PROPERTIES OF HEAVY BARYONS IN THE RELATIVISTIC QUARK MODEL

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"My conclusion is that if you want to know the mass of a particle and if you have little time (in years!) and little money you should forget all your prejudices and use potential models. This is, in fact, even true to a large extent for systems containing light quarks, which is still more mysterious."

André Martin (CERN) CERN-TH/96-318

Heavy baryons (qqQ)

$J = 1/2$	Λ_c	Λ_b	Ξ_c	Ξ_b	Σ_c	Σ_b	Ξ'_c	Ξ'_b	Ω_c	Ω_b
$J = 3/2$					Σ_c^*	Σ_b^*	Ξ_c^*	Ξ_b^*	Ω_c^*	Ω_b^*
	$[ud]c$	$[ud]b$	$[us]c$	$[us]b$	$\{ud\}c$	$\{ud\}b$	$\{us\}c$	$\{us\}b$	$\{ss\}c$	$\{ss\}b$
			$[ds]c$	$[ds]b$	$\{dd\}c$	$\{dd\}b$	$\{ds\}c$	$\{ds\}b$		
					$\{uu\}c$	$\{uu\}b$				
isospin	0	0	1/2	1/2	1	1	1/2	1/2	0	0

INTRODUCTION

- Mass spectrum of heavy baryons B_Q (qqQ), $(q = u, d, s)$

Main assumption: heavy-quark–light-diquark picture of heavy baryons

Three-body calculation $\longrightarrow$ two-step two-body calculation

Diquark is a composite system with $S = 0, 1$:

- light diquark is not point-like object: Its interaction with gluons is smeared by the form factor expressed through the overlap integral of diquark wave functions

Difference in dynamics of heavy and light quarks:

- slow relative motion of heavy quark Q
- fast motion of light diquarks d ($v/c \sim 0.6 - 0.7$) $\rightarrow$ light diquark should be treated fully relativistically

Pauli principle for ground state diquarks:

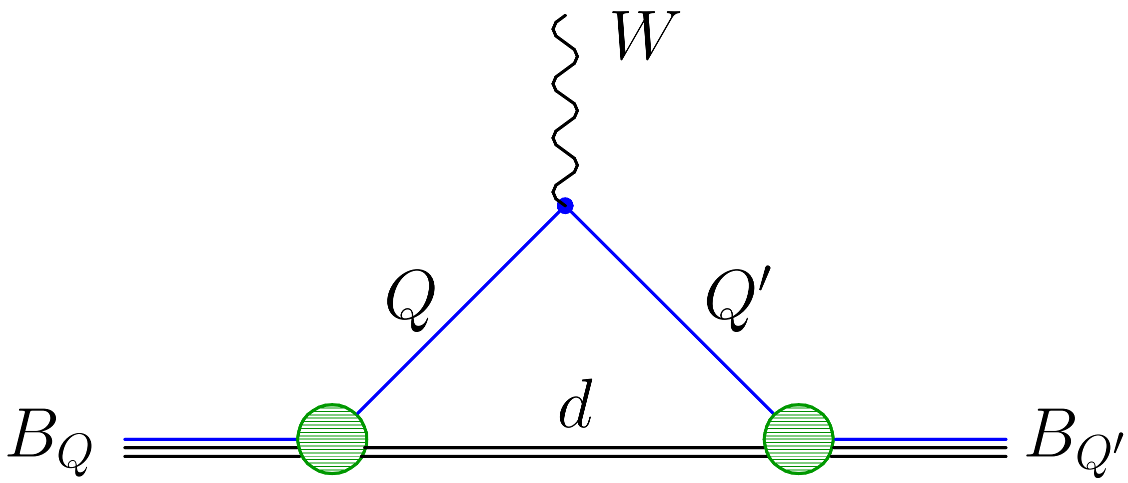
- (qq') diquark can have $S = 0, 1$ (scalar $[q, q']$, axial vector $\{q, q'\}$)
- (qq) diquarks can have only $S = 1$ (axial vector $\{q, q\}$)

Two types of heavy baryons:

- Λ_Q type – light scalar diquark: Λ_b, Λ_c, Ξ_b, Ξ_c – spin 1/2
- $\Omega_Q(\Sigma_Q)$ type – light axial vector diquark: Ω_b, Ω_c, Ξ'_b, Ξ'_c, Σ_b, Σ_c – spin 1/2;
  Ω_b^*, Ω_c^*, Ξ_b^*, Ξ_c^*, Σ_b^*, Σ_c^* – spin 3/2

- Heavy-to-heavy semileptonic decays of baryons: $B_Q \to B_{Q'} e\nu$ $(Q = b, c)$

Additional source for the determination of V_{cb}.



Active heavy quark and spectator light diquark.

HQS ($m_Q \to \infty$):

heavy quark spin and mass decouple $\longrightarrow$ heavy baryon properties are determined by light diquarks $\longrightarrow$

- masses of ground state baryons with spin 1/2 and 3/2 containing the axial vector diquark are degenerate
- for $\Lambda_b \longrightarrow \Lambda_c$ one universal form factor $\zeta(w)$ (Isgur-Wise function)
- for $\Omega_b \longrightarrow \Omega_c$ two universal form factors $\zeta_1(w)$ and $\zeta_2(w)$
- isospin violating decay amplitudes, e.g. $\Lambda_b \longrightarrow \Sigma_c$, vanish

$1/m_Q$ order:

- for $\Lambda_Q \longrightarrow \Lambda_{Q'}$ one additional mass parameter $\bar{\Lambda}$ and one additional function $\chi(w)$
- for $\Omega_Q \longrightarrow \Omega_{Q'}$ one additional mass parameter $\bar{\Lambda}$ and five additional functions

RELATIVISTIC QUARK MODEL

Quasipotential equation of Schrödinger type:

$$\left(\frac{b^2(M)}{2\mu_R} - \frac{\mathbf{p}^2}{2\mu_R}\right)\Psi_M(\mathbf{p}) = \int \frac{d^3q}{(2\pi)^3} V(\mathbf{p},\mathbf{q};M)\Psi_M(\mathbf{q})$$

$\mathbf{p}$ - relative momentum of quarks (diquarks)
M - bound state mass ($M = E_1 + E_2$)
μ_R - relativistic reduced mass:

$$\mu_R = \frac{E_1E_2}{E_1+E_2} = \frac{M^4 - (m_1^2 - m_2^2)^2}{4M^3}$$

$b(M)$ - on-mass-shell relative momentum in cms:

$$b^2(M) = \frac{[M^2 - (m_1+m_2)^2][M^2 - (m_1-m_2)^2]}{4M^2}$$

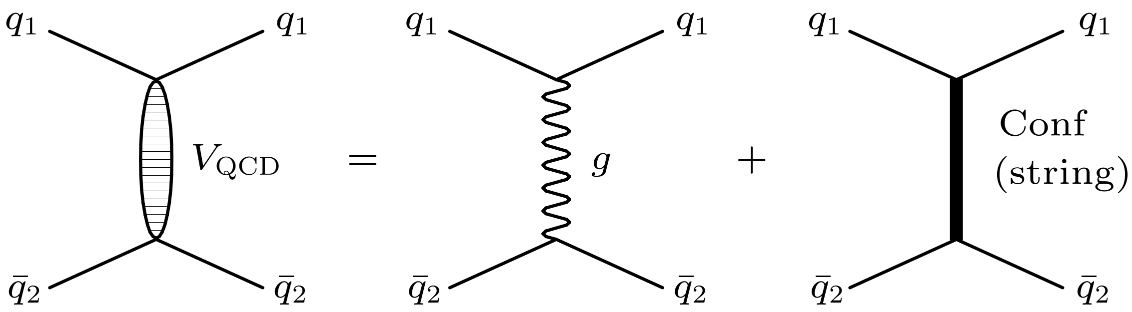
$E_{1,2}$ - center of mass energies:

$$E_1 = \frac{M^2 - m_2^2 + m_1^2}{2M}, \quad E_2 = \frac{M^2 - m_1^2 + m_2^2}{2M}$$

- Parameters of the model fixed from meson sector

- $q\bar{q}$ **quasipotential** (meson sector)

(Constructed with the help of off-mass-shell scattering amplitude projected onto positive-energy states)



$$V(\mathbf{p},\mathbf{q};M) = \bar{u}_1(p)\bar{u}_2(-p)\left\{\frac{4}{3}\alpha_S D_{\mu\nu}(\mathbf{k})\gamma_1^\mu\gamma_2^\nu + V^V_{\rm conf}(\mathbf{k})\Gamma_1^\mu\Gamma_{2;\mu} + V^S_{\rm conf}(\mathbf{k})\right\}u_1(q)u_2(-q)$$

$\mathbf{k} = \mathbf{p} - \mathbf{q}$

$D_{\mu\nu}(\mathbf{k})$ - (perturbative) gluon propagator

$\Gamma_\mu(\mathbf{k})$ - effective long-range vertex with Pauli term:

$$\Gamma_\mu(\mathbf{k}) = \gamma_\mu + \frac{i\kappa}{2m}\sigma_{\mu\nu}k^\nu,$$

κ - anomalous chromomagnetic moment of quark,

$$u^{\lambda}(p) = \sqrt{\frac{\epsilon(p)+m}{2\epsilon(p)}} \begin{pmatrix} 1 \\ \dfrac{\boldsymbol{\sigma}\mathbf{p}}{\epsilon(p)+m} \end{pmatrix} \chi^{\lambda},$$

with $\epsilon(p) = \sqrt{\mathbf{p}^2 + m^2}$.

- Lorentz structure of $V_{\rm conf} = V_{\rm conf}^{V} + V_{\rm conf}^{S}$

In nonrelativistic limit

$$\left.\begin{array}{rcl} V_{\rm conf}^{V} & = & (1-\varepsilon)(Ar+B) \\ V_{\rm conf}^{S} & = & \varepsilon(Ar+B) \end{array}\right\} \quad \text{Sum :} \quad (Ar+B)$$

ε - mixing parameter

Parameters A, B, κ, ε and quark masses fixed from analysis of meson masses and radiative decays:

$\varepsilon = -1$ from heavy quarkonium radiative decays ($J/\psi \to \eta_c + \gamma$) and HQET

$\kappa = -1$ from fine splitting of heavy quarkonium ^3P_J states and HQET

$(1 + \kappa) = 0 \Longrightarrow$ vanishing long-range chromomagnetic interaction !

Freezing of α_s for light quarks (Simonov, Badalyan)

$$\alpha_s(\mu) = \frac{4\pi}{\beta_0 \ln \frac{\mu^2 + M_0^2}{\Lambda^2}}, \quad \beta_0 = 11 - \frac{2}{3} n_f, \quad \mu = \frac{2 m_1 m_2}{m_1 + m_2},$$

$$M_0 = 2.24 \sqrt{A} = 0.95 \text{ GeV}$$

Quasipotential parameters:

$A = 0.18$ GeV^2, $B = -0.30$ GeV,

$\Lambda = 0.413$ GeV (from M_ρ)

Quark masses:

$m_b = 4.88$ GeV	$m_s = 0.50$ GeV
$m_c = 1.55$ GeV	$m_{u,d} = 0.33$ GeV

- **Heavy baryons in quark-diquark picture**

(qq)-interaction: $$V_{qq} = \frac{1}{2} V_{q\bar{q}}$$

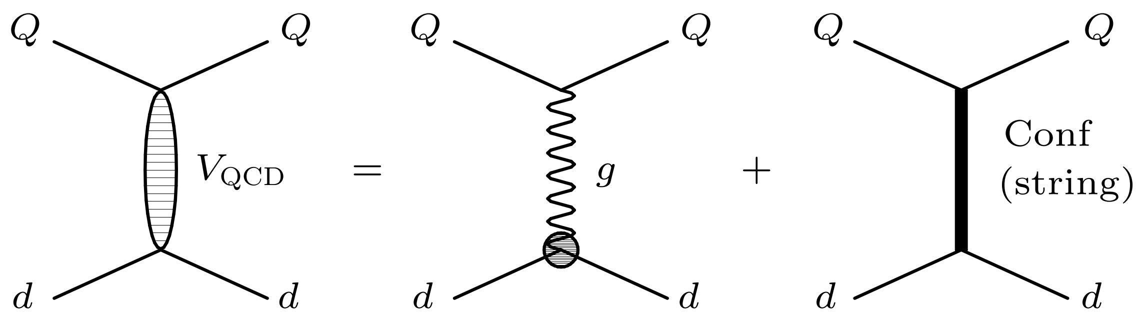
$$V(\mathbf{p}, \mathbf{q}; M) = \bar{u}_1(p)\bar{u}_2(-p)\mathcal{V}(\mathbf{p}, \mathbf{q}; M)u_1(q)u_2(-q),$$

where

$$\mathcal{V}(\mathbf{p}, \mathbf{q}; M) = \frac{2}{3}\alpha_S D_{\mu\nu}(\mathbf{k})\gamma_1^\mu \gamma_2^\nu + \frac{1}{2} V^V_{\rm conf}(\mathbf{k})\Gamma_1^\mu \Gamma_{2;\mu} + \frac{1}{2} V^S_{\rm conf}(\mathbf{k})$$

(dQ)-interaction: $$d = (qq')$$

$$V(\mathbf{p}, \mathbf{q}; M) = \frac{\langle d(P)|J_\mu|d(Q)\rangle}{2\sqrt{E_d(p)E_d(q)}}\bar{u}_Q(p)\frac{4}{3}\alpha_S D_{\mu\nu}(\mathbf{k})\gamma^\nu u_Q(q)$$
$$+\psi_d^*(P)\bar{u}_Q(p)J_{d;\mu}\Gamma_Q^\mu V^V_{\rm conf}(\mathbf{k})u_Q(q)\psi_d(Q) + \psi_d^*(P)\bar{u}_Q(p)V^S_{\rm conf}(\mathbf{k})u_Q(q)\psi_d(Q)$$



$J_{d,\mu}$ – effective long-range vector vertex of diquark:

$$J_{d;\mu} = \begin{cases} \dfrac{(P+Q)_\mu}{2\sqrt{E_d(p)E_d(q)}} & \text{for scalar diquark} \\ \dfrac{(P+Q)_\mu}{2\sqrt{E_d(p)E_d(q)}} + \dfrac{i\mu_d}{2M_d}\Sigma_\mu^\nu k_\nu & \text{for axial vector diquark } (\mu_\mathrm{d}=0) \end{cases}$$

μ_d - total chromomagnetic moment of axial vector diquark
diquark spin matrix: $(\Sigma_{\rho\sigma})_\mu^\nu = -i(g_{\mu\rho}\delta_\sigma^\nu - g_{\mu\sigma}\delta_\rho^\nu)$
$\mathbf{S}_d$ - axial vector diquark spin: $(S_{d;k})_{il} = -i\varepsilon_{kil}$

$\psi_d(P)$ – diquark wave function:

$$\psi_d(p) = \begin{cases} 1 & \text{for scalar diquark} \\ \varepsilon_d(p) & \text{for axial vector diquark} \end{cases}$$

$\varepsilon_d(p)$ – polarization vector of axial vector diquark

$\langle d(P)|J_\mu|d(Q)\rangle$ – vertex of diquark-gluon interaction:

$$\langle d(P)|J_\mu(0)|d(Q)\rangle = \int \frac{d^3p\, d^3q}{(2\pi)^6} \bar{\Psi}^d_P(\mathbf{p})\Gamma_\mu(\mathbf{p},\mathbf{q})\Psi^d_Q(\mathbf{q}) \Rightarrow F(k^2)$$

Γ_μ – two-particle vertex function of the diquark-gluon interaction:

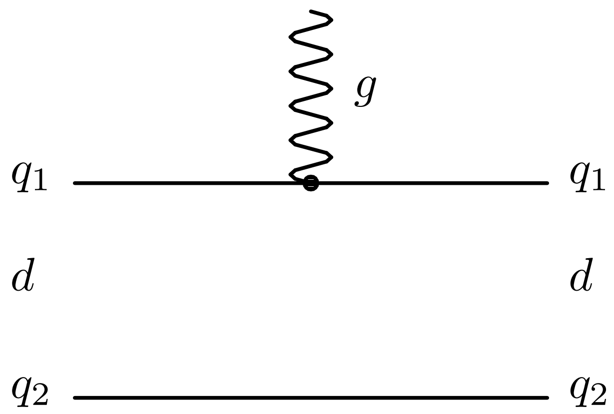


Figure 1: The vertex function Γ of the diquark-gluon interaction in the impulse approximation. The gluon interaction only with one quark is shown.

Table 1: Masses of light ground state diquarks (in MeV). S and A denotes scalar and axial vector diquarks antisymmetric $[q, q']$ and symmetric $\{q, q'\}$ in flavour, respectively.

Quark content	Diquark type	Mass				
		our RQM	Ebert et al. NJL	Burden et al. BSE	Maris BSE	Hess et al. Lattice
$[u, d]$	S	710	705	737	820	694(22)
$\{u, d\}$	A	909	875	949	1020	806(50)
$[u, s]$	S	948	895	882	1100	
$\{u, s\}$	A	1069	1050	1050	1300	
$\{s, s\}$	A	1203	1215	1130	1440	

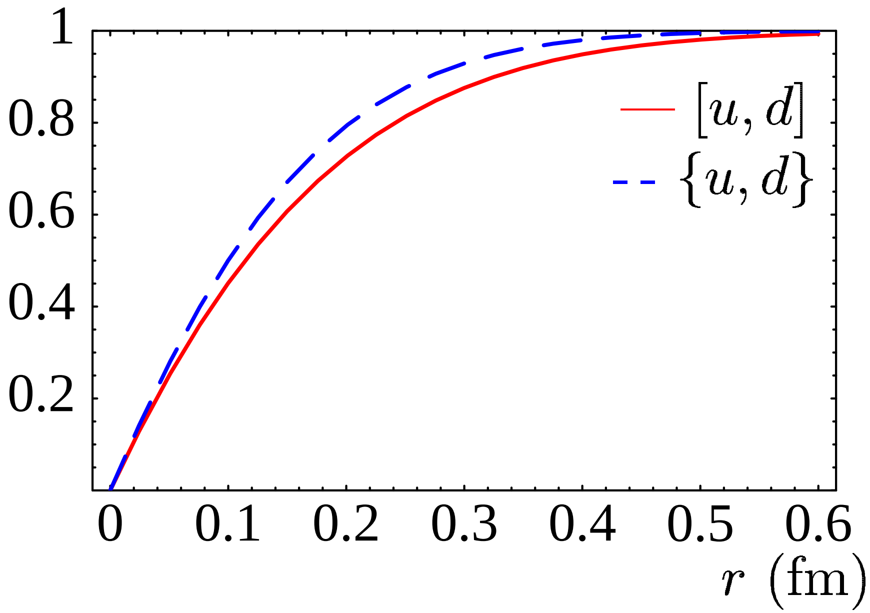


Figure 2: The form factors $F(r)$ for the scalar $[u, d]$ (solid line) and axial vector $\{u, d\}$ (dashed line) diquarks.

MASSES OF HEAVY BARYONS

⋆ p/m_Q expansion for heavy quark
⋆ relativistic treatment of light diquark $d = (qq)$

- leading order in p/m_Q

for scalar diquark

$$V^{(0)}(r) = \hat{V}_{\rm Coul}(r) + V_{\rm conf}(r),$$

for axial vector diquark

$$V^{(0)}(r) = \hat{V}_{\rm Coul}(r)+V_{\rm conf}(r)+\frac{1}{E_d(E_d+M_d)}\frac{1}{r}\left[\frac{M_d}{E_d}\hat{V}'_{\rm Coul}(r)-V'_{\rm conf}(r)+\mu_d\frac{E_d+M_d}{2M_d}V'^{V}_{\rm conf}(r)\right]\mathbf{L}\mathbf{S}_d,$$

$$\hat{V}_{\rm Coul}(r) = -\frac{4}{3}\alpha_s\frac{F(r)}{r}, \qquad V_{\rm conf}(r) = Ar + B,$$

where $\hat{V}_{\rm Coul}(r)$ is the smeared Coulomb potential (which accounts for diquark structure).

- $\delta V(r)$ corrections up to second order in p/m_Q (spin-independent + $\mathbf{L}\mathbf{S}_Q$, $\mathbf{S}_d\mathbf{S}_Q$, T terms)

Mass formula

$$\frac{b^2(M)}{2\mu_R} = \frac{\langle \mathbf{p}^2\rangle}{2\mu_R} + \langle V^{(0)}(r)\rangle + \langle \delta V(r)\rangle.$$

Table 2: Masses of the ground state heavy baryons (in MeV).

Baryon	$I(J^P)$	Theory						Experiment
		our (2005)	Capstick Isgur	Roncaglia et al.	Savage	Jenkins	Mathur* et al.	PDG
Λ_c	$0(\frac{1}{2}^+)$	2297	2265	2285			2290	2286.46(14)
Σ_c	$1(\frac{1}{2}^+)$	2439	2440	2453			2452	2453.76(18)
Σ_c^*	$1(\frac{3}{2}^+)$	2518	2495	2520	2518		2538	2518.0(5)
Ξ_c	$\frac{1}{2}(\frac{1}{2}^+)$	2481		2468			2473	2471.0(4)
Ξ_c'	$\frac{1}{2}(\frac{1}{2}^+)$	2578		2580	2579	2580.8(2.1)	2599	2578.0(2.9)
Ξ_c^*	$\frac{1}{2}(\frac{3}{2}^+)$	2654		2650			2680	2646.1(1.2)
Ω_c	$0(\frac{1}{2}^+)$	2698		2710			2678	2697.5(2.6)
Ω_c^*	$0(\frac{3}{2}^+)$	2768		2770	2768	2760.5(4.9)	2752	2768.3(3.0)†
Λ_b	$0(\frac{1}{2}^+)$	5622	5585	5620			5672	5620.2(1.6)
Σ_b	$1(\frac{1}{2}^+)$	5805	5795	5820		5824.2(9.0)	5847	5807.5(3.6)‡
Σ_b^*	$1(\frac{3}{2}^+)$	5834	5805	5850		5840.0(8.8)	5871	5829.0(3.3)‡
Ξ_b	$\frac{1}{2}(\frac{1}{2}^+)$	5812		5810		5805.7(8.1)	5788	5774(26)⋆
Ξ_b'	$\frac{1}{2}(\frac{1}{2}^+)$	5937		5950		5950.9(8.5)	5936	
Ξ_b^*	$\frac{1}{2}(\frac{3}{2}^+)$	5963		5980		5966.1(8.3)	5959	
Ω_b	$0(\frac{1}{2}^+)$	6065		6060		6068.7(11.1)	6040	
Ω_b^*	$0(\frac{3}{2}^+)$	6088		6090		6083.2(11.0)	6060	

* error estimates of lattice calculations — ∼50 MeV for charmed, ∼100 MeV for bottom baryons

† BaBar 2006; ‡ CDF 2006; ⋆ D0 2007

Table 3: Masses of the excited Λ_Q ($Q=c,b$) heavy baryons (in MeV) (scalar diquark)

		$Q=c$		$Q=b$		
$I(J^P)$	Qd state	M(our)	$M^{\rm exp}$ PDG	M(our)	$M^{\rm exp}$ PDG	$M^{\rm exp}$ CDF
$0(\frac{1}{2}^+)$	$1S$	2297	2286.46(14)	5622	5624(9)	5619.7(2.4)
$0(\frac{1}{2}^-)$	$1P$	2598	2595.4(6)	5930		
$0(\frac{3}{2}^-)$	$1P$	2628	2628.1(6)	5947		
$0(\frac{1}{2}^+)$	$2S$	2772	2766.6(2.4)?	6086		
$0(\frac{3}{2}^+)$	$1D$	2874		6189		
$0(\frac{5}{2}^+)$	$1D$	2883	2882.5(2.2)	6197		
$0(\frac{1}{2}^-)$	$2P$	3017		6328		
$0(\frac{3}{2}^-)$	$2P$	3034		6337		

Table 4: Masses of the excited Σ_Q ($Q=c,b$) heavy baryons (in MeV) (axial vector diquark)

		$Q=c$				$Q=b$	
$I(J^P)$	Qd state	M(our)	$M^{\rm exp}$ PDG	$M^{\rm exp}$ BaBar	$M^{\rm exp}$ Belle	M(our)	$M^{\rm exp}$ CDF
$1(\frac{1}{2}^+)$	$1S$	2439	2453.76(18)			5805	5807.5(3.6)
$1(\frac{3}{2}^+)$	$1S$	2518	2518.0(5)			5834	5829.0(3.3)
$1(\frac{1}{2}^-)$	$1P$	2805				6122	
$1(\frac{1}{2}^-)$	$1P$	2795				6108	
$1(\frac{3}{2}^-)$	$1P$	2799	$2802(^4_7)$			6106	
$1(\frac{3}{2}^-)$	$1P$	2761	2766.6(2.4)?			6076	
$1(\frac{5}{2}^-)$	$1P$	2790				6083	
$1(\frac{1}{2}^+)$	$2S$	2864				6202	
$1(\frac{3}{2}^+)$	$2S$	2912		2939.8(2.3)?	$2938(^3_5)$?	6222	

Table 5: Masses of the excited Ξ_Q $(Q = c, b)$ heavy baryons with scalar diquark (in MeV).

		$Q=c$		$Q=b$		
$I(J^P)$	Qd state	M(our)	$M^{\rm exp}$ PDG	M(our)	$M^{\rm exp}$ D0	$M^{\rm exp}$ CDF
$\frac{1}{2}(\frac{1}{2}^+)$	$1S$	2481	2471.0(4)	5812	5774(26)	5793(4)
$\frac{1}{2}(\frac{1}{2}^-)$	$1P$	2801	2791.9(3.3)	6119		
$\frac{1}{2}(\frac{3}{2}^-)$	$1P$	2820	2818.2(2.1)	6130		
$\frac{1}{2}(\frac{1}{2}^+)$	$2S$	2923		6264		

Table 6: Masses of the excited Ξ_Q $(Q = c, b)$ heavy baryons with axial vector diquark (in MeV).

		$Q=c$				$Q=b$
$I(J^P)$	Qd state	M(our)	$M^{\rm exp}$ PDG	$M^{\rm exp}$ Belle	$M^{\rm exp}$ BaBar	M(our)
$\frac{1}{2}(\frac{1}{2}^+)$	$1S$	2578	2578.0(2.9)			5937
$\frac{1}{2}(\frac{3}{2}^+)$	$1S$	2654	2646.1(1.2)			5963
$\frac{1}{2}(\frac{1}{2}^-)$	$1P$	2934				6249
$\frac{1}{2}(\frac{1}{2}^-)$	$1P$	2928				6238
$\frac{1}{2}(\frac{3}{2}^-)$	$1P$	2931				6237
$\frac{1}{2}(\frac{3}{2}^-)$	$1P$	2900				6212
$\frac{1}{2}(\frac{5}{2}^-)$	$1P$	2921				6218
$\frac{1}{2}(\frac{1}{2}^+)$	$2S$	2984		2978.5(4.1)	2967.1(2.9)	6327
$\frac{1}{2}(\frac{3}{2}^+)$	$2S$	3035				6341
$\frac{1}{2}(\frac{1}{2}^+)$	$1D$	3132				6420
$\frac{1}{2}(\frac{3}{2}^+)$	$1D$	3127				6410
$\frac{1}{2}(\frac{3}{2}^+)$	$1D$	3131				6412
$\frac{1}{2}(\frac{5}{2}^+)$	$1D$	3123				6403
$\frac{1}{2}(\frac{5}{2}^+)$	$1D$	3087		3082.8(3.3)	3076.4(1.0)	6377
$\frac{1}{2}(\frac{7}{2}^+)$						

SEMILEPTONIC DECAYS

• Matrix elements of weak current

Matrix element of weak current $J^W_\mu = \bar{Q}'\gamma_\mu(1-\gamma_5)Q$:

$$\langle B_{Q'}(p_{B_{Q'}})|J^W_\mu|B_Q(p_{B_Q})\rangle = \int \frac{d^3p\, d^3q}{(2\pi)^6}\bar{\Psi}_{B_{Q'}\,\mathbf{p}_{B_{Q'}}}(\mathbf{p})\Gamma_\mu(\mathbf{p},\mathbf{q})\Psi_{B_Q\,\mathbf{p}_{B_Q}}(\mathbf{q}),$$

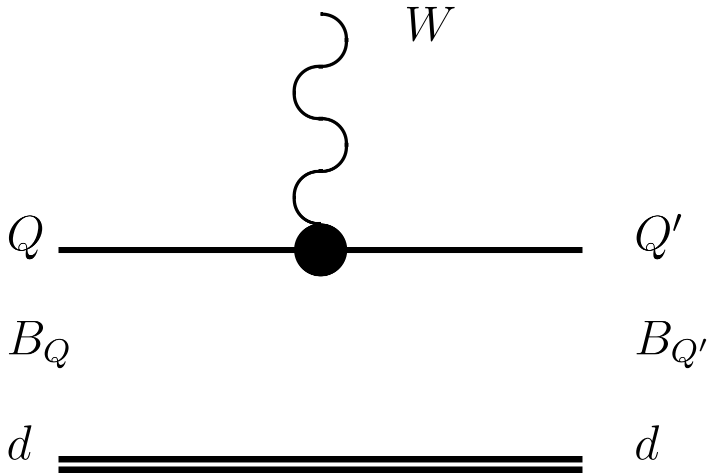


Figure 3: Lowest order vertex function $\Gamma^{(1)}$ contributing to the current matrix element.

Wave function $\Psi_{B_{Q'}\mathbf{\Delta}}$ of the moving baryon is connected with the rest-frame wave function $\Psi_{B_{Q'}\mathbf{0}} \equiv \Psi_{B_{Q'}}$ by the transformation

$$\Psi_{B_{Q'}\mathbf{\Delta}}(\mathbf{p}) = D^{1/2}_{Q'}(R^W_{L_{\mathbf{\Delta}}}) D^{\mathcal{I}}_{d}(R^W_{L_{\mathbf{\Delta}}}) \Psi_{B_{Q'}\mathbf{0}}(\mathbf{p}), \qquad \mathcal{I} = 0, 1,$$

where R^W – Wigner rotation, $L_{\mathbf{\Delta}}$ – Lorentz boost from the baryon rest frame to a moving one.

- Rotation matrix $D^{1/2}_{Q'}(R)$ of heavy quark spin:

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} D^{1/2}_{Q'}(R^W_{L_{\mathbf{\Delta}}}) = S^{-1}(\mathbf{p}_{Q'}) S(\mathbf{\Delta}) S(\mathbf{p}),$$

$$S(\mathbf{p}) = \sqrt{\frac{\epsilon(p) + m}{2m}} \left(1 + \frac{\boldsymbol{\alpha}\mathbf{p}}{\epsilon(p) + m}\right).$$

- Rotation matrix $D^{\mathcal{I}}_{d}(R)$ of diquark with spin $\mathcal{I} = 0, 1$:

  $D^0_d(R^W) = 1$ for scalar diquark

  $D^1_d(R^W) = R^W$ for axial vector diquark.

• Form factors of heavy baryons with scalar diquark

Hadronic matrix elements for $\Lambda_Q \to \Lambda_{Q'} e\nu$:

$$\langle \Lambda_{Q'}(v', s')|V^\mu|\Lambda_Q(v, s)\rangle = \bar{u}_{\Lambda_{Q'}}(v', s')\Big[F_1(w)\gamma^\mu + F_2(w)v^\mu + F_3(w)v'^\mu\Big]u_{\Lambda_Q}(v, s),$$
$$\langle \Lambda_{Q'}(v', s')|A^\mu|\Lambda_Q(v, s)\rangle = \bar{u}_{\Lambda_{Q'}}(v', s')\Big[G_1(w)\gamma^\mu + G_2(w)v^\mu + G_3(w)v'^\mu\Big]\gamma_5 u_{\Lambda_Q}(v, s),$$

$$w = v \cdot v' = \frac{M^2_{\Lambda_Q} + M^2_{\Lambda_{Q'}} - q^2}{2M_{\Lambda_Q}M_{\Lambda_{Q'}}}.$$

• In heavy quark limit $m_Q \to \infty$

$$F_1(w) = G_1(w) = \zeta(w), \qquad F_2(w) = F_3(w) = G_2(w) = G_3(w) = 0.$$

• At $1/m_Q$ order in HQET

$$F_1(w) = \zeta(w) + \left(\frac{\bar{\Lambda}}{2m_Q} + \frac{\bar{\Lambda}}{2m_{Q'}}\right)[2\chi(w) + \zeta(w)],$$
$$G_1(w) = \zeta(w) + \left(\frac{\bar{\Lambda}}{2m_Q} + \frac{\bar{\Lambda}}{2m_{Q'}}\right)\left[2\chi(w) + \frac{w-1}{w+1}\zeta(w)\right],$$
$$F_2(w) = G_2(w) = -\frac{\bar{\Lambda}}{2m_{Q'}}\frac{2}{w+1}\zeta(w), \qquad F_3(w) = -G_3(w) = -\frac{\bar{\Lambda}}{2m_Q}\frac{2}{w+1}\zeta(w).$$

In our model up to $1/m_Q$ order

$$F_1(w) = \zeta(w) + \left(\frac{\bar\Lambda}{2m_Q} + \frac{\bar\Lambda}{2m_{Q'}}\right)[2\chi(w) + \zeta(w)]$$
$$+4(1-\varepsilon)(1+\kappa)\left[\frac{\bar\Lambda}{2m_{Q'}}\frac{1}{w-1} - \frac{\bar\Lambda}{2m_Q}(w+1)\right]\chi(w),$$
$$G_1(w) = \zeta(w) + \left(\frac{\bar\Lambda}{2m_Q} + \frac{\bar\Lambda}{2m_{Q'}}\right)\left[2\chi(w) + \frac{w-1}{w+1}\zeta(w)\right] - 4(1-\varepsilon)(1+\kappa)\frac{\bar\Lambda}{2m_Q}w\chi(w),$$
$$F_2(w) = -\frac{\bar\Lambda}{2m_{Q'}}\frac{2}{w+1}\zeta(w) - 4(1-\varepsilon)(1+\kappa)\left[\frac{\bar\Lambda}{2m_{Q'}}\frac{1}{w-1} + \frac{\bar\Lambda}{2m_Q}w\right]\chi(w),$$
$$G_2(w) = -\frac{\bar\Lambda}{2m_{Q'}}\frac{2}{w+1}\zeta(w) - 4(1-\varepsilon)(1+\kappa)\frac{\bar\Lambda}{2m_{Q'}}\frac{1}{w-1}\chi(w),$$
$$F_3(w) = -G_3(w) = -\frac{\bar\Lambda}{2m_Q}\frac{2}{w+1}\zeta(w) + 4(1-\varepsilon)(1+\kappa)\frac{\bar\Lambda}{2m_Q}\chi(w).$$

⋆ For $(1-\varepsilon)(1+\kappa) = 0$ HQET results are reproduced ($\kappa = -1$ in our model)!

The $\Lambda_b \to \Lambda_c$ differential decay rate near zero recoil:

$$\lim_{w\to 1} \frac{1}{\sqrt{w^2-1}} \frac{d\Gamma(\Lambda_b \to \Lambda_c e\nu)}{dw} = \frac{G_F^2|V_{cb}|^2}{4\pi^3} M_{\Lambda_c}^3 (M_{\Lambda_b} - M_{\Lambda_c})^2 |G_1(1)|^2$$

$$G_1(w) = 1 - \hat{\rho}^2(w-1) + \cdots$$

In our model with $1/m_Q$ corrections: $\hat{\rho}^2 = 1.51$

Experiment (DELPHI Collaboration): $\hat{\rho}^2 = 2.03 \pm 0.46^{+0.72}_{-1.00}$

Lattice QCD (Bowler et al.): $\hat{\rho}^2 = 1.1 \pm 1.0$

Our prediction for the branching ratio ($|V_{cb}| = 0.041$, $\tau_{\Lambda_b} = 1.23 \times 10^{-12}$s)

$$Br^{\rm theor}(\Lambda_b \to \Lambda_c l\nu) = 6.9\%$$

Experiment

$$Br^{\rm exp}(\Lambda_b \to \Lambda_c l\nu) = \begin{cases} (5.0^{+1.1+1.6}_{-0.8-1.2})\ \% & \text{DELPHI} \\ (8.1 \pm 1.2^{+1.1}_{-1.6} \pm 4.3)\ \% & \text{CDF} \end{cases}$$

$$Br^{\rm exp}(\Lambda_b \to \Lambda_c l\nu + \text{anything}) = (9.1 \pm 2.1)\%. \qquad \text{PDG}$$

Table 7: Comparison of different theoretical predictions for semileptonic decay rates Γ (in $10^{10}\mathrm{s}^{-1}$) of bottom baryons.

Decay	Our RQM	Singleton NRQM	Cheng NRQM	Körner NRQM	Ivanov RTQM	Ivanov BS	Cardarelli LF	Albertus NRQM	Huang sum rule
$\Lambda_b \to \Lambda_c e\nu$	5.64	5.9	5.1	5.14	5.39	6.09	5.08 ± 1.3	5.82	5.4 ± 0.4
$\Xi_b \to \Xi_c e\nu$	5.29	7.2	5.3	5.21	5.27	6.42	5.68 ± 1.5	4.98	
$\Sigma_b \to \Sigma_c e\nu$	1.44	4.3			2.23	1.65			
$\Xi_b' \to \Xi_c' e\nu$	1.34								
$\Omega_b \to \Omega_c e\nu$	1.29	5.4	2.3	1.52	1.87	1.81			
$\Sigma_b \to \Sigma_c^* e\nu$	3.23				4.56	3.75			
$\Xi_b' \to \Xi_c^* e\nu$	3.09								
$\Omega_b \to \Omega_c^* e\nu$	3.03			3.41	4.01	4.13			

SUMMARY

- Mass spectra and semileptonic decays of heavy baryons qqQ are studied in heavy-quark–light-diquark picture
- Light quarks and light diquarks are treated fully relativistically
- $1/m_Q$ expansion is used only for heavy quarks
- Light diquarks are considered as nonlocal objects. Diquark wave functions are used to calculate diquark-gluon vertex describing diquark structure
- Predictions for the masses of the Ω_c^*, Σ_b, Σ_b^* and Ξ_b were recently confirmed experimentally
- Calculated masses of ground state and excited heavy baryons are in good agreement with available experimental data
- All presently available data on masses of heavy baryons can be accommodated in the picture treating a heavy baryon as the bound state of light diquark and heavy quark, experiencing orbital and radial excitations
- Diquark and baryon wave functions obtained in calculating heavy baryon masses are used for the calculation of semileptonic decays
- Structure of weak decay matrix elements agrees with model independent predictions of HQET both at leading and subleading orders of heavy quark expansion
- Leading and subleading Isgur-Wise functions for heavy baryon decays are explicitly expressed through the overlap integrals of wave functions in the whole accessible kinematic range
- Calculated decay rates agree with available experimental data