On two-point correlation functions in AdS/QCD

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References

This talk is mainly based on the work A.K., [hep-th] 0801.4215

Model under consideration is developed in J. Erlich, E. Katz, D. T. Son, M. Stephanov, Phys.Rev.Lett. 95 (2005) 261602. [arXiv:hep-ph/0501128] and L. Da Rold, A. Pomarol, Nucl.Phys. B721 (2005) 79-97. [arXiv:hep-ph/0501218]

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Introduction

The AdS/QCD models are the way to apply methods of the AdS/CFT correspondence to describe QCD in strong coupling regime.
The feature of AdS/CFT is the relation between AdS curvature radius and t'Hooft constant in the gauge theory.

$$\frac{R^4}{4\pi\alpha'^2} = \lambda' = N_c g_{ym}^2$$

Our aim is to compute some two-point correlation functions in QCD in the simplest model with hard wall cut of the AdS and reveal their dependence of parameters of the QCD, such as number of colors N_c and t'Hooft constant λ'

The model

- We consider AdS_5 space with the hard wall, placed at some radial coordinate z_m

$$ds^2 = \frac{R^2}{z^2}(-dz^2 + dx^\mu dx_\mu); \qquad 0 < z \leq z_m$$

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- The fields in AdS are in correspondence with QCD currents:

$$\begin{aligned} L^a_\mu &\leftrightarrow \bar{q}_L \gamma^\mu t^a q_L \\ R^a_\mu &\leftrightarrow \bar{q}_R \gamma^\mu t^a q_R \\ \left(\frac{2}{z}\right) X^{\alpha\beta} &\leftrightarrow \bar{q}^\alpha_R q^\beta_L \end{aligned}$$

here t^a are generators of $SU(N_f)$ in the adjoint representation. $X^{\alpha\beta}$ is bifundamental in $SU_L(N_f)xSU_R(N_f)$ and we take $N_f = 2$ in the case of two light quarks.

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- Boundary conditions at z_m are: $\partial_z L(z_m) = 0 \;\; ; \;\; \partial_z R(z_m) = 0.$

The model

The action is

$$S = \int d^5x \sqrt{g}\, Tr \left\{ \Lambda^2 (|DX|^2 + \frac{3}{R^2}|X|^2) - \frac{1}{4g_5^2}(F_L^2 + F_R^2) \right\}$$

where

$$D_B X = \partial_B X - \imath L_B X + \imath X R_B$$

$$L(R) = L(R)^a t^a$$

$$F_{(L)BD} = \partial_B L_D - \partial_D L_B - \imath [L_B, L_D],$$

and we introduce the normalization constant Λ of field X

The model

The classical solution for $X^{\alpha\beta}$ has the form

$$X_0(z) = \frac{1}{2}Mz + \frac{1}{2}\Sigma z^3.$$

By the AdS/CFT conjecture, one relates M to the quark mass matrix, and Σ to the VEV of operator $\langle \bar{q}q \rangle$, i.e. quark condensates. We choose normalization such as $M = m\mathbf{1}; \Sigma = \sigma\mathbf{1}$, assuming the equality of quark masses.

It is convenient to decompose:

$$X = X_0 e^{\imath 2\pi^a (t^a)} = \mathbf{1}\frac{v(z)}{2} e^{\imath 2\pi^a t^a} \qquad v(z) = mz + \sigma z^3$$

One can see, that in the quadratic order X interacts only with axial field $2A = L - R$ and not with vector one ($2V = R + L$). That means X breaks chiral symmetry.

The model

We take the transverse gauge for V_μ and decompose.A_μ on longitudinal and transverse parts.:

$$\partial_\mu V_\mu = 0 \qquad A_\mu = A_{\perp\mu} + \partial_\mu \phi$$

One can relate the pseudoscalar current $\bar{q}\gamma_5 q$ with axial vector current $\bar{q}\gamma_5\gamma_\mu q$ via:

$$\partial_\mu \left(\bar{q}\gamma_5\gamma_\mu q\right) = 2m \left(\bar{q}\gamma_5 q\right)$$

The model

This allows us to write down convenient table of correspondence:

$$
\begin{aligned}
V_\mu &\leftrightarrow \bar{q}\gamma^\mu q = J_V \\
A_\mu &\leftrightarrow \bar{q}\gamma_5\gamma^\mu q = J_A \\
\frac{Q^2}{2m}\phi &\leftrightarrow \bar{q}\gamma_5 q = J_\pi
\end{aligned}
$$

Using this correspondance we can compute some current correlators in QCD via AdS/CFT recipe, for example:

$$\langle J_V(q_1) J_V(q_2)\rangle = \frac{\delta}{\delta V_0(q_1)}\frac{\delta}{\delta V_0(q_2)} S(V_{classic})|_{V_0=0}.$$

$$V_0(q) = V_{classic}(q,z)|_{z=0}$$

Parameter fixing

Equations of motion:

$$\left[\partial_z\left(\frac{1}{z}\partial_z V_\mu^a\right)+\frac{q^2}{z}V_\mu^a\right]_\perp=0$$

$$\left[\partial_z\left(\frac{1}{z}\partial_z A_\mu^a\right)+\frac{q^2}{z}A_\mu^a-\frac{R^2g_5^2\Lambda^2v^2}{z^3}A_\mu^a\right]_\perp=0$$

$$\partial_z\left(\frac{1}{z}\partial_z\phi^a\right)+\frac{R^2g_5^2\Lambda^2v^2}{z^3}(\pi^a-\phi^a)=0$$

$$-q^2\partial_z\phi^a+\frac{R^2g_5^2\Lambda^2v^2}{z^2}\partial_z\pi^a=0$$

Parameter fixing

Equation for V is exactly solvable and gives

$$V(Q,z) = -V_0(Q)\frac{1}{I_0(Qz_m)}Qz[K_0(Qz_m)I_1(Qz) - I_0(Qz_m)K_1(Qz)]$$

The variation of metric with respect to boundary value V_0 gives

$$\delta S_V = -\int d^4x \frac{R}{g_5^2}\left[\delta V_\mu \frac{\partial_z V_\mu}{z}\right]_{z=\epsilon}$$

And the result for current correlator is:

$$\langle J^a_{V\mu}(q) J^b_{V\nu}(q)\rangle = \delta^{ab}(q_\mu q_\nu - q^2 g_{\mu\nu})\Pi_V(q^2)$$

Parameter fixing

where

$$\Pi_V(Q^2) = -\frac{R}{2g_5^2} lnQ^2\epsilon^2$$

This result can be compared with the QCD sum rules leading term:

$$\Pi_V(Q^2) = -\frac{N_c}{24\pi^2} lnQ^2\epsilon^2$$

And this fixes g_5

$$\boxed{\frac{g_5^2}{R} = \frac{12\pi^2}{N_c}}$$

Parameter fixing

To compute correlator of J_π we find solutions for coupled ϕ and π near the boundary

$$\phi(z) = \phi_0(q) Qz K_1(Qz).$$

$$\pi(z) = -\phi_0(q) \frac{Q^2}{g_5^2 R^2 \Lambda^2 m^2} Qz K_1(Qz).$$

The variation of action with respect to $\phi_0(q)$ gives

$$\delta S_\pi = \int d^4x \ \ \frac{R}{g_5^2} \left[\delta \partial_\mu \phi \frac{\partial_z \partial_\mu \phi}{z} \right]_{z=\epsilon} - \Lambda^2 R^3 \left[\delta \pi \frac{v^2}{z^3} \partial_z \pi \right]_{z=\epsilon}$$

Parameter fixing

And we get for correlator:

$$\langle J_\pi(q), J_\pi(q)\rangle = 2\frac{R}{g_5^2}\frac{1}{g_5^2 R^2 \Lambda^2} Q^2 ln(Q^2\epsilon^2)$$

Comparison with the sum rules leading term

$$\langle J_\pi(q), J_\pi(q)\rangle_{QCD} = \frac{N_c}{16\pi^2} Q^2 ln(Q^2\epsilon^2)$$

gives the value of Λ

$$\boxed{\Lambda^2 = \frac{8}{3}\frac{1}{g_5^2 R^2} = \frac{2N_c}{9\pi^2}\frac{1}{R^3}}$$

Parameter fixing

It remains us to fix relation between σ and condensate.
In QCD

$$\langle \bar{q}q \rangle = \frac{\delta \varepsilon_{QCD}}{\delta m_q}|_{m_q=0}$$

In AdS it corresponds to:

$$\langle \bar{q}q \rangle = \frac{\delta \ S(X_0)}{\delta m}\bigg|_{m=0} = 3R^3\Lambda^2\sigma = \frac{2N_c}{3\pi^2}\sigma$$

Results

Now all parameters are fixed. And the action looks:

$$S = \frac{N_c}{12\pi^2} \int d^5x \left\{ -\frac{1}{4z}(F_A^2 + F_V^2) + \frac{4}{3z^3} v(z)^2 (\partial\pi - A)^2 + \frac{4}{z^5} v(z)^2 \right\}$$

We can obtain the axial current correlator:

$$\Pi_A(Q^2) = -\frac{N_c}{24\pi^2} \left[lnQ^2 + \frac{128}{15} \frac{\sigma^2}{Q^6} - \frac{64}{9} \frac{\sigma m}{Q^4} \right]$$

Results

The interesting object is "left-right" correlator $\Pi_{LR} = \Pi_A - \Pi_V$

$$\Pi_{LR} = -\frac{N_c}{9\pi^2}\left[\frac{16}{5}\frac{\sigma^2}{Q^6} - \frac{8}{3}\frac{\sigma m_q}{Q^4}\right]$$

Note here, that it has not powers of R, namely it has the order λ'^0. If we denote coefficients in this formla as f and ρ, we find that at $\lambda' \to \infty$ our calculation predicts:

$$f(\lambda') \sim \rho(\lambda') \sim \lambda'^0$$

while at weak coupling regime(sum rules):

$$\rho(\lambda') \sim \lambda'^0 \qquad f(\lambda') = -4\pi\alpha_s \sim \lambda'.$$

Conclusion

The model under consideration has several free parameters, but still has some predictive power. It gives qualitatively satisfactory results, but numbers differ. Anyway this can be the sign, that sum rules result changes at strong coupling regime, at which the model is valid.

Thank you for your attention!