ($E^2 \neq m^2 + k^2$)
- modification of gravity on the brane
- new cosmology on the brane (late time accelerated expansion)
- escape from the brane
- "deposition" at the brane
- "trans-Plankian" problem
- ...

- All of these features are very interesting and promising from the phenomenological point of view...

In most known interesting cases matter in the bulk

- violates energy conditions S.Dubovsky 2001
- is not specified. Lorentz violating metric is introduced '*ad hoc*' M.L., V.Rubakov 2005

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? What are the properties of the bulk matter? Is the matter satisfies the *Null Energy Conditions (NEC)*? If 'YES' what are the properties of the metric?

Null Energy Conditions

NEC: $T_{AB}\xi^A\xi^B \geq 0$ for $\forall$ null vector ξ^A: $g_{AB}\xi^A\xi^B = 0$

✓ The NEC is the *weakest* of the energy conditions

✓ The NEC is *not violated* by Λ, by any known matter, or by unitary two-derivative QFT

✓ The NEC violation leads to

– superluminal propagation
– instabilities
– violations of unitarity

Cline,Jeon,Moore 2004; Hsu,Jenkins,Wise 2004; Dubovsky,Gregoire,Nicolis,Rattazzi 2006; Buniy,Hsu,Murray 2006

✓ The NEC forbids "strange" solutions to Einstein's equations

– traversable wormholes — Morris,Thorne 1988; Visser,Kar,Dadhich 2003
– superluminal "warp drives" — Alcubierre 1994; S.Krasnikov 1998
– time machines — Morris,Thorne,Yurtsever 1998; Hawking 1992
– universes with big rip singularities — Caldwell 2002; Caldwell,Kamionkowski,N.Weinberg 2003

The matter should respects the NEC

Metric Properties

- study of properties of a generic metric is a formidable task
- static metric $\Longrightarrow A(t,z,\vec{x}), B(t,z,\vec{x}), C(t,z,\vec{x})$ do not depend on t
- $SO(3) \Longrightarrow A(z,\vec{x}), B(z,\vec{x}), C(z,\vec{x})$ are depending on $|\vec{x}|$ only
- $T^3 \Longrightarrow A(z,|\vec{x}|), B(z,|\vec{x}|), C(z,|\vec{x}|)$ do not depend on $|\vec{x}| \Longrightarrow$

$$ds^2 = \mathrm{e}^{-2a(z)}dt^2 - \mathrm{e}^{-2b(z)}d\vec{x}^{\,2} - dz^2$$

- $\mathbb{Z}_2$ (for simplicity) $\Longrightarrow a(z) = a(-z), b(z) = b(-z)$
- brane is located at $z = 0$

$$T^A_{B,b} = \mathrm{diag}(\rho_b + \sigma, -p_b + \sigma, -p_b + \sigma, -p_b + \sigma, 0)\delta(z)$$

- by appropriate rescaling $a(0) = b(0) = 0$

The No-Go Theorem

Let the spatial curvature of the brane be equal to zero. Then no generic background of type

$$ds^2 = \mathrm{e}^{-2a(z)}dt^2 - \mathrm{e}^{-2b(z)}d\vec{x}^2 - dz^2$$

with $a(z) \neq b(z)$ without bulk physical singularities is possible provided that NECs on the brane and in the bulk are satisfied and total brane energy density (including brane tension) is positive

$$\rho_b + \sigma \geq 0.$$

It is impossible to shield the singularities from the brane by a horizon [Cline, Firouzjahi 2002].

If $a(z) = b(z)$ then the solution respecting the above conditions exists.

The Proof

- The NEC inequalities

$$g_{00}T_0^0(\xi^0)^2 - |g_{11}|T_1^1\sum_{i=1}^{3}(\xi^i)^2 - T_5^5(\xi^5)^2 \geq 0$$

$$g_{00}(\xi^0)^2 - |g_{11}|\sum_{i=1}^{3}(\xi^i)^2 = (\xi^5)^2$$

$$\Downarrow$$

$$|g_{11}|\left(T_0^0 - T_1^1\right)\sum_{i=1}^{3}(\xi^i)^2 + \left(T_0^0 - T_5^5\right)(\xi^5)^2 \geq 0$$

$$\Downarrow$$

$T_0^0 - T_1^1 \geq 0$ and $T_0^0 - T_5^5 \geq 0$

- Einstein's equations $(8\pi G_N = 1)$

$$T_0^0 - T_1^1 = b'' - a'' - 3b'^2 + a'^2 + 2a'b' \geq 0$$

$$T_0^0 - T_5^5 = 3(b'' - b'^2 + b'a') \geq 0$$

$$\Downarrow$$

$$b'' - a'' - 3(a' - b')^2 - 4a'(b' - a') \geq 0$$

$$b'' - b'^2 + b'a' \geq 0$$

- Israel's junction condition and brane NEC

$$p_b + \rho_b = 2(b'(0) - a'(0)) \geq 0$$

$$\rho_b + \sigma = 6b'(0) \geq 0$$

- Assume $b'(0) > a'(0) \geq 0 \Rightarrow \exists\ z_c > 0$:

$$b'(z) - a'(z) \geq 0 \text{ for } 0 < z < z_c$$

- In this region

$$b'' - a'' - 3(a' - b')^2 - 4a'(b' - a') \geq 0 \longrightarrow$$

$$b'' - a'' - 3(a' - b')^2 - 4a'(b' - a') = \phi(b' - a')$$

where $\phi = \dfrac{T_0^0 - T_1^1}{b' - a'} \geq 0$

- This equation can be solved in terms of b' generically as follows

$$b'(z) = a'(z) - \frac{\exp\left(4a(z) + 4\int\limits_0^z \phi(y)\, dy\right)}{3\int\limits_0^z dy\, \exp\left(4a(y) + 4\int\limits_0^y \phi(t)\, dt\right) - C}$$

where $\frac{1}{C} = b'(0) - a'(0) > 0$

- Let $z_c < \infty \;\Rightarrow\; b'(z_c) = a'(z_c) \;\Rightarrow$ at z_c
  - the numerator vanishes $\;\Rightarrow\; a(z_c) \to -\infty \; (\phi \geq 0)$
  - the denominator turns to infinity $\;\Rightarrow\; \exists\; z_*,\; 0 < z_* < z_c$, such that

$$3\int\limits_0^{z_*} dy\, \exp\left(4a(y) + 4\int\limits_0^y \phi(t)\, dt\right) = C$$

and

$$b'(z_*) - a'(z_*) \to \infty$$

- $z_c \to \infty$

$$b'' - b'^2 + b'a' \geq 0 \to$$

$$b'' - b'^2 + b'a' = \chi b' \; , \quad \chi \geq 0 \Rightarrow$$

$$b'(z) = \frac{\exp\left(- a(z) + \int\limits_0^z \chi(y)\, dy \right)}{- \int\limits_0^z dy \, \exp\left(- a(y) + \int\limits_0^y \chi(t)\, dt \right) + \frac{1}{b'(0)}}$$

In order to keep both functions $b' - a'$ and b' finite in the bulk the following integrals

$$\int\limits_0^\infty e^{4a(z)} dz < \infty, \quad \text{and} \quad \int\limits_0^\infty e^{-a(z)} dz < \infty$$

have to converge which cannot be provided simultaneously

$\mathcal{NB}$: $a'(z)$ and $b'(z)$ cannot have singularities at the same point

- Assume $b'(0) = a'(0) \geq 0$ but $b'(z) \neq a'(z) \;\Rightarrow$

$$a'(z) = a_0 + a_1 z + \mathcal{O}(z^2), \quad b'(z) = a_0 + b_1 z + \mathcal{O}(z^2)$$

Then the NEC inequalities become

$$b_1 - a_1 - 4a_0(b_1 - a_1)z + O(z^2) \geq 0, \quad b_1 - a_0(b_1 - a_1)z + O(z^2) \geq 0$$

$$\Downarrow$$

$$b_1 \geq 0, \quad b_1 \geq a_1$$

$$\Downarrow$$

$$\exists\, z_n > 0 : \quad b'(z_n) > a'(z_n) \geq 0$$

$$\Downarrow$$

We return to the previous case $b'(0) > a'(0) \geq 0$

- $a'(z) = b'(z)$ — Lorentz invariant case.

  An example:

$$b'(z) = \begin{cases} (z - z_0)^3 + \sqrt{-\frac{\Lambda}{6}}, & 0 \le z < z_0 \le \sqrt[6]{-\frac{\Lambda}{6}}, \\ \sqrt{-\frac{\Lambda}{6}}, & z \ge z_0. \end{cases}$$

$$b'(0) = \sqrt{-\frac{\Lambda}{6}} - z_0^3 \ge 0$$

$$T_0^0 = 3b'' - 6b'^2 - \Lambda \ge 0$$

Are the found singularities physical?

$$T_5^5 = -3b'(b' + a')$$

Since a' and b' cannot have singularities simultaneously then T_5^5 has a pole as soon as one of the functions a', b' turns to infinity. Thus all singularities are physical and cannot be screened by a horizon.

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The Theorem Is Proved

Conclusion

✓ Our statement does not allow generic smooth flat braneworld static backgrounds with positively defined energy density satisfying NECs in the bulk and on the brane to exist.

✓ The statement of the theorem does not depend on the finiteness of the volume of the extra dimension which claims that the integral $\int_0^\infty dz\sqrt{g} < \infty$ converges.

✓ **The Way Out** — Lorentz invariant setup

✓ **The Way Out** — positive brane curvature is of the same order as the Anti-de-Sitter scale Λ

✓ **The Way Out?** — time depending metric may (or may not) help to evade the theorem. From the qualitative reasons on may think that the scale factors should change fast enough with time (of the same order as Λ).