Anomalously large tunneling times in multidimensional quantum mechanics

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DL, A. Panin, S. Sibiryakov, PRL **99**, 170407 (2007)
DL, A. Panin, *To be published*

Why tunneling time?

Leaving aside the natural curiosity... Experimental signature!

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Quasistationary state

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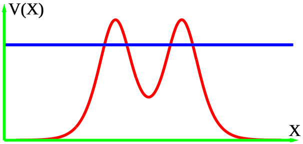
  Yamamoto, Miyamoto, Hayashi, Phys.Stat.Solid. (b) **209**, 305 (1998)]



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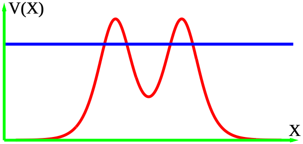
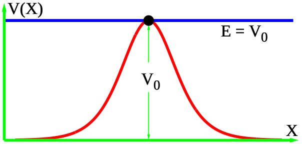


Classically unstable "state"

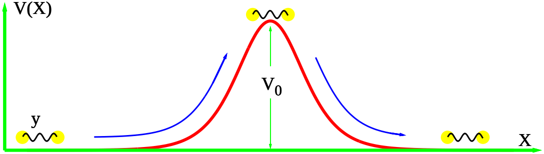
- (Not a quantum state at all)
  [Takahashi, Ikeda, PRL **97**, 240403 (2006)]



Surprise

Studying the semiclassical solutions:

Classically unstable intermediate "states" form in the generic tunneling processes.



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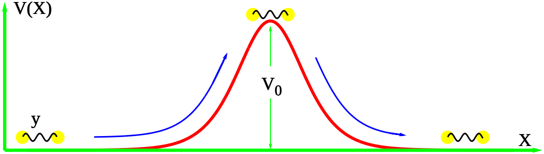

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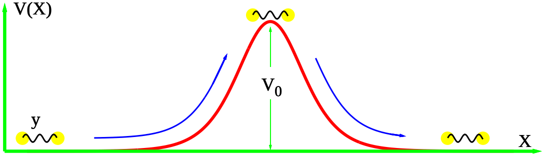

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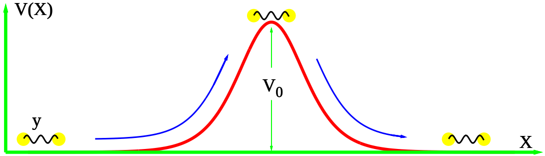

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- High total energy $E > E_c$
  (E_c — critical energy, $E_c > V_0$)

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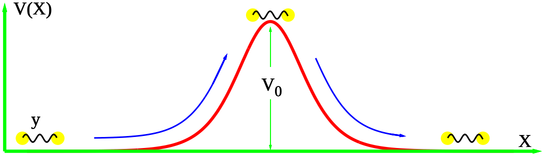


Conditions:

- Several degrees of freedom ($N \geq 2$)
- Nonlinear coupling between d.o.f.
- High total energy $E > E_c$
  (E_c — critical energy, $E_c > V_0$)
- Fixed initial state

Numerous examples:

- Tunneling in regular quantum systems ($N = 2$)

  [Bonini, Cohen, Rebbi, Rubakov, PRD **60**, 076004 (1999)

  Bezrukov, DL, ArXiv:quant-ph/0301022(2003); JETP **98**, 820 (2004)]

- Tunneling in systems with dynamical chaos ($N = 1.5,\ \ 2$)

  [Takahashi, Ikeda, J. Phys. A **36**, 7953 (2003)

  DL, Panin, Sibiryakov, PRE **76**, 046209 (2007)]

- Topology-changing transitions in Electroweak theory ($N = \infty$)

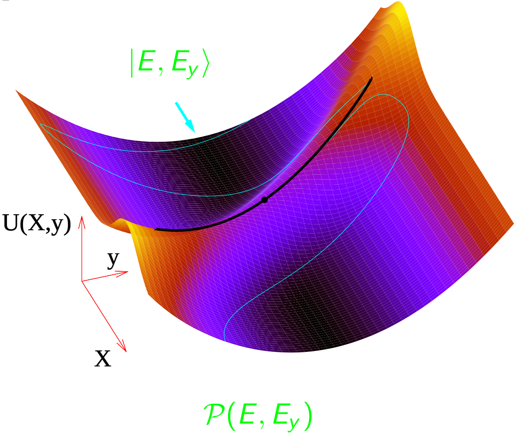
  Bezrukov, DL, Rebbi, Rubakov, Tinyakov, PRD **68**, 036005 (2003)

- Collision–induced soliton creation in scalar field theory ($N = \infty$)

  DL, Sibiryakov, PRD **71**, 025001 (2005)

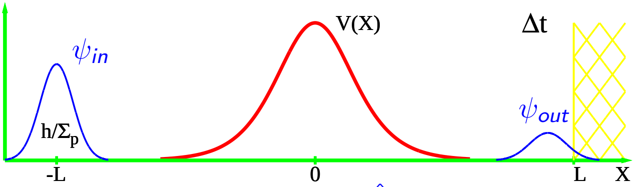
The model

$$S = \int dt \left[(\dot{X}^2 + \dot{y}^2 - \omega^2 y^2)/2 - \exp(-(X+y)^2/2) \right]$$



$\hbar \to 0$ $\Rightarrow$ Semiclassical description

Definition of tunneling time



$$\psi_{out}(X,y) = \mathrm{e}^{-i\hat{\mathcal{H}}\Delta t/\hbar}\,\psi_{in}(X,y)$$

Probability of tunneling into the region $X > L$ in time Δt:

$$\mathcal{P}(\Delta t) = \int_L^{+\infty} dX \int dy\, |\psi_{out}(X,y)|^2$$

Probability distribution over Δt:

$$\rho(\Delta t) = \mathcal{N}\frac{d\mathcal{P}}{d\Delta t}\,, \qquad \int_0^{+\infty} \rho(\Delta t) d\Delta t = 1\,.$$

Average time of passing:

$$\langle \Delta t \rangle = \int_0^{+\infty} d\Delta t \cdot \Delta t \, \rho(\Delta t) \; .$$

Dispersion of Δt

$$\sigma_t^2 = \langle \Delta t^2 \rangle - \langle \Delta t \rangle^2 \; .$$

Tunneling time delay (compared to the free motion):

$$t_D = \langle \Delta t \rangle - \frac{L}{\dot{X}_{in}} - \frac{L}{\dot{X}_{out}}$$

Interaction time functional

Counts a time that trajectory $X(t)$, $y(t)$ spends in the region $X < L$:

$$T_{int}[X(t)] = \int_0^{\Delta t} dt\, \theta(L - X(t)) \,, \qquad\qquad T_{int} < \Delta t \,.$$

Faddeev–Popov unity

$$\underbrace{1 = \int_0^{\Delta t} d\tau\, \delta(\tau - T_{int}[X(t)])}$$

$$\psi_{out} = \int [dX(t)\, dy(t)] \cdot \mathrm{e}^{iS/\hbar} \cdot \psi_{in}(X_{in}, y_{in})$$

$\hbar \longrightarrow 0$: evaluate all integrals except for $\int d\tau$ in the saddle-point approximation!

Result: $\mathcal{P}(\Delta t) = \int_0^{\Delta t} d\tau \underbrace{A(\tau) \cdot \mathrm{e}^{-F(\tau)/\hbar}}$

Contribution of paths that leave the region $X < L$ in time $\tau < \Delta t$.
Does not depend on Δt!

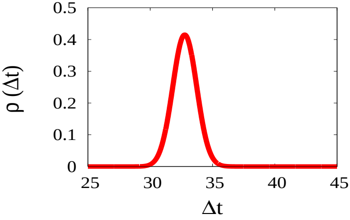
$$\Downarrow$$

$$\rho(\Delta t) = \mathcal{N} \cdot A(\Delta t) \cdot \mathrm{e}^{-F/\hbar} .$$

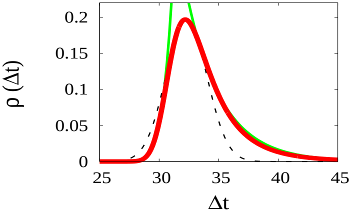
We can calculate $\langle \Delta t \rangle$, t_D , σ_t^2 !

Results

$E = 0.91 < E_c$



$E = 1.09 > E_c$



$(E_y = 0.05,\ \Sigma = 0.1,\ \hbar = 0.02,\ L = 20)$

$$\rho = \mathcal{N} \cdot \mathrm{e}^{-(\Delta t - \langle \Delta t \rangle)/2\sigma_t^2}$$

$$\langle \Delta t \rangle \sim O(\hbar^0)$$

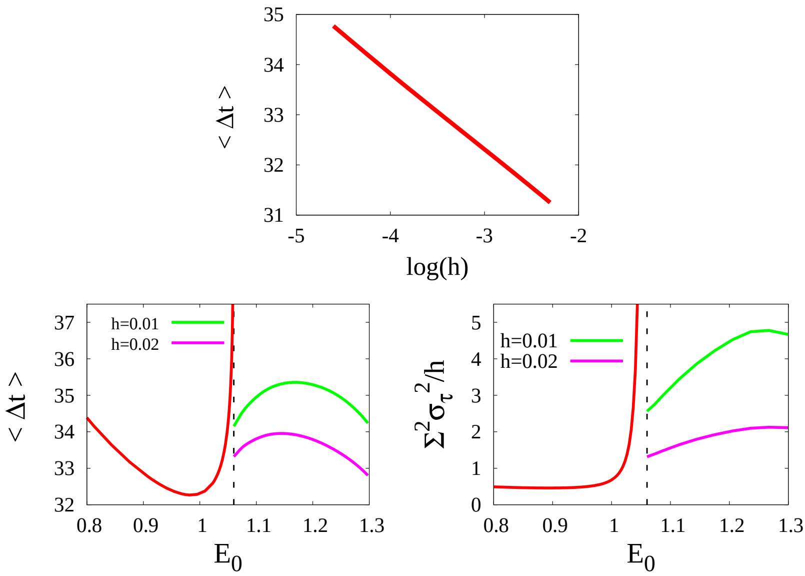
$$\sigma_t^2 \sim O(\hbar)$$

$$\rho \sim \mathcal{N} \exp\left\{ -\frac{\omega_- \Delta t}{2} - \frac{\alpha}{\hbar} \mathrm{e}^{-\omega_- \Delta t/2} \right\}$$

$$\langle \Delta t \rangle \sim -\frac{2}{\omega_-} \log \hbar$$

$$\sigma_t^2 \sim O(\hbar^0)$$

$$\langle \Delta t \rangle \sim c_1 \log \hbar + c_2$$



Conclusions

Tunneling via formation of classically unstable states (Sphaleron – driven tunneling) is a new general mechanism of tunneling.

Experimental signatures:

- Anomalously large tunneling time.
- Different formula for the tunneling probability.
- Wide final–state distributions.

We are hoping for the new mechanism to be discovered experimentally in the near future!