Neutrinos and electrons in dense matter: a new approach

QUARKS-2008
Sergiev Posad,
23-29 May, 2008

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A.Studenikin,
"Method of exact solutions in studies of neutrinos and electrons in dense matter"
J.Phys.A:Math.Theor. 41 (2008) 164047 (20 pp)

A.Grigoriev, A.Savochkin, A.Studenikin,
"Quantum states of the neutrino in non-uniformly moving medium"
Russ.Phys.J. 50 (2007) 845-852

… in a series of our papers …

A.Studenikin, "Neutrinos and electrons in background matter: a new approach", **Ann.Fond. de Broglie 31** (2006) 289-316

- We develop quite **powerful method** for description of **neutrinos** (and **electrons**) motion in **background matter** which implies use of **modified Dirac equations exact solutions for ν (and e) wave functions**

…method of exact solutions…

Interaction of particles in external electromagnetic fields

(**Furry representation** in quantum electrodynamics)

$\mathrm{B}_\perp$

$e \rightarrow e + \gamma$

synchrotron radiation

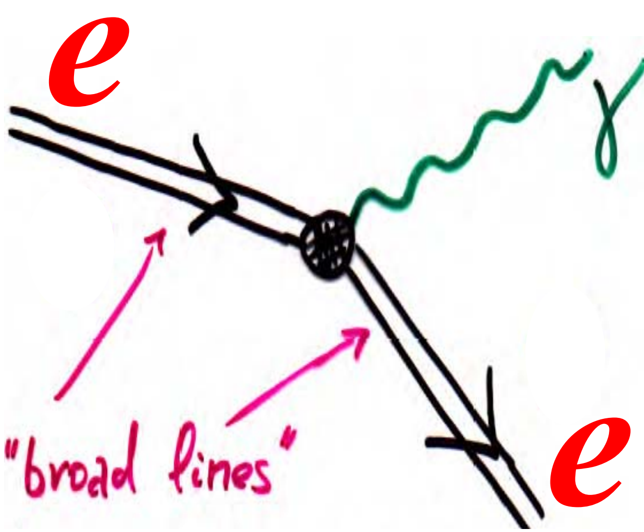
Potential of electromagnetic field

$$\boxed{A_\mu(x) = A^q_\mu(x) + A^{ext}_\mu(x)},$$

quantized part of potential

evolution **operator**

$$\boxed{U_F(t_1, t_2) = Texp\Big[-i\int_{t_1}^{t_2} j^\mu(x) A^q_\mu(x) dx\Big]},$$



charged particles **current** $\boxed{j_\mu(x) = \frac{e}{2}[\overline{\Psi}_F\gamma_\mu, \Psi_F]}$,

Dirac equation in **external classical (non-quantized) field** $A^{ext}_\mu(x)$

$$\boxed{\Big\{\gamma^\mu\Big(i\partial_\mu - eA^{ext}_\mu(x)\Big) - m_e\Big\}\Psi_F(x) = 0}$$

Neutrino decay in matter

- Z.Berezhiani, M.Vysotsky,
  Neutrino **decay** in matter, Phys.Lett.B 199 (1987) 281;
- Z.Berezhiani, A.Smirnov,
  Matter-induced neutrino **decay** and supernova 1987A,
  Phys.Lett.B 220 (1989) 279;
- C.Giunti, C.W.Kim, U.W.Lee, W.P.Lam,
  Majoron **decay** of neutrinos in matter,
  Phys.Rev.D 45 (1992) 1557.

Matter can induce the neutrino **decay** into antineutrino and a light scalar particle (**majoron**):

$\boxed{\nu \rightarrow \tilde{\nu} + \chi}$, or vice versa $\boxed{\tilde{\nu} \rightarrow \nu + \chi.}$

- Z.Berezhiani, A.Rossi,
  Majoron **decay** in matter, Phys.Lett.B 336 (1994) 439:

$$\boxed{\chi \rightarrow \nu + \nu, \quad \chi \rightarrow \tilde{\nu} + \tilde{\nu}.}$$

- … beyond the Standard Model …

ν and e in matter are treated within the «method of exact solutions» of quantum wave equations

A.Studenikin, A.Ternov,
Phys.Lett.B 608 (2005) 107;
“Neutrino quantum states and spin light in matter ”;

hep-ph/0410296 (Proc. of Quarks-2004),
“Generalized Dirac-Pauli equation and neutrino quantum states in matter”

A.Grigoriev, A.Studenikin, A.Ternov,
Phys.Lett.B 608 622 (2005) 199
“Spin light of neutrino in dense matter ”

A.Studenikin,

J.Phys.A: Math.Gen.39 (2006) 6769
“Quantum treatment of neutrino in background matter”;

Ann. Fond. de Broglie 31 (2006) 289,
“Neutrinos and electrons in background matter: a new approach”;

J.Phys. A: Math. Theor. 41 (2008) 164047
“Method of wave equations exact solutions in studies of neutrinos and electrons interaction in dense matter”

A.Studenikin, **J.Phys.A: Math.Theor. 41** (2008) 164047
A.Studenikin, **J.Phys.A: Math.Gen. 39** (2006) 6769; **Ann.Fond. de Broglie 31** (2006) 289
A.Studenikin, **Phys.Atom.Nucl. 70** (2007) 1275; *ibid* **67** (2004)1014
A.Grigoriev, A.Savochkin, A.Studenikin, **Russ.Phys. J. 50** (2007) 845
A.Grigoriev, S.Shinkevich, A.Studenikin, A.Ternov, I.Trofimov, **Russ.Phys. J. 50** (2007) 596
A.Studenikin, A.Ternov, **Phys.Lett.B 608** (2005) 107; **Grav. & Cosm. 14** (2008)
A.Grigoriev, A.Studenikin, A.Ternov, **Phys.Lett.B 622** (2005) 199
Grav. & Cosm. 11 (2005) 132 ; **Phys.Atom.Nucl. 6 9** (2006)1940
K.Kouzakov, A.Studenikin, **Phys.Rev.C 72** (2005) 015502
M.Dvornikov, A.Grigoriev, A.Studenikin, **Int.J Mod.Phys.D 14** (2005) 309
S.Shinkevich, A.Studenikin, **Pramana 64** (2005) 124
A.Studenikin, **Nucl.Phys.B** (Proc.Suppl.) **143** (2005) 570
M.Dvornikov, A.Studenikin, **Phys.Rev.D 69** (2004) 073001
Phys.Atom.Nucl. 64 (2001) 1624
Phys.Atom.Nucl. 67 (2004) 719
JETP 99 (2004) 254; **JHEP 09** (2002) 016
A.Lobanov, A.Studenikin, **Phys.Lett.B 601** (2004) 171
Phys.Lett.B 564 (2003) 27
Phys.Lett.B 515 (2001) 94
A.Grigoriev, A.Lobanov, A.Studenikin, **Phys.Lett.B 535** (2002) 187
A.Egorov, A.Lobanov, A.Studenikin, **Phys.Lett.B 491** (2000) 137

Standard model electroweak interaction of a flavour neutrino in matter (*f* = *e*)

Interaction Lagrangian (it is supposed that **matter contains only electrons**)

$$L_{int} = -\frac{g}{4\cos\theta_W}\left[\bar{\nu}_e\gamma^\mu(1+\gamma_5)\nu_e - \bar{e}\gamma^\mu(1-4\sin^2\theta_W+\gamma_5)e\right]Z_\mu$$

$$-\frac{g}{2\sqrt{2}}\bar{\nu}_e\gamma^\mu(1+\gamma_5)eW_\mu^+ - \frac{g}{2\sqrt{2}}\bar{e}\gamma^\mu(1+\gamma_5)\nu_e W_\mu^-$$

Charged current interactions contribution to neutrino potential in matter

$$\Delta L_{eff}^{CC} = \sqrt{2}\,G_F\left\langle \bar{e}\gamma^\mu(1+\gamma_5)e\right\rangle\left(\bar{\nu}_e\gamma^\mu\frac{1+\gamma_5}{2}\nu_e\right)$$

Neutral current interactions contribution to neutrino potential in matter

$$\Delta L_{eff}^{NC} = -\frac{G_F}{\sqrt{2}}\left\langle \bar{e}\gamma^\mu[(1-4\sin\theta_W^2)+\gamma_5]e\right\rangle\left(\bar{\nu}_e\gamma^\mu\frac{1+\gamma_5}{2}\nu_e\right)$$

Matter current and polarization

When the **electron field bilinear**

$$\left\langle \bar{e}\gamma^{\mu}(1+\gamma_5)e \right\rangle$$

is **averaged** over the background

$$\langle \bar{e}\gamma_0 e\rangle \sim density\,,$$

$$\langle \bar{e}\gamma_i e\rangle \sim velocity\,, \qquad i = 1, 2, 3$$

$$\langle \bar{e}\gamma_\mu\gamma_5 e\rangle \sim spin\,,$$

it can be replaced by the **matter** (electrons) **current**

☆ $$\boxed{j^{\mu} = (n, n\mathbf{v}),}$$

invariant number density

speed of matter

and **polarization**

☆ $$\boxed{\lambda^{\mu} = \left(n(\boldsymbol{\zeta}\mathbf{v}), n\boldsymbol{\zeta}\sqrt{1-v^2} + \frac{n\mathbf{v}(\boldsymbol{\zeta}\mathbf{v})}{1+\sqrt{1-v^2}} \right)}$$

Modified Dirac equation for neutrino in matter

Addition to the vacuum neutrino Lagrangian

matter current

$$\Delta L_{eff} = \Delta L_{eff}^{CC} + \Delta L_{eff}^{NC} = -f^{\mu}\left(\bar{\nu}\gamma_{\mu}\frac{1+\gamma^{5}}{2}\nu\right)$$

where $f^{\mu} = \frac{G_F}{\sqrt{2}}\left((1+4\sin^{2}\theta_W)j^{\mu} - \lambda^{\mu}\right)$

matter polarization

$$\boxed{\left\{i\gamma_{\mu}\partial^{\mu} - \frac{1}{2}\gamma_{\mu}(1+\gamma_{5})f^{\mu} - m\right\}\Psi(x) = 0}$$

A.Studenikin, A.Ternov,
hep-ph/0410297, ***Phys.Lett.B* 608 (2005) 107**
A.Grigoriev, A.Studenikin, A.Ternov,
***Phys.Lett.B* 622 (2005) 199**

It is supposed that there is a macroscopic amount of electrons in the scale of a neutrino de Broglie wave length. Therefore, **the interaction of a neutrino with the matter (electrons) is coherent.**

L.Chang, R.Zia,'88; J.Panteleone,'91; K.Kiers, N.Weiss, M.Tytgat,'97-'98; P.Manheim,'88; D.Nötzold, G.Raffelt,'88; J.Nieves,'89; V.Oraevsky, V.Semikoz, Ya.Smorodinsky,89; W.Naxton, W-M.Zhang'91; M.Kachelriess,'98; A.Kusenko, M.Postma,'02.

This is the most general equation of motion of a neutrino in which the effective potential accounts for both the **charged** and **neutral-current** interactions with the background matter and also for the possible effects of the matter **motion** and **polarization**.

Neutrino wave function in matter (II)

$$\boxed{\Psi_{\varepsilon,\mathbf{p},s}(\mathbf{r},t)=\frac{e^{-i(E_\varepsilon t-\mathbf{pr})}}{2L^{\frac{3}{2}}}\begin{pmatrix}\sqrt{1+\frac{m}{E_\varepsilon-\alpha m}}\sqrt{1+s\frac{p_3}{p}}\\ s\sqrt{1+\frac{m}{E_\varepsilon-\alpha m}}\sqrt{1-s\frac{p_3}{p}}\;e^{i\delta}\\ s\varepsilon\eta\sqrt{1-\frac{m}{E_\varepsilon-\alpha m}}\sqrt{1+s\frac{p_3}{p}}\\ \varepsilon\eta\sqrt{1-\frac{m}{E_\varepsilon-\alpha m}}\sqrt{1-s\frac{p_3}{p}}\;e^{i\delta}\end{pmatrix}}$$

A.Studenikin, A.Ternov, hep-ph/0410297;
***Phys.Lett.B* 608 (2005) 107;**

$$\eta=\mathrm{sign}\left(1-s\alpha\frac{m}{p}\right),\ \delta=\arctan\left(p_2/p_1\right)$$

A.Grigoriev, A.Studenikin, A.Ternov,
***Phys.Lett.B* 622 (2005) 199**

$$\boxed{E_\varepsilon=\varepsilon\eta\sqrt{\mathbf{p}^2\left(1-s\alpha\frac{m}{p}\right)^2+m^2}+\alpha m}$$

The quantity $\boxed{\varepsilon=\pm1}$ splits the solutions into the two branches that

in the limit of **vanishing matter density,** $\boxed{\alpha\rightarrow0,}$

reproduce the **positive** and **negative-frequency** solutions, respectively.

Neutrino flavour oscillations in matter

Consider the two flavour neutrinos, ν_e and ν_μ , propagating in electrically neutral matter of **electrons** , **protons** and **neutrons** : $n_e = n_p$.

The matter density parameters are

$$\boxed{\alpha_{\nu_e} = \frac{1}{2\sqrt{2}}\frac{G_F}{m}\Big(n_e(1+4\sin^2\theta_W) + n_p(1-4\sin^2\theta_W) - n_n\Big)}$$

and

$$\boxed{\alpha_{\nu_\mu,\nu_\tau} = \frac{1}{2\sqrt{2}}\frac{G_F}{m}\Big(n_e(4\sin^2\theta_W - 1) + n_p(1-4\sin^2\theta_W) - n_n\Big)}$$

The energies of the **relativistic** **active** neutrinos are

$$\boxed{E^{s=-1}_{\nu_e,\nu_\mu} \approx E_0 + 2\alpha_{\nu_e,\nu_\mu} m_{\nu_e,\nu_\mu}},$$

and the energy difference

$$\boxed{\Delta E \equiv E^{s=-1}_{\nu_e} - E^{s=-1}_{\nu_\mu} = \sqrt{2} G_F n_e} \Rightarrow \boxed{\textbf{MSW effect}}$$

Modified Dirac equation for matter composed of electrons, protons and neutrons (I)

The generalizations of the modified Dirac equation for more complicated matter compositions and the other flavour neutrinos are just straightforward.

For matter composed of **electrons** , **protons** and **neutrons** :

$$f^{\mu} = \sqrt{2} G_F \sum_{f=e,p,n} j_f^{\mu} q_f^{(1)} + \lambda_f^{\mu} q_f^{(2)} ,$$

where

$$q_f^{(1)} = (I_{3L}^{(f)} - 2Q^{(f)} \sin^2\theta_W + \delta_{ef}), \quad q_f^{(2)} = -(I_{3L}^{(f)} + \delta_{ef}), \quad \delta_{ef} = \begin{cases} 1 & \text{for} f = e, \\ 0 & \text{for} f = n, p \end{cases}$$

isospin third component

electric charge of a fermion ***f***

current

polarization

$$\boxed{\left\{ i\gamma_{\mu}\partial^{\mu} - \frac{1}{2}\gamma_{\mu}(1+\gamma_5)f^{\mu} - m \right\}\Psi(x) = 0}$$

Modified Dirac equation for matter composed of electrons, protons and neutrons (II)

Neutrino **energy spectrum** in matter composed of **electrons** , **protons** and **neutrons** :

$$\boxed{E_\varepsilon = \varepsilon\sqrt{\mathbf{p}^2\Big(1 - s\alpha\frac{m}{p}\Big)^2 + m^2} + \alpha m} \qquad \begin{aligned} \varepsilon &= \pm 1 \\ s &= \pm 1 . \end{aligned}$$

Density parameter α is determined by particles number densities $n_{e,p,n}$.

★ For **electron neutrino** ν_e :

$$\boxed{\alpha_{\nu_e} = \frac{1}{2\sqrt{2}}\frac{G_F}{m}\Big(n_e(1 + 4\sin^2\theta_W) + n_p(1 - 4\sin^2\theta_W) - n_n\Big)} ,$$

- in electrically **neutral** and **neutron** reach matter $\alpha_{\nu_e} < 0$,
- for **electron antineutrino** $\alpha_{\tilde{\nu}_e} \rightarrow -\alpha_{\nu_e}$.

★ For **muon** and **tau neutrino** ν_μ , ν_τ :

$$\boxed{\alpha_{\nu_\mu,\nu_\tau} = \frac{1}{2\sqrt{2}}\frac{G_F}{m}\Big(n_e(4\sin^2\theta_W - 1) + n_p(1 - 4\sin^2\theta_W) - n_n\Big)} .$$

Neutrino processes in matter

- **Neutrino reflection from interface between vacuum and matter**
- **Neutrino trapping in matter**
- **Neutrino-antineutrino pair annihilation at interface between vacuum and matter**
- **Spontaneous neutrino-antineutrino pair creation in matter**

L.Chang, R.Zia,’88
A.Loeb,’90
J.Panteleone,’91
K.Kiers, N.Weiss, M.Tytgat,’97-’98
M.Kachelriess,’98
A.Kusenko, M.Postma,’02 H.Koers,’04
A.Studenikin, A.Ternov,’04
A.Grigoriev, S.Shinkevich, A.Studenikin, A.Ternov, ’05
I.Pivovarov, A.Studenikin,’05
A.Ivanov, A.Studenikin, ‘05

Neutrino reflection from interface between vacuum and matter

If the neutrino energy in **vacuum** E

is less than the neutrino minimal energy in **medium** $\alpha_1 m + m$

$$1 < \alpha_1 < 2$$

$$\alpha = \frac{1}{2\sqrt{2}} \tilde{G}_F \frac{n}{m}$$

matter density parameter

$$m \leq E < \alpha_1 m + m$$

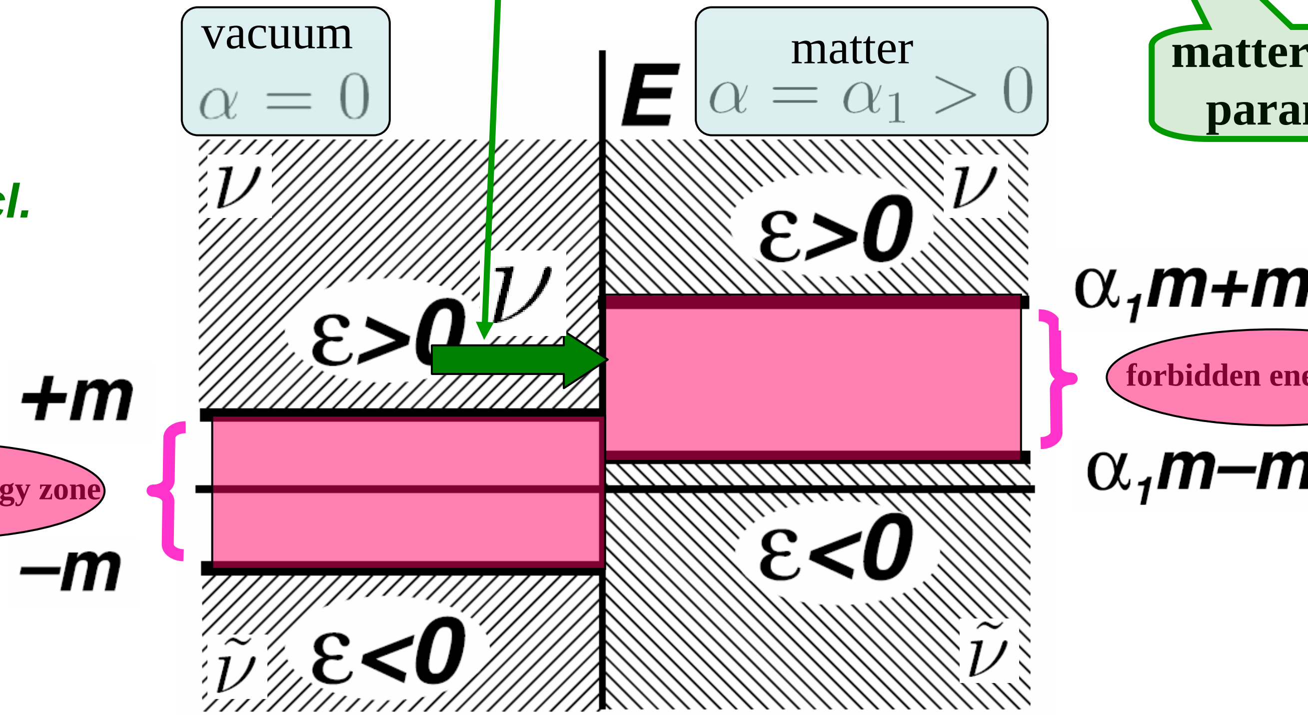
A.Grigoriev,

A.Studenikin,

A.Ternov,

Phys.Atom.Nucl.

69 (2006) 1940



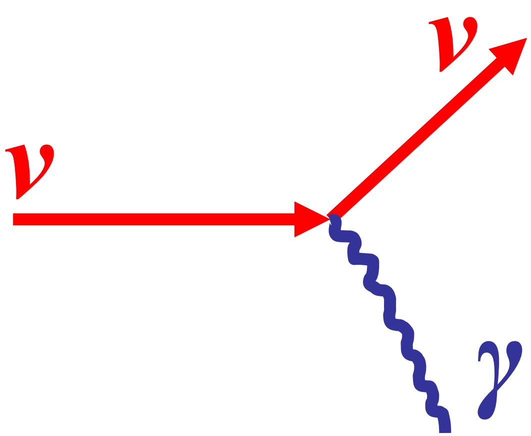
then the appropriate energy level inside the medium is **not accessible** for neutrino

neutrino **is reflected** from the interface.

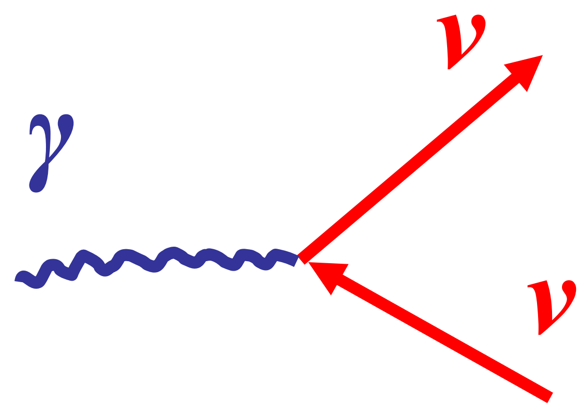
Other quantum effects

- **Spin light of** ν
- **Spin light of** e
- **Neutrino bound states in rotating medium**

Neutrino – photon couplings (I)

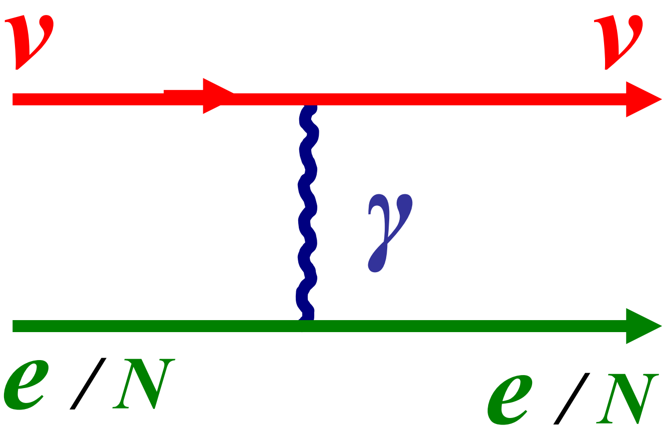


ν decay, Cherenkov radiation

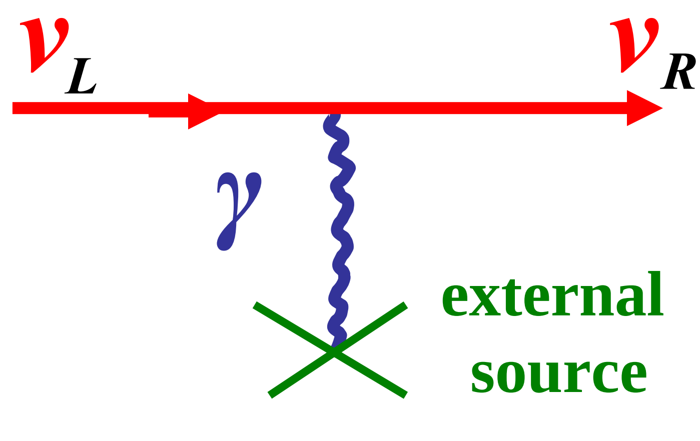


γ decay in plasma

!!!

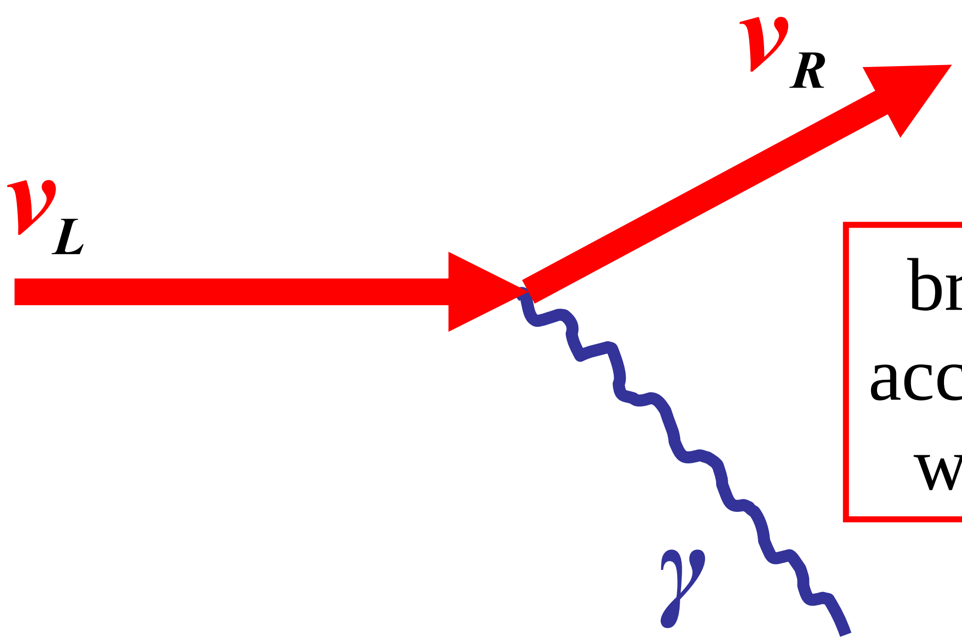


Scattering



Spin precession

Neutrino – photon couplings (II)



broad neutrino lines account for interaction with environment

“Spin light of neutrino in matter”

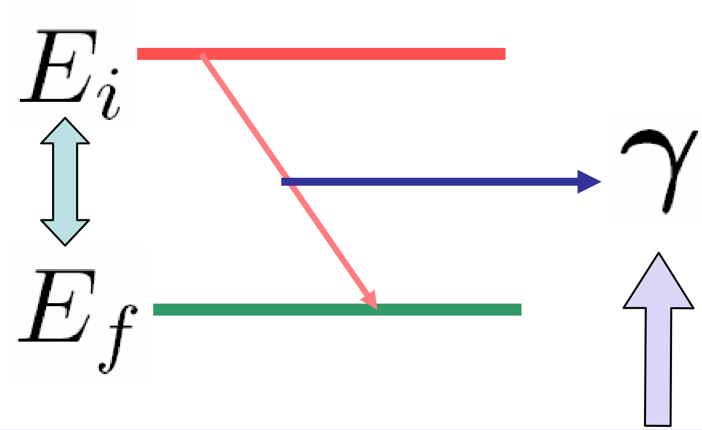
…within the quantum treatment…

Quantum theory of spin light of neutrino (I)

$SL\nu$

Quantum treatment of *spin light of neutrino* in matter
showns that this process originates from the **two subdivided phenomena:**

- the **shift** of the neutrino **energy levels** in the presence of the background matter, which is different for the two opposite **neutrino helicity states**,



$$\boxed{E_\varepsilon = \varepsilon\eta\sqrt{\mathbf{p}^2\left(1 - s\alpha\frac{m}{p}\right)^2 + m^2} + \alpha m}$$

$$\boxed{\eta = \mathrm{sign}\left(1 - s\alpha\frac{m}{p}\right)}$$

$$\boxed{s = \pm 1}$$

- the radiation of the photon in the process of the neutrino transition from the **"excited" helicity state** to the **low-lying helicity state** in matter

A.Studenikin, A.Ternov, Phys.Lett.B 608 (2005) 107;
A.Grigoriev, A.Studenikin, A.Ternov, Phys.Lett.B 622 (2005) 199;
Grav. & Cosm. 14 (2005) 132;

hep-ph/0507200, hep-ph/0502210,

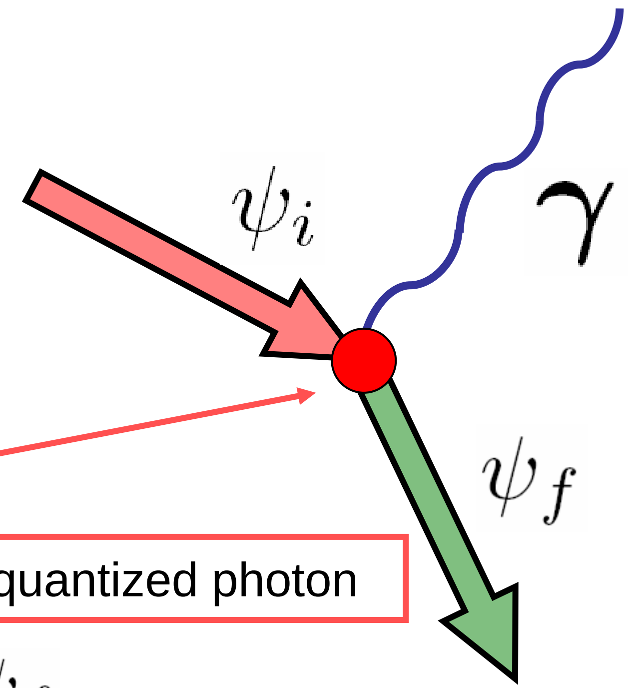
hep-ph/0502231

neutrino-spin self-polarization effect in the matter

A.Lobanov, A.Studenikin, Phys.Lett.B 564 (2003) 27;
Phys.Lett.B 601 (2004) 171

Quantum theory of spin light of neutrino $SL\nu$

Within the **quantum approach**, the corresponding Feynman diagram is the one-photon emission diagram with the initial and final neutrino states described by the **“broad lines”** that account for the neutrino interaction with matter.



Neutrino magnetic moment interaction with quantized photon

the amplitude of the transition $\psi_i \longrightarrow \psi_f$

$$\boxed{S_{fi} = -\mu\sqrt{4\pi}\int d^4x\,\bar{\psi}_f(x)(\hat{\mathbf{\Gamma}}\mathbf{e}^*)\frac{e^{ikx}}{\sqrt{2\omega L^3}}\psi_i(x)},$$

$$\boxed{\hat{\mathbf{\Gamma}} = i\omega\{[\mathbf{\Sigma}\times\boldsymbol{\varkappa}] + i\gamma^5\mathbf{\Sigma}\}},\qquad k^\mu = (\omega, \mathbf{k}), \boldsymbol{\varkappa} = \mathbf{k}/\omega \quad \text{momentum}$$

$\mathbf{e}^*$ polarization of photon

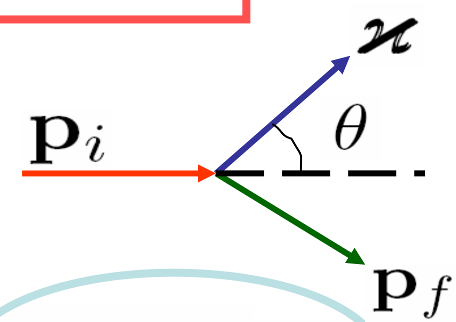
Spin light of neutrino photon's energy

$SL\nu$ transition amplitude after integration :

$$S_{fi} = -\mu\sqrt{\frac{2\pi}{\omega L^3}}\, 2\pi\delta(E_f - E_i + \omega) \int d^3x \bar{\psi}_f(\mathbf{r})(\hat{\mathbf{\Gamma}}\mathbf{e}^*)e^{i\mathbf{kr}}\psi_i(\mathbf{r})$$

Energy-momentum conservation

$$E_i = E_f + \omega, \qquad \mathbf{p}_i = \mathbf{p}_f + \boldsymbol{\varkappa}$$



For **electron neutrino** moving in matter composed of **electrons**

$$\omega = \frac{2\alpha m p_i\left[(E_i - \alpha m) - (p_i + \alpha m)\cos\theta\right]}{(E_i - \alpha m - p_i\cos\theta)^2 - (\alpha m)^2}$$

$$\alpha = \frac{1}{2\sqrt{2}}\tilde{G}_F\frac{n}{m} > 0$$

photon energy

- In the radiation process: $s_i = -1 \Longrightarrow s_f = +1$ **neutrino self-polarization**

- For not very high densities of matter , $\tilde{G}_F n/m \ll 1$, in the linear approximation over α

$$\omega = \frac{\beta}{1-\beta\cos\theta}\omega_0 \quad , \qquad \omega_0 = \frac{\tilde{G}_F}{\sqrt{2}}n\beta$$

neutrino speed in vacuum

Spin light transition rate (III)

$SL\nu$ transition rate for different neutrino momentum p
and **matter density parameter** $\alpha = \frac{1}{2\sqrt{2}}\tilde{G}_F \frac{n}{m} > 0$

"relativistic" case

$p \gg m$

$$\Gamma = \begin{cases} \frac{64}{3}\mu^2\alpha^3 p^2 m, & \text{for } \alpha \ll \frac{m}{p}, \\ 4\mu^2\alpha^2 m^2 p, & \text{for } \frac{m}{p} \ll \alpha \ll \frac{p}{m}, \\ 4\mu^2\alpha^3 m^3, & \text{for } \alpha \gg \frac{p}{m}. \end{cases}$$

"non-relativistic" case

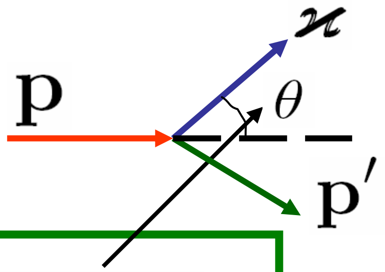
$p \ll m$

$$\Gamma = \begin{cases} \frac{64}{3}\mu^2\alpha^3 p^3, & \text{for } \alpha \ll 1, \\ \frac{512}{5}\mu^2\alpha^6 p^3, & \text{for } 1 \ll \alpha \ll \frac{m}{p}, \\ 4\mu^2\alpha^3 m^3, & \text{for } \alpha \gg \frac{m}{p}. \end{cases}$$

Spin light radiation power

$SL\nu$

radiation power angular distribution :



$$I=\mu^2\int_0^{\pi}\omega^4\left[(\tilde\beta\tilde\beta'+1)(1-y\cos\theta)-(\tilde\beta+\tilde\beta')(\cos\theta-y)\right]\frac{\sin\theta}{1+\tilde\beta' y}d\theta$$

$$\tilde\beta=\frac{p+\alpha m}{E-\alpha m},\quad \tilde\beta'=\frac{p'-\alpha m}{E'-\alpha m},\quad y=\frac{\omega-p\cos\theta}{p'},\quad K=\frac{E-\alpha m-p\cos\theta}{\alpha m},\quad \omega=\frac{2\alpha mp\left[(E-\alpha m)-(p+\alpha m)\cos\theta\right]}{(E-\alpha m-p\cos\theta)^2-(\alpha m)^2}$$

"relativistic" case

$\boxed{p\gg m}$

$$I=\begin{cases}\frac{128}{3}\mu^2\alpha^4p^4, & \text{for } \alpha\ll\frac{m}{p},\\ \frac{4}{3}\mu^2\alpha^2m^2p^2, & \text{for } \frac{m}{p}\ll\alpha\ll\frac{p}{m},\\ 4\mu^2\alpha^4m^4, & \text{for } \alpha\gg\frac{p}{m}.\end{cases}$$

"non-relativistic" case

$\boxed{p\ll m}$

$$I=\begin{cases}\frac{128}{3}\mu^2\alpha^4p^4, & \text{for } \alpha\ll 1,\\ \frac{1024}{3}\mu^2\alpha^8p^4, & \text{for } 1\ll\alpha\ll\frac{m}{p},\\ 4\mu^2\alpha^4m^4, & \text{for } \alpha\gg\frac{m}{p}.\end{cases}$$

Spin light photon average energy

$$\langle \omega \rangle = \frac{\text{radiation power}}{\text{transition rate}} = \frac{I}{\Gamma}$$

See also:
A.Lobanov,
Phys.Lett.B 619
(2005) 136

"relativistic" case

$$\boxed{p \gg m}$$

$$\langle \omega \rangle \simeq \begin{cases} 2\alpha \frac{p^2}{m}, & \text{for } \alpha \ll \frac{m}{p}, \\ \frac{1}{3}p, & \text{for } \frac{m}{p} \ll \alpha \ll \frac{p}{m}, \\ \alpha m, & \text{for } \alpha \gg \frac{p}{m}. \end{cases}$$

"non-relativistic" case

$$\boxed{p \ll m}$$

$$\langle \omega \rangle \simeq \begin{cases} 2p\alpha, & \text{for } \alpha \ll 1, \\ \frac{10}{3}p\alpha^2, & \text{for } 1 \ll \alpha \ll \frac{m}{p}, \\ \alpha m, & \text{for } \alpha \gg \frac{m}{p}. \end{cases}$$

$\alpha \ll \frac{m}{p}$

$$\omega = 2.37 \times 10^{-7} \left(\frac{n}{10^{30} cm^{-3}} \right) \left(\frac{E}{m_\nu} \right)^2 eV$$

energy range of $SL\nu$

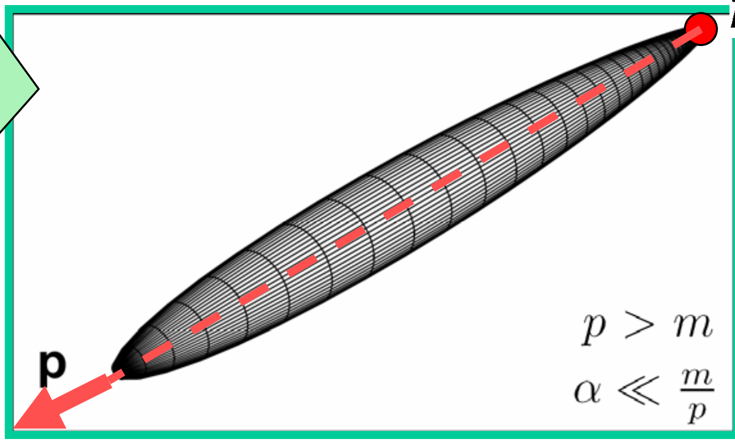
span up to

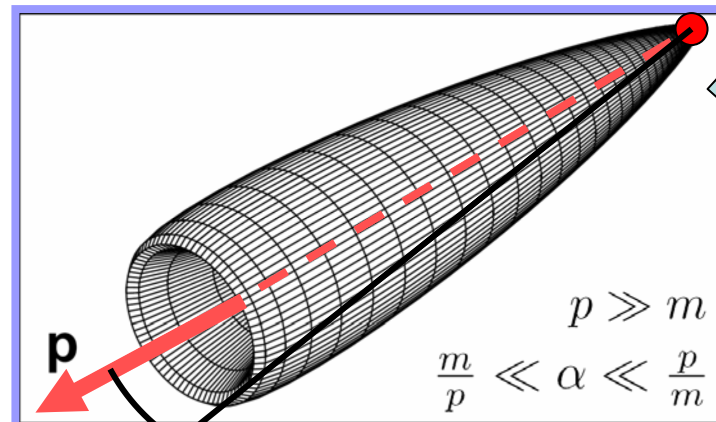
gamma-rays

Spatial distribution of radiation power

From the **angular distribution** of $\boxed{SL\nu}$

$$I = \mu^2 \int_0^{\pi} \omega^4 \left[(\tilde{\beta}\tilde{\beta}' + 1)(1 - y\cos\theta) - (\tilde{\beta} + \tilde{\beta}')(\cos\theta - y) \right] \frac{\sin\theta}{1 + \tilde{\beta}' y} d\theta$$





for $p/m = 5$ **and** $\alpha = 0.01$

$\boxed{n \approx 10^{35} cm^{-3}}$

neutrino momentum

mass

matter density

$$\cos\theta_{max} \simeq 1 - \frac{2}{3}\alpha\frac{m}{p}$$

maximum in radiation power distribution

for $p/m = 10^3$ **and** $\alpha = 100$

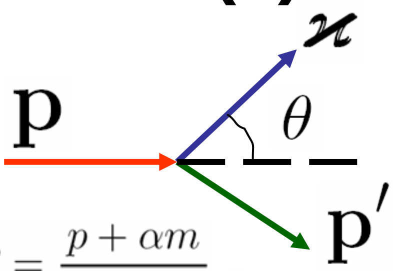
$\boxed{n \approx 10^{39} cm^{-3}}$

increase of matter density

projector-like distribution → **cap-like distribution**

Polarization properties of $SL\nu$ photons (II)

Radiation power of **circularly polarized** photons:



$$\boxed{I^{(l)} = \mu^2 \int_0^\pi \frac{\omega^4}{1+\beta' y} S_l \sin\theta d\theta} , \quad \tilde{\beta}' = \frac{p' - \alpha m}{E' - \alpha m}, \quad \tilde{\beta} = \frac{p + \alpha m}{E - \alpha m},$$

$$y = \frac{\omega - p\cos\theta}{p'}, \quad K = \frac{E - \alpha m - p\cos\theta}{\alpha m},$$

$$\omega = \frac{2(E - \alpha m)(K\beta - 1)}{K^2 - 1}$$

where

$$\boxed{S_l = \frac{1}{2}\left(1 + l\beta'\right)\left(1 + l\beta\right)\left(1 - l\cos\theta\right)\left(1 + ly\right)}$$

$l = \pm 1$ correspond to the photon **right** and **left circular polarizations** .

In the limit of **low matter density** $\alpha \ll 1$: $E_0 = \sqrt{p^2 + m^2}$

$$\boxed{I^{(l)} \simeq \frac{64}{3}\mu^2 \alpha^4 p^4 \left(1 - l\frac{p}{2E_0}\right)}, \quad I^{(+1)} > I^{(-1)}, \text{ however } I^{(+1)} \sim I^{(-1)} .$$

In **dense matter** $\left(\alpha \gg \frac{m}{p} \text{ for } p \gg m, \text{and } \alpha \gg 1 \text{ for } p \ll m\right)$:

$$\boxed{\begin{array}{lcl} I^{(+1)} & \simeq & I \\ I^{(-1)} & \simeq & 0 \end{array}}$$

In a dense matter $SL\nu$ is right-circular polarized.

It is possible to have

$$\boxed{\tau = \frac{1}{\Gamma} << \text{age of the Universe}}$$

Consider the case $\frac{m}{p} \ll \alpha \ll \frac{p}{m}$

$$\alpha m_\nu = \frac{1}{2\sqrt{2}} G_F n (1 + sin^2\theta_W)$$

$$\begin{cases} \Gamma = 4\mu^2\alpha^2 m^2 p \\ I = \frac{128}{3}\mu^2\alpha^4 p^4 \\ \langle\omega\rangle \simeq \frac{1}{3} p \end{cases}$$

For ultra-relativistic ν
with momentum $p \sim 10^{20} eV$
and magnetic moment $\mu \sim 10^{-10}\mu_B$
in very dense matter $n \sim 10^{40} cm^{-3}$

$$\boxed{p \gg m_{plasmon}}$$

recently also discussed by A.Kuznetsov, N.Mikheev, 2006

it follows that $$\boxed{\tau = \frac{1}{\Gamma} = 1.5 \times 10^{-8} s}$$

A.Lobanov, A.S., PLB 2003; PLB 2004
A.Grigoriev, A.S., PLB 2005
A.Grigoriev, A.S., A.Ternov, PLB 2005

SLe

Spin **L**ight of **E**lectron in matter

… a method of studying charged particles interaction in matter…

A.Studenikin,
J.Phys.A: Math. Gen. 39 (2006) 6769

Grigoriev, Shinkevich, Studenikin, Ternov, Trofimov, hep-ph/0611128,
Russ.Phys.J 50 (2007) 596,
Grav.&Cosm. 14 (2008)

Modified Dirac equation for electron in matter

$$\boxed{\left\{ i\gamma_\mu\partial^\mu - \frac{1}{2}\gamma_\mu(1 - 4\sin^2\theta_W + \gamma_5)\tilde{f}^\mu - m_e \right\}\Psi_e(x) = 0,}$$

where

$$\tilde{f}^\mu = -f^\mu = \frac{G_F}{\sqrt{2}}(j_n^\mu - \lambda_n^\mu)$$

matter polarization

matter current

It is suppose that there is a macroscopic amount of neutrons in the scale of an electron de Broglie wave length. Therefore, **the interaction of electron with the matter (neutrons) is coherent.**

This is the most general equation of motion of an neutrino in which the effective potential accounts for **neutral-current** interactions with the background electrically neutral matter and also for the possible effects of matter **motion** and **polarization**.

Electron wave function in matter (II)

$$\boxed{\Psi_{\varepsilon,\mathbf{p},s}(\mathbf{r},t)=\frac{e^{-i(E_\varepsilon^{(e)}t-\mathbf{pr})}}{2L^{\frac{3}{2}}}\begin{pmatrix}\sqrt{1+\frac{m_e}{E_\varepsilon^{(e)}-c\alpha_n m_e}}\sqrt{1+s\frac{p_3}{p}}\\ s\sqrt{1+\frac{m_e}{E_\varepsilon^{(e)}-c\alpha_n m_e}}\sqrt{1-s\frac{p_3}{p}}\;e^{i\delta}\\ s\varepsilon\eta\sqrt{1-\frac{m_e}{E_\varepsilon^{(e)}-c\alpha_n m_e}}\sqrt{1+s\frac{p_3}{p}}\\ \varepsilon\eta\sqrt{1-\frac{m_e}{E_\varepsilon^{(e)}-c\alpha_n m_e}}\sqrt{1-s\frac{p_3}{p}}\;e^{i\delta}\end{pmatrix}}$$

A.Grigoriev, S.Shinkevich, A.Studenikin, A.Ternov, I.Trofimov, hep-ph/0611128, Russ.Phys.J 50 (2007) 596

$\eta = \mathrm{sign}\left(1-s\alpha\frac{m_e}{p}\right)$, $\delta = \arctan\left(p_2/p_1\right)$

$$\boxed{E_\varepsilon^{(e)}=\varepsilon\eta\sqrt{\mathbf{p}^2\left(1-s\alpha_n\frac{m_e}{p}\right)^2+m_e^2}+c\alpha_n m_e}\;,\quad \text{where } c=1-4\sin^2\theta_W$$

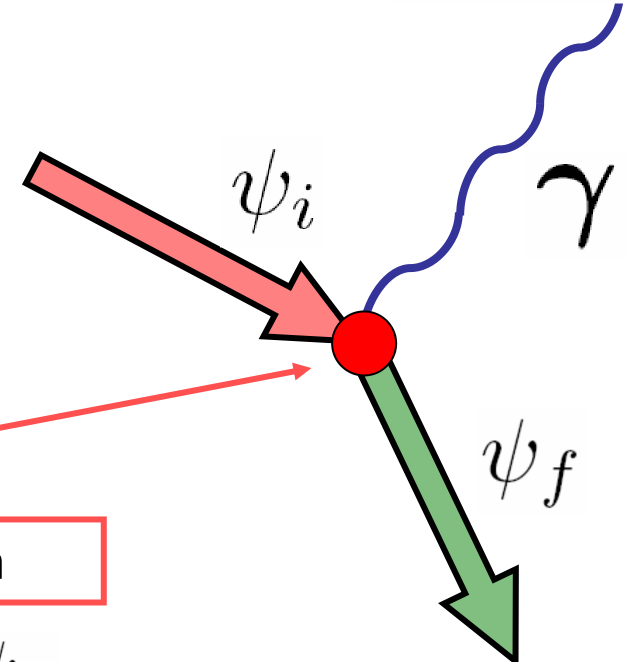
A.Studenikin, J.Phys.A: Math. Gen. 39 (2006) 6769

$$\alpha_n=\frac{1}{2\sqrt{2}}G_F\frac{n_n}{m_e},$$

The quantity $\boxed{\varepsilon = \pm 1}$ splits the solutions into the two branches that in the limit of **vanishing matter density,** $\boxed{\alpha \to 0,}$ reproduce the **positive** and **negative-frequency** solutions, respectively.

Theory of spin light of electron

SLe



The corresponding Feynman diagram is the one-photon emission diagram with the **initial** and **final** electron states described by the **“broad lines”** that account for the electron interaction with matter.

Electron interaction with quantized photon

the amplitude of the transition $\psi_i \longrightarrow \psi_f$

$$\boxed{S_{fi} = -ie\sqrt{4\pi}\int d^4x \bar{\psi}_f(x)\gamma^\mu e_\mu{}^* \frac{e^{ikx}}{\sqrt{2\omega L^3}}\psi_i(x)},$$

$k^\mu = (\omega, \mathbf{k})$, $\boldsymbol{\varkappa} = \mathbf{k}/\omega$ momentum

$\mathbf{e}^*$ polarization of photon

Order-of-magnitude estimation :

$$\boxed{R = \frac{\Gamma_{SLe}}{\Gamma_{SL\nu}} \sim \frac{e^2}{\omega^2\mu^2}},$$

A.S.,
J.Phys.A: Math. Gen.
39 (2006) 6769

then for $\mu \sim 10^{-10}\mu_0$ **and** $\omega \sim 5\ MeV$

$$\boxed{R \sim 10^{18}},$$

under these conditions $\boxed{SLe}$

is more effective than $\boxed{SL\nu}$

Grigoriev, Shinkevich, Studenikin, Ternov, Trofimov, hep-ph/0611128, Russ.Phys.J 50 (2007) 596; Grav.&Cosmology 14 (2008)

Exact calculations of $\boxed{SLe}$

Transition rate

$$\Gamma = \frac{e^2}{2}\int_0^{\pi} \frac{\omega}{1+\tilde{\beta}'_e\, y}\, S\sin\theta\, d\theta$$

and power

$$\mathrm{I} = \frac{e^2}{2}\int_0^{\pi} \frac{\omega^2}{1+\tilde{\beta}'_e\, y} S\sin\theta\, d\theta$$

where

$$S = (1 - y\cos\theta)\left(1 - \tilde{\beta}_e\tilde{\beta}'_e - \frac{{m_e}^2}{\tilde{E}\tilde{E}'}\right),$$

$$\tilde{\beta}_e = \frac{p+\alpha_n m_e}{\tilde{E}}, \quad \tilde{\beta}'_e = \frac{p'-\alpha_n m_e}{\tilde{E}'}, \quad \tilde{E} = E - c\alpha_n m_e,$$

energy and momentum of final neutrino

$$E' = E - \omega, \quad p' = K_e\,\omega - p,$$

$$K_e = \frac{\tilde{E} - p\cos\theta}{\alpha_n m_e}, \quad y = \frac{\omega - p\cos\theta}{p'}.$$

Spin light of electron in matter (n)

SLe

Transition rate

$$\boxed{\Gamma = \frac{e^2 m^3}{4p^2}\frac{(1+2a)\left[(1+2b)^2\ln(1+2b) - 2b(1+3b)\right]}{(1+2b)^2\sqrt{1+a+b}}},$$

and power

$$\boxed{\mathrm{I} = \frac{e^2 m^4}{6p^2}\frac{(1+a)\left[3(1+2b)^3\ln(1+2b) - 2b(3+15b+22b^2)\right] - 8b^4}{(1+2b)^3}},$$

where $a = \alpha_n^2 + p^2/m_e^2,\ b = 2\alpha_n p/m_e.$

Spin light photon average energy

SLe

$$\langle \omega \rangle = \frac{\text{radiation power}}{\text{transition rate}} = \frac{I}{\Gamma}$$

In case $\boxed{\alpha_n \gg m_e/p}$

$$\boxed{\langle \omega \rangle \simeq \begin{cases} p, & \text{for } \frac{m_e}{p} \ll \alpha_n \ll \frac{p}{m_e}, \\ \alpha_n m_e, & \text{for } \alpha_n^{-1} \ll \frac{p}{m_e} \ll \alpha_n, \end{cases}}$$

(it is supposed $\ln \frac{4\alpha_n p}{m_e} \gg 1$**)**

- **For relativistic electrons the emitted photon energy is in the range of gamma-rays**

- SLe **photon caries away nearly the whole of the initial electron energy**

From exact calculations of $\boxed{SLe}$

$$n \sim 10^{37} \div 10^{40} \ \text{cm}^{-3}$$
$$p \sim 1 \div 10^{3} \ \text{MeV}$$
$$m_\nu = 1 \ \text{eV}$$
$$\mu = 10^{-10}\mu_0$$

$$R_\Gamma = \frac{\Gamma_{SLe}}{\Gamma_{SL\nu}} \sim 10^{16} \div 10^{19}$$

$$R_\text{I} = \frac{\text{I}_{SLe}}{\text{I}_{SL\nu}} \sim 10^{15} \div 10^{19}$$

Grigoriev, Shinkevich, Studenikin, Ternov, Trofimov, hep-ph/0611128, Russ.Phys.J 50 (2007) 596

New mechanism of electromagnetic radiation

? **Why** **Spin Light** **of neutrino** $SL\nu$ **of electron** SLe **in matter.**

Analogies with:

* classical electrodynamics

an object with charge $Q=0$ and magnetic moment $\vec{m}=\frac{1}{2}\sum_i e_i[\vec{r}_i \times \vec{v}_i]\neq 0$

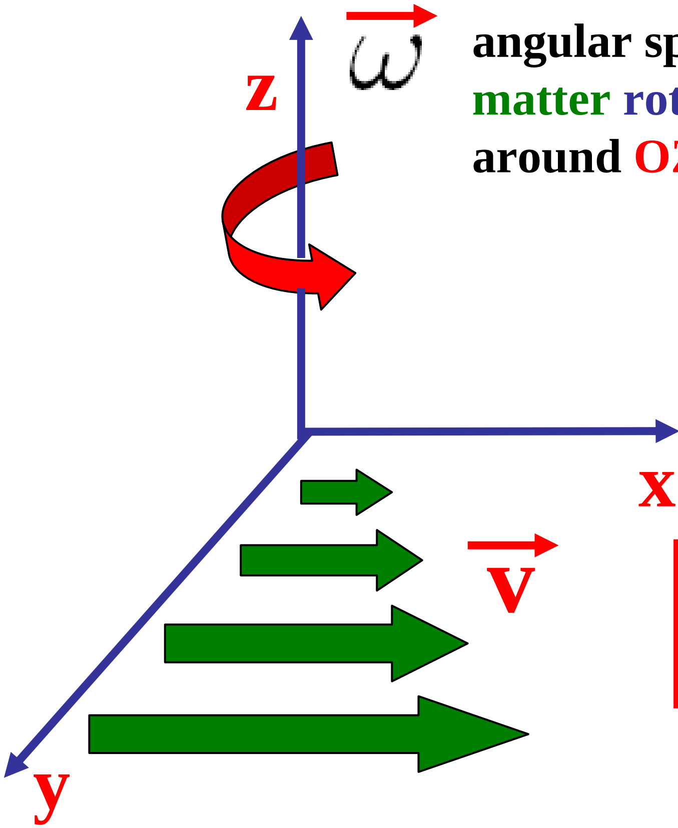
$$\boxed{I^{cl.el.}=\frac{2}{3}\ddot{\vec{m}}^{\,2}}$$

magnetic dipole radiation power

Neutrino bound states in rotating medium

- ***A.Grigoriev, A.Savochkin, A.Studenikin,***
  "Quantum states of the neutrino in non-uniformly moving medium"
  Russ.Phys.J. 50 (2007) 845-852

Neutrino energy quantization in moving matter



angular speed of matter rotation around OZ

Consider ν moving in rotating medium composed of neutrons (generalization s.f.):

$$\boxed{\left\{ i\gamma_\mu \partial^\mu - \frac{1}{2}\gamma_\mu (1+\gamma_5) f^\mu - m \right\} \Psi(x) = 0}$$

A.Ternov, A.Studenikin, Phys.Lett.B 608 (2005) 107

where matter potential $f^\mu = -G(n, n\mathbf{v}), \quad \mathbf{v} = (\omega y, 0, 0), \qquad G = \frac{G_F}{\sqrt{2}}$

neutron number density **speed of matter** **angular speed of rotation**

For ν wave function components $\Psi(x)$:

$$\boxed{\begin{aligned} \left[i\left(\partial_0-\partial_3\right)+Gn\right]\Psi_1+\left[-\left(i\partial_1+\partial_2\right)+Gn\omega y\right]\Psi_2&=m\Psi_3\\ \left[\left(-i\partial_1+\partial_2\right)+Gn\omega y\right]\Psi_1+\left[i\left(\partial_0+\partial_3\right)+Gn\right]\Psi_2&=m\Psi_4\\ i\left(\partial_0+\partial_3\right)\Psi_3+\left(i\partial_1+\partial_2\right)\Psi_4&=m\Psi_1\\ \left(i\partial_1-\partial_2\right)\Psi_3+i\left(\partial_0-\partial_3\right)\Psi_4&=m\Psi_2 \end{aligned}}$$

Method of exact solutions $\Longrightarrow$ exact solution ?

The problem is reasonably simplified in case of relativistic ν :

$\boxed{m/p_0 \ll 1}$ **and** $\boxed{Gn\omega \gg m^2}$

Two pairs of wave function components decouple from each other

and 4 equations $\Longrightarrow$ 2 x 2 equations

that couple wave function components in pairs: $\boxed{(\Psi_1,\ \Psi_2)}$ **and** $\boxed{(\Psi_3,\ \Psi_4)}$

Second pair of equations (in relativistic case) $\boxed{(\Psi_3,\ \Psi_4)}$

$$\boxed{\begin{aligned} (p_0 - p_3)\,\Psi_3 - (p_1 - ip_2)\,\Psi_4 &= 0 \\ -(p_1 + ip_2)\,\Psi_3 + (p_0 + p_3)\,\Psi_4 &= 0 \end{aligned}}$$

does not contain matter term and describes sterile right-handed ν state Ψ_R, can be written in plain-wave form

$$\boxed{\Psi_R \sim L^{-\frac{3}{2}} \exp\{i(-p_0 t + p_1 x + p_2 y + p_3 z)\}\psi}$$

Exact solution of second pair of equation is

$$\boxed{\Psi_R = \frac{e^{-ipx}}{L^{3/2}\sqrt{2p_0(p_0 - p_3)}} \begin{pmatrix} 0 \\ 0 \\ -p_1 + ip_2 \\ p_3 - p_0 \end{pmatrix}}$$

where: $px = p_\mu x^\mu, \ p_\mu = (p_0, p_1, p_2, p_3)$, $x_\mu = (t, x, y, z)$

with vacuum dispersion relation $\Rightarrow$ **sterile** Ψ_R .

$(\Psi_1, \ \Psi_2)$

The first pair of equations does contain matter term

$$\left(p_0 + p_3 + Gn\right)\Psi_1 - \sqrt{\rho}\left(\frac{\partial}{\partial\eta} - \eta\right)\Psi_2 = 0$$
$$\sqrt{\rho}\left(\frac{\partial}{\partial\eta} + \eta\right)\Psi_1 + \left(p_0 - p_3 + Gn\right)\Psi_2 = 0$$

$\eta = \sqrt{\rho}\left(x_2 + \frac{p_1}{\rho}\right), \quad \rho = Gn\omega$

describes left-handed active ν , Ψ_L .

Equations are similar to those describing electron in magnetic field B :

ν in rotating medium ⟵ **correspondence** ⟶ **e in magnetic field**

matter current

$nv \Rightarrow A$

vector potential

$A = (By, 0, 0)$

Exact solution Ψ_L of

$$\begin{aligned}\left(p_0+p_3+Gn\right)\Psi_1-\sqrt{\rho}\left(\frac{\partial}{\partial\eta}-\eta\right)\Psi_2=0\\ \sqrt{\rho}\left(\frac{\partial}{\partial\eta}+\eta\right)\Psi_1+\left(p_0-p_3+Gn\right)\Psi_2=0\end{aligned}$$

where

$$\eta=\sqrt{\rho}\left(x_2+\frac{p_1}{\rho}\right),\quad \rho=Gn\omega$$

can be found in plain-wave form ,

$$\Psi_L\sim\frac{1}{L}\exp\{i(-p_0t+p_1x+p_3z)\}\psi(y)\quad ,$$

and is then written as

$$\Psi_L=\frac{\rho^{\frac{1}{4}}e^{-ip_0t+ip_1x+ip_3z}}{L\sqrt{(p_0-p_3+Gn)^2+2\rho N}}\begin{pmatrix}(p_0-p_3+Gn)\,u_N(\eta)\\ -\sqrt{2\rho N}u_{N-1}(\eta)\\ 0\\ 0\end{pmatrix}$$

$u_N(\eta)$ are Hermite functions of order N

Energy spectrum of active left-handed neutrino

$$\boxed{p_0 = \sqrt{p_3^2 + 2\rho N} - Gn, \quad N = 0, 1, 2, \ldots}$$

$$\rho = Gn\omega$$

Antineutrino → **"negative sign" energy eigenvalues**

$$\boxed{\tilde{p}_0 = \sqrt{p_3^2 + 2\rho N} + Gn, \quad N = 0, 1, 2, \ldots}$$

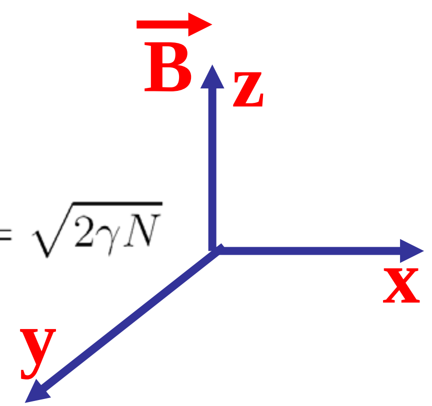
→ ν energy quantization

Lev Landau (1908-2008) **jubilee**

Transversal motion of active relativistic ν is quantized in rotating medium like electron motion is quantized in magnetic field (Landau energy levels):

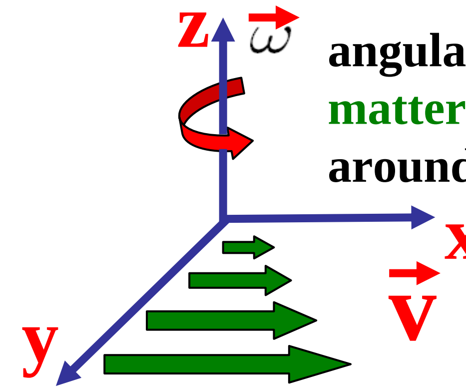
$$\boxed{p_0^{(e)} = \sqrt{m_e^2 + p_3^2 + 2\gamma N}, \quad \gamma = eB, \quad N = 0, 1, 2, \ldots}$$

$$p_\perp^{(e)} = \sqrt{2\gamma N}$$



Consider antineutrino $\tilde{\nu}$
in rotating neutron matter,
then energy of transversal motion

$$\boxed{\tilde{p}_{\perp} = \sqrt{2\rho N}} \qquad \rho = Gn\omega$$



angular speed of matter rotation around OZ

Quantum number N also determines **radius** of antineutrino quasi-classical orbit in moving matter:

$$\boxed{R = \sqrt{\frac{2N}{Gn\omega}}}$$

⇨ **binding orbits inside a Neutron Star !?**

NS:

$$R_{NS} = 10\ km$$
$$n = 10^{37} cm^{-3}$$
$$\omega = 2\pi \times 10^{3}\ s^{-1}$$

for this set $R = \sqrt{\frac{2N}{Gn\omega}}$ **<** $R_{NS} = 10\ km$

if $\boxed{N \leq N_{max} = 10^{10}}$ **,** $\tilde{\nu}$ **with** $N \leq 10^{10}$
can be bound
inside the star

thus, $\tilde{\nu}$ **with energy** $\tilde{p}_0 \lesssim 10^3$ eV **can be bound inside NS**

$N \gg 1$ and $p_3 = 0$

ν quantum states in rotating matter

A.Studenikin, J.Phys.A: Math.Theor. 41 (2008) 164047

quasi-classical circular orbits due to central force

$$\boxed{\mathbf{F}_m^{(\nu)} = q_m^{(\nu)} \boldsymbol{\beta} \times \mathbf{B}_m}$$

$$\mathbf{B}_m = \boldsymbol{\nabla} \times \mathbf{A}_m, \ \ \mathbf{A}_m = n\mathbf{v}$$

"magnetic field" **vector potential**

"charge"

$$q_m^{(\nu)} = -G$$

matter-induced "Lorentz force",

$$\boxed{\mathbf{F}_m^{(\nu)} \perp \boldsymbol{\beta}}$$

Generalization to non-constant matter density:

$$\boxed{\mathbf{F}_m^{(\nu)} = q_m^{(\nu)}\mathbf{E} + q_m^{(\nu)}\boldsymbol{\beta} \times \mathbf{B}_m,}$$

L.Silva, R.Bingham, J.Dawson, J.Mendoca, P.Shukla, Phys.Plasma 7 (2000) 2166

"magnetic field" $\boxed{\mathbf{B}_m = n\boldsymbol{\nabla} \times \mathbf{v} - \mathbf{v} \times \boldsymbol{\nabla} n}$

"electric field" $\boxed{\mathbf{E} = -\boldsymbol{\nabla} n}$

Variation scale of matter density should be less then de Broglie wave length

$$\boxed{\left| \frac{\boldsymbol{\nabla} n}{np} \right| \ll 1}$$

Method of exact solutions

Modified **Dirac equations** for ν (and e)

(containing the correspondent effective matter potentials)

+

exact solutions (particles wave functions)

↓

a basis for investigation of different phenomena which can proceed when **neutrinos** and **electrons** move in dense media

(**astrophysical** and **cosmological** **environments**).