Gauged supergravity and hidden symmetries

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Based on:

- E. Bergshoeff, O.H., T. Nutma
  A note on E_{11} and three-dimensional gauged supergravity,
  arXiv:0803.2989 [hep-th]
- E. Bergshoeff, O.H., A. Kleinschmidt, H. Nicolai, T. Nutma, J. Palmkvist,
  work in progress

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Overview:

Part I: Supergravity and its Symmetries

- 11-dimensional Supergravity and Dimensional Reduction
- Hidden Symmetries
- E_{10} and E_{11}: Unifying Symmetries?

Part II: Gauged Supergravity

- Gauged Supergravity and the Embedding tensor
- Deformation and Top-form Potentials
- E_{11} Origin of Gauged Supergravity
- E_{10} Coset Model: Full Dynamics?
- Conclusions

11-dimensional Supergravity:

Local supersymmetry: $\delta_\epsilon \psi^I_\mu = D_\mu \epsilon^I + \cdots, \qquad I = 1, \ldots, \mathcal{N}$

Maximal supergravity:

$D = 11$: $\mathcal{N} = 1$ maximal, largest dimension compatible with SUSY

$$128_{\mathsf{B}} + 128_{\mathsf{F}} = (g_{MN} \times 44, A_{MNP} \times 84)_{\mathsf{B}} + (\psi_M \times 128)_{\mathsf{F}}$$

Action: [Cremmer & Julia (1978)]

$$S_{11} = \int d^{11}x \Big(-\frac{1}{4\kappa^2}\sqrt{-g}R - \frac{1}{48}\sqrt{-g}F_{MNKL}F^{MNKL} + \frac{2\kappa}{144^2}\varepsilon^{M_1 \ldots M_{11}} F_{M_1 \ldots M_4} F_{M_5 \ldots M_8} A_{M_9 \ldots M_{11}} + \mathcal{L}_{\text{fermions}} \Big)$$

Dimensional reduction to $D = 4$: $\qquad M = (\mu, m), \quad m = 1, \ldots, 7$

$$g_{MN} = \begin{pmatrix} g_{\mu\nu} & B_{\mu\, m} \\ B_{\nu\, n} & \phi_{mn} \end{pmatrix}, \qquad A_{MNP} = (A_{\mu\nu\rho}, A_{\mu\nu\, m}, A_{\mu\, mn}, A_{mnk})$$

Hidden Symmetries:

Dualizing field strengths of 2-forms: $(F_{\mu\nu\rho\, m} = \partial_{[\mu} A_{\nu\rho]\, m})$

$$\varepsilon^{\mu\nu\rho\sigma} F_{\nu\rho\sigma\, m} \;=\; \tilde{F}^{\mu}{}_{m} \;\equiv\; \partial^{\mu} \tilde{A}_m$$

Scalar content:

$$\underbrace{\phi_{mn} + A_{mnk} + \tilde{A}_m}_{28+35+7=70} \quad \Rightarrow \quad E_{7(7)}/SU(8) \quad [133 - 63 = 70]$$

After dualization, exceptional symmetry groups!

$$E_6 \quad (D=5)\,, \qquad E_7 \quad (D=4)\,, \qquad E_8 \quad (D=3)\,, \qquad E_9 \quad (D=2)$$

$\Rightarrow$ Only $SL(d) \subset E_d$ manifest (space-time) symmetries

$\Rightarrow$ 3-form in $D = 11$ predicted by requiring, say, $E_{8(8)}$

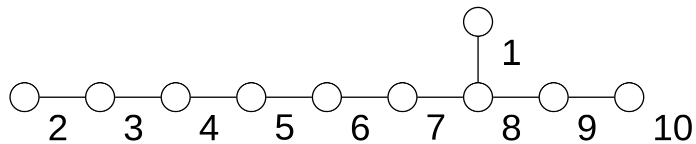
Where do these symmetries come from?

Even more?

$$E_{10} \quad (D=1)\,, \qquad E_{11} \quad (D=0)$$

E_{10} and E_{11}: Unifying symmetries?

Conjecture: E_{10} [Damour, Henneaux & Nicolai (2002)] resp. E_{11} [West (2001)]
are symmetries of (maximal) supergravities or maybe even M-theory.

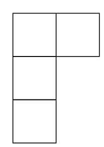


Level decomposition:

$\Rightarrow$ E_{10}/E_{11} are very intricate, ∞-dimensional Kac-Moody algebras

$\Rightarrow$ decomposition with respect to duality/space-time subgroup

Example: E_{11} decomposed w.r.t. $SL(11)$

$\ell = 0:$	$K^a{}_b$	$[\mathfrak{sl}(11)]$	graviton
$\ell = 1:$	$R^{a_1 \cdots a_3}$		$3-$form
$\ell = 2:$	$R^{a_1 \cdots a_6}$		dual $6-$form
$\ell = 3:$	$R^{a_1 \cdots a_8, b}$		dual graviton

Part II: Gauged Supergravity

Goal: (e.g. in $D = 3$)
Promote a subgroup $G_0 \subset E_8$ to a local (gauge) symmetry

Minimal coupling:

$$\partial_\mu \to D_\mu = \partial_\mu - g\Theta_{MN} A_\mu{}^M t^N$$

with embedding tensor Θ_{MN}, [Nicolai & Samtleben (2000)]

$$\mathfrak{g}_0 = \{X_M = \Theta_{MN} t^N\}\,, \qquad \dim(\mathfrak{g}_0) = \mathrm{rank}(\Theta)$$

Lagrangian:

$$\mathcal{L} = \mathcal{L}_{\mathrm{min}} - \tfrac{1}{4} g \varepsilon^{\mu\nu\rho} A_\mu{}^M \Theta_{MN} \Big(\partial_\nu A_\rho{}^N - \tfrac{1}{3} g \Theta_{KL} f^{NK}{}_P A_\nu{}^L A_\rho{}^P\Big) - \tfrac{1}{32} g^2 G^{MN,KL} \Theta_{MN} \Theta_{KL} + \mathcal{L}_{\mathrm{fermions}}$$

Constraints:

Gauge invariance of Θ_{MN}: $\qquad \Theta_{KP}\Theta_{L(M} f^{KL}{}_{N)} = 0$

Linear constraint (SUSY): $\qquad (\mathbf{248} \otimes \mathbf{248})_{\mathrm{sym}} = \underline{\mathbf{1}} \oplus \underline{\mathbf{3875}} \oplus \mathbf{27000}$

Top-form and Deformation potentials

Gauging takes duality-covariant form: Θ_{MN} carries $E_{8(8)}$ indices

However:

specific gauging $\Leftrightarrow$ *constant* Θ_{MN} $\Leftrightarrow$ $E_{8(8)}$ broken to G_0

[Alternatively: $E_{8(8)}$ rotates different theories into each other.]

Recover duality symmetry: [de Wit, Nicolai & Samtleben (2008)]

$\Rightarrow$ promote $\Theta_{MN} \to \Theta_{MN}(x)$

$\Rightarrow$ introduce Def.- and Top-forms $B_{\mu\nu}{}^{MN}$, $C_{\mu\nu\rho}{}^{MN,P}$

$$\mathcal{L}_{\text{tot}} = \mathcal{L}_g + \tfrac{1}{4} g \varepsilon^{\mu\nu\rho} D_\mu \Theta_{MN} B_{\nu\rho}{}^{MN} - \tfrac{1}{6} g^2 \Theta_{KP} \Theta_{L(M} f^{KL}{}_{N)} \varepsilon^{\mu\nu\rho} C_{\mu\nu\rho}{}^{MN,P}$$

Equations of motion for B, C: Linear and quadratic constraint.

Gauge symmetries leave action invariant, but: only off-shell closure.

E_{11} Origin of Gauged Supergravity:

E_{11} w.r.t. $SL(D)\times G_D$: [Bergshoeff, De Baetselier & Nutma (2007), Riccioni & West (2007)]

$(D-1)$– and D–form potentials, but no embedding tensor

Level	$SL(3)\times E_{8(8)}$ representation	Generator
1	$(\mathbf{3},\mathbf{248})$	$X^{\mu}{}_{\mathcal{M}}$
2	$(\bar{\mathbf{3}},\mathbf{1}\oplus\mathbf{3875})$	$Y^{\mu\nu}{}_{\mathcal{M}\mathcal{N}}$
3	$(\mathbf{1},\mathbf{248}\oplus\mathbf{3875}\oplus\mathbf{147250})$	$Z^{\mu\nu\rho}{}_{\mathcal{M}\mathcal{N},\mathcal{P}}$

agreement with linear and quadratic constraints

$\Rightarrow$ Symmetries: Non-linear realisation $\mathcal{V}\to g\,\mathcal{V}\,h(x)$, $g\in E_{11}$

$\Rightarrow$ $\mathcal{V} \,=\, \exp\left(A_{\mu}{}^{M}X^{\mu}{}_{M}+B_{\mu\nu}{}^{MN}Y^{\mu\nu}{}_{MN}+C_{\mu\nu\rho}{}^{MN,P}Z^{\mu\nu\rho}{}_{MN,P}\right)$

$\Rightarrow$ agreement with gauge symmetries in ungauged limit

E_{10} Coset Model: Full Dynamics?

Dynamics: 1-dimensional σ–model with $E_{10}/K(E_{10})$ target space

$$S_{E_{10}/K(E_{10})} = \frac{1}{4}\int dt\, n(t)^{-1}\left(\mathcal{P}(t)|\mathcal{P}(t)\right) ,$$

with Maurer-Cartan forms:

$$\mathcal{V}^{-1}\partial_t\mathcal{V} = \mathcal{P}(t) + \mathcal{Q}(t) , \quad \mathcal{P} \in \mathfrak{e}_{10} \ominus \mathfrak{k}(\mathfrak{e}_{10}) , \quad \mathcal{Q} \in \mathfrak{k}(\mathfrak{e}_{10})$$

In decomposition w.r.t. $SL(2) \times E_8$:

$$\mathcal{V}^{-1}\partial_t\mathcal{V} = \cdots + \left(\partial_t B_{ij}{}^{MN} - A_{[i}{}^{M}\partial_t A_{j]}{}^{N}\right) Y^{ij}{}_{MN}$$

Same structure as supergravity expression

$$G_{\mu\nu\rho}{}^{MN} = \partial_{[\mu}B_{\nu\rho]}{}^{MN} + A_{[\mu}{}^{M}\partial_{\nu}A_{\rho]}{}^{N} - \frac{2}{3}g\Theta_{KL}f^{MK}{}_{P}A_{[\mu}{}^{N}A_{\nu}{}^{L}A_{\rho]}{}^{P} + \cdots$$

after gauge-fixing $A_0 = 0$.

Precise match possible $\Rightarrow$ work in progress

Summary & Outlook:

- The Kac-Moody algebras E_{10} and E_{11} contain a lot of information about supergravity without using supersymmetry
- even for *gauged* supergravity:
  predicts linear and quadratic constraints on embedding tensor
- corresponding gauge symmetries of deformation and top-form potentials in $D = 3$ determined $\Rightarrow$ only on-shell closure
- non-trivial ungauged limit which coincides with E_{11}
- E_{10} sigma model potentially able to predict even dynamics of (truncated) supergravity $\Rightarrow$ falsifiable!
- Is there space-time covariant description of E_{11}–invariant dynamics?