Superluminal Travel

in two dimensions

with Sergey Sibiryakov
arXiv:0806.****

Whether superluminal signals are possible in a *consistent Lorentz invariant quantum* field theory?

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0-th order motivation
(excuse?):

The answer is *Yes!*

At least in 1+1 dimensions and if the spatial parity is broken

Some more motivations:

- Many people feel that locality is an approximate notion in gravitational theories (no local observables, information recovery from black holes). Instantaneous signal propagation definitely qualifies as a non-local effect.

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Theories described in this talk provide a (first?) example of non-local Lorentz invariant microscopic QFT's.

▸ Interesting candidate habitants of the swampland

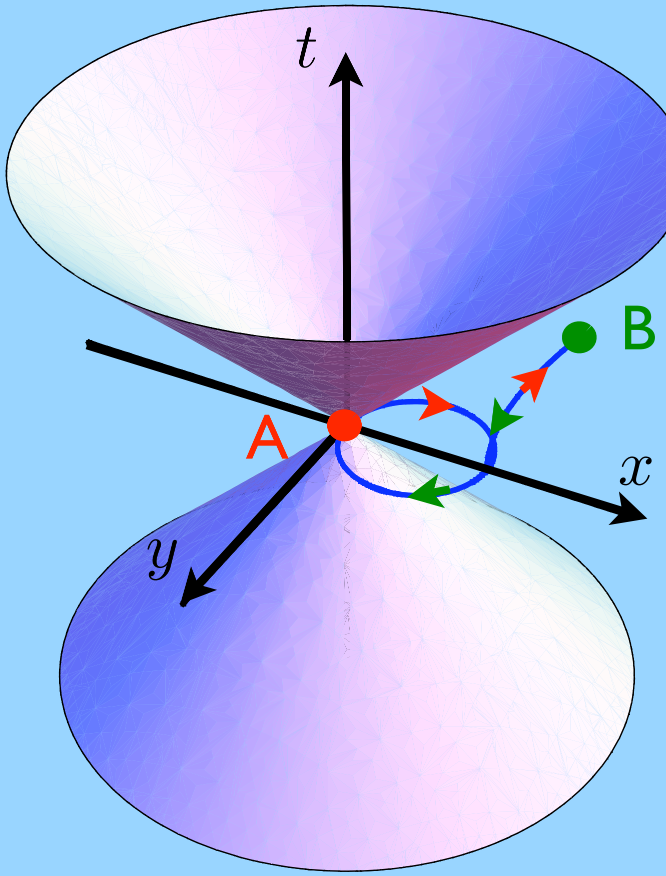
- DGP model
- 4d Higgs phases of gravity (ghost condensate and more general models of massive gravity)

Rich unexpected phenomenology: anomalous precession of the Moon perihelion, gravitational wave signal from primordial massive gravitons clumped in the halo,...

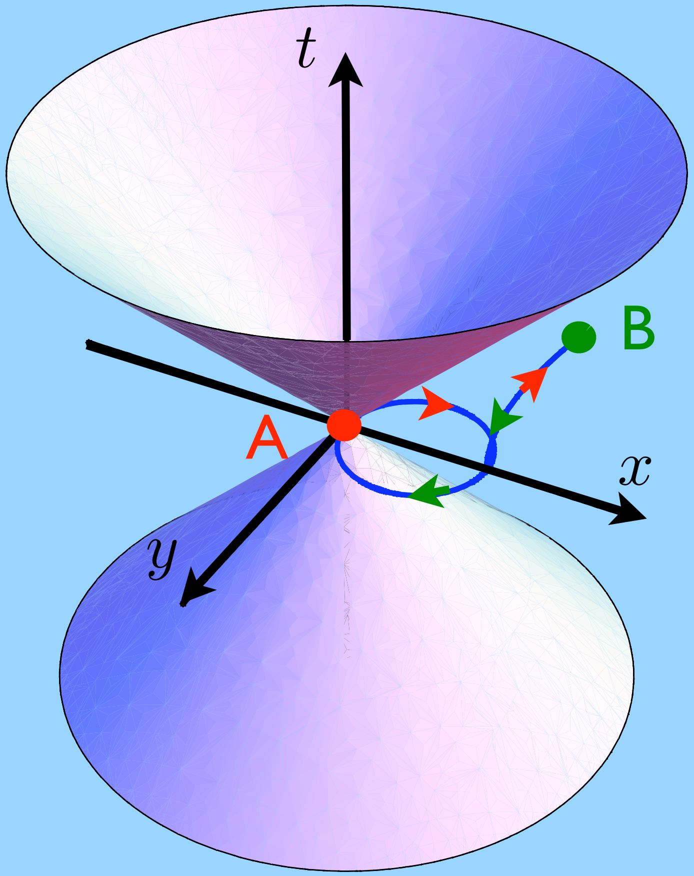
New theoretical opportunities: bouncing cosmologies,...

Tensions with causality and physics of horizons

High school argument why superluminal signals are impossible



High school argument why superluminal signals are impossible

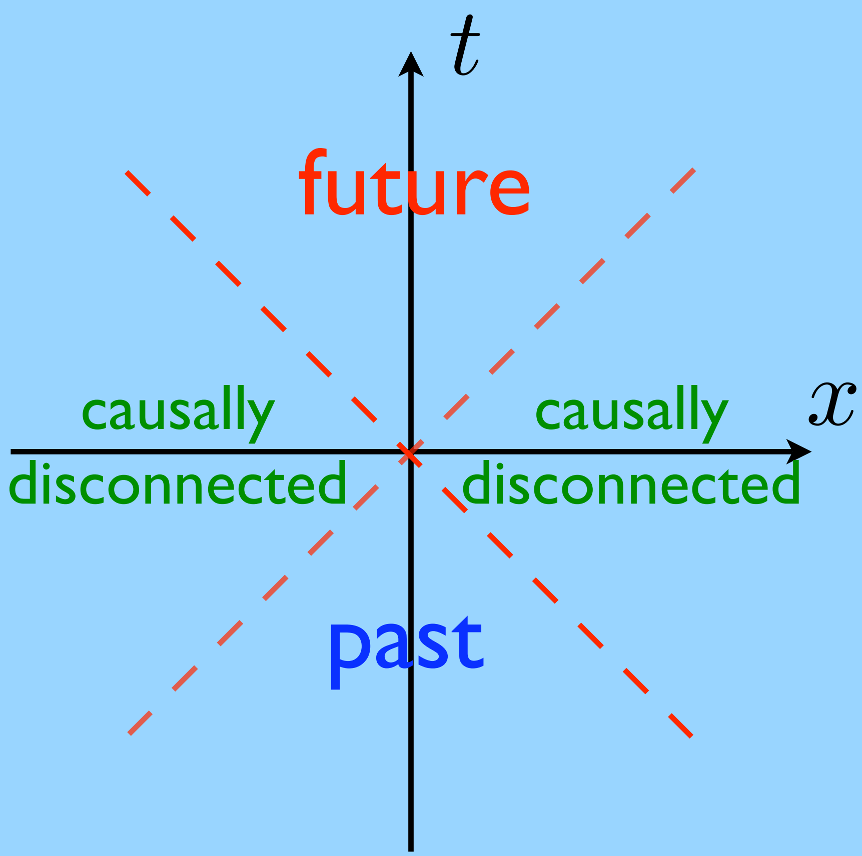


doesn't work in (1+1)d if

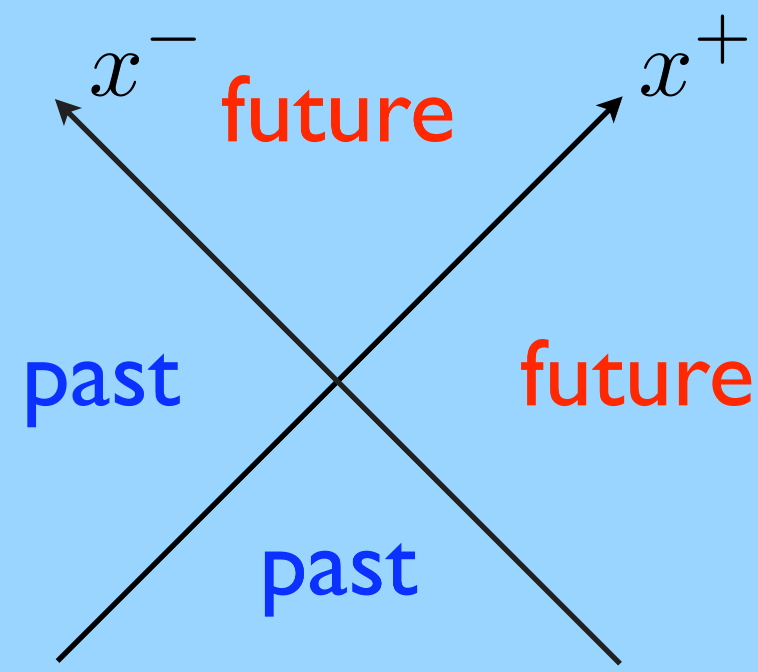
$$x \longrightarrow -x$$

is broken

Equivalently, in (1+1)d one has *two* causal structures compatible with the Poincare group



standard



“instantaneous”

Are there QFT’s with instantaneous causal structure?

Straightforward way...

$$S = -\int d^2x (\partial_- A_+)^2$$

No propagating degrees of freedom, c.f. grav. potential in Newton's theory of gravity

A bit less trivial model:

$$S = \int d^2x \left\{ -(\partial_- A_+)^2 + (\partial_+ A_-)^2 - m^2 (A_+ A_-)^2 \right\}$$

NB: "Wick rotation" $x^+ \to -ix^+$ *gives positive Euclidean action*

We ended up studying a bit different class of models.

One reason:

It seems hard to keep instantaneous effects small...

conventional massive field:

$$\partial_+\partial_-\phi = m^2\phi$$

constant x^+- surfaces are not good Cauchy slices

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The trick is to get a vector with a vev $\langle V^+\rangle \neq 0$

$$S = \int d^2x(\partial\phi)^2 - m^2\phi^2 + (V^+\partial_+\phi)^2$$

$SO(1,1)$ *nonlinear sigma-model*

aka Einstein-aether theory
aka (Lorentzian) nematic liquid crystal

$$\int d^2x \left(-\alpha_1 \partial_\mu V^\nu \partial^\mu V_\nu - \alpha_2 \partial_\mu V^\mu \partial_\nu V^\nu - \alpha_3 \partial_\mu V^\mu \epsilon^{\nu\lambda} \partial_\nu V_\lambda + \lambda(V^\mu V_\mu - 1)\right)$$

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$$V_\pm = \frac{1}{\sqrt{2}} \mathrm{e}^{\mp\psi}$$

$$\int dx_+ dx_- \left\{ \frac{1}{g^2} \partial_+ \psi \partial_- \psi + \frac{\beta_+}{2g^2} (\partial_+ \psi)^2 \mathrm{e}^{2\psi} + \frac{\beta_-}{2g^2} (\partial_- \psi)^2 \mathrm{e}^{-2\psi} \right\}$$

Three cases:

$$\beta_+ = \beta_- \; , \quad \beta_+ = -\beta_- \; , \quad \text{or } \beta_- = 0$$

$$\beta_+ = -\beta_-$$

$$S = \int dx_+ dx_- \left\{ \frac{1}{g^2} \partial_+ \psi \partial_- \psi + \frac{\beta}{2g^2} \left((\partial_+ \psi)^2 \mathrm{e}^{2\psi} - (\partial_- \psi)^2 \mathrm{e}^{-2\psi} \right) \right\}$$

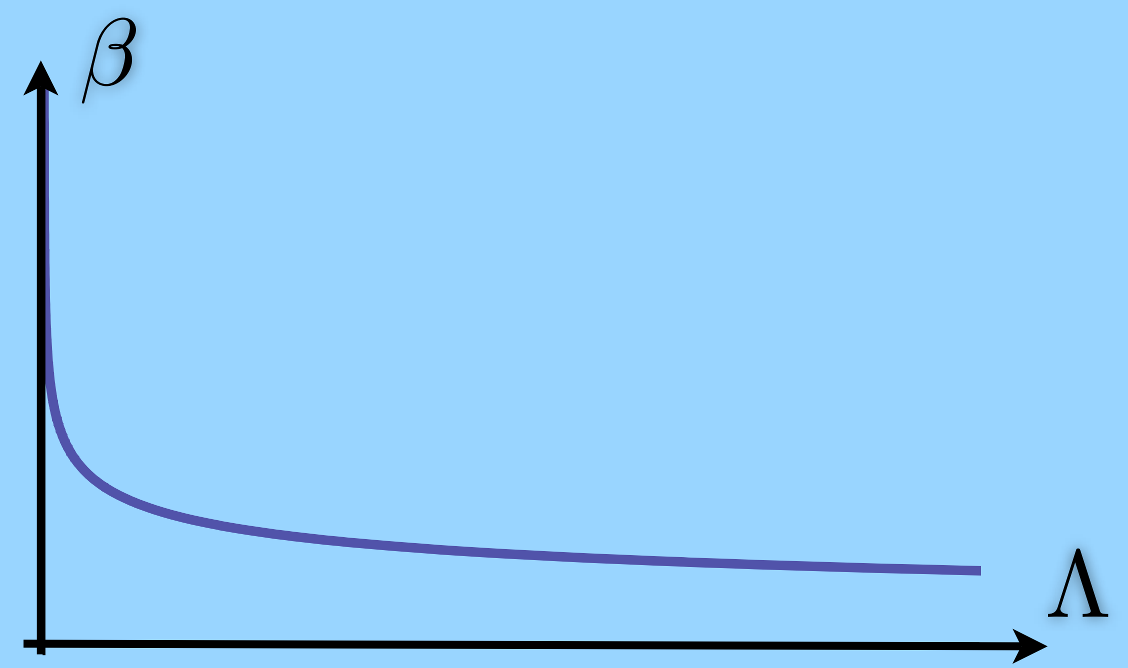
- $\psi(x^+, x^-) \to \psi(\mathrm{e}^{\gamma} x^+, \mathrm{e}^{-\gamma} x^-) + \gamma$
- Renormalizable
- $x^+ \to -i x^+$ gives $\mathrm{Re}(S_E) > 0$
- Positive definite Hamiltonian
- Asymptotically free

$$S = \int dx_+ dx_- \left\{ \frac{1}{g^2} \partial_+ \psi \partial_- \psi + \frac{\beta}{2g^2} \left((\partial_+ \psi)^2 e^{2\psi} - (\partial_- \psi)^2 e^{-2\psi} \right) \right\}$$

1-loop RGE (all orders in β):

$$g = const$$

$$\frac{d\beta}{d \log \Lambda} = -\frac{g^2 \beta}{\pi \sqrt{1 + \beta^2}}$$



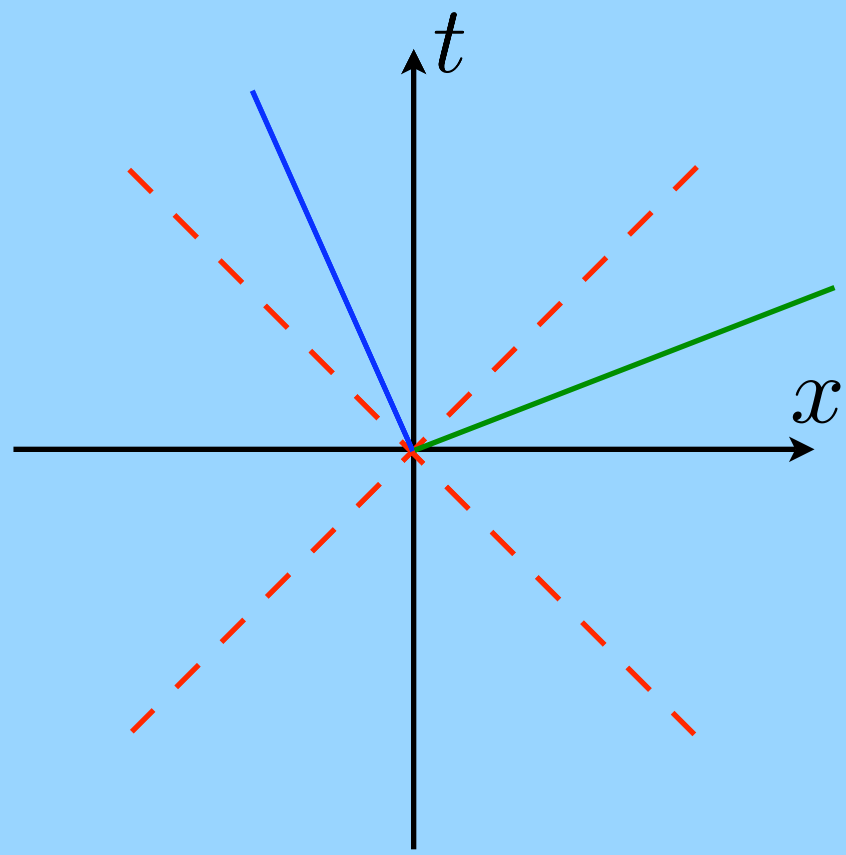
UV: $\frac{1}{g^2} \partial_+ \psi \partial_- \psi$

IR: $\frac{1}{2\varkappa^2} \left\{ (\partial_+ \psi)^2 e^{2\psi} - (\partial_- \psi)^2 e^{-2\psi} \right\}$

$$\varkappa^2 = \frac{g^2}{\beta} \to 0$$

Dispersion relation around $\psi = 0$

$$c_R = \beta + \sqrt{1+\beta^2}\ , \quad c_L = -\beta + \sqrt{1+\beta^2}$$



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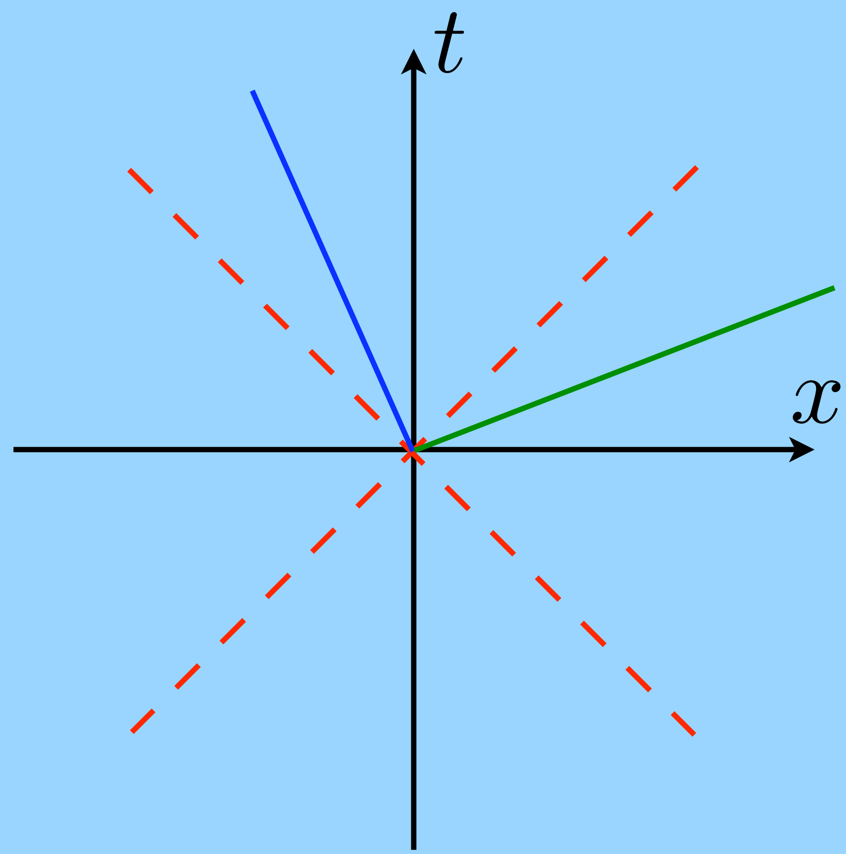
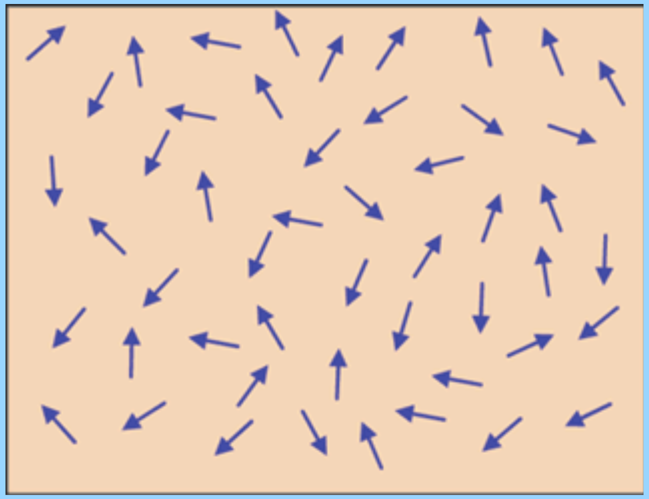


How does this agree with Coleman-Mermin-Wagner?



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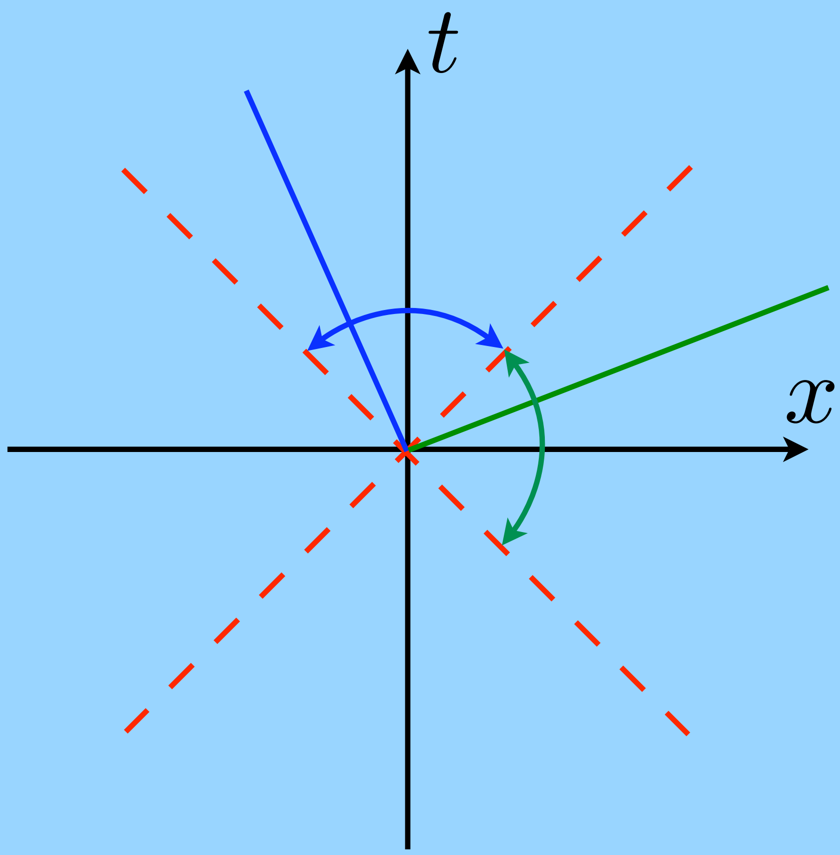

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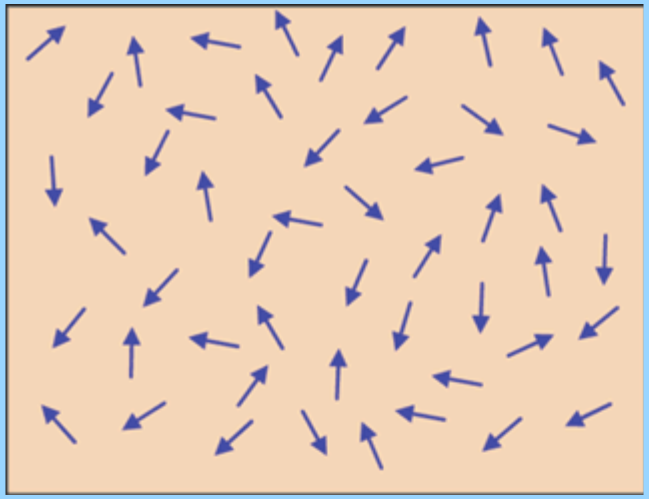

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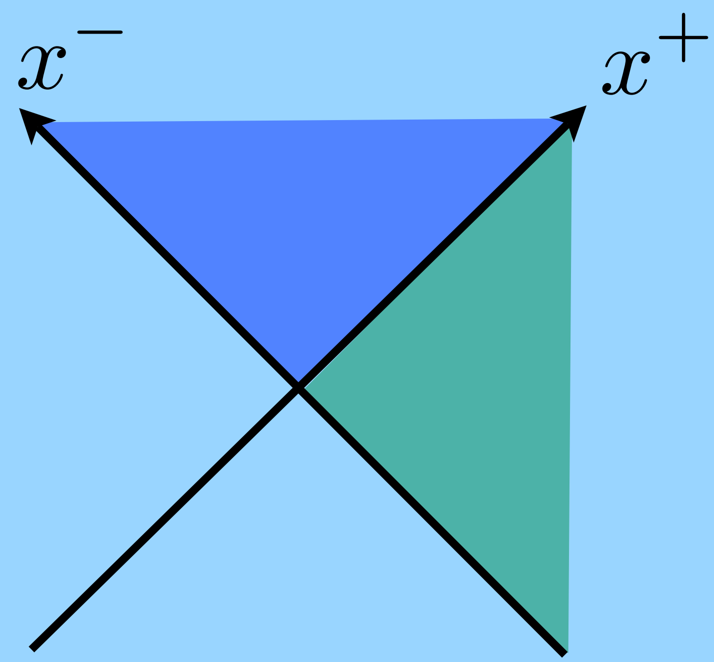

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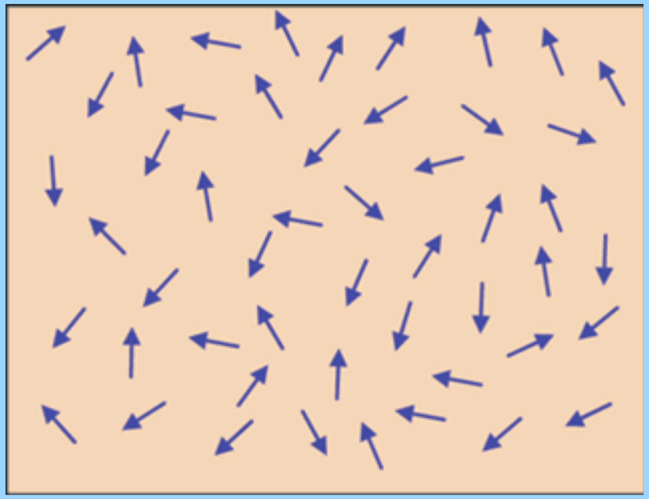


At long distances directions of the spin average out, but perturbation theory is a good guide for other properties

The following picture emerges:

- Lorentz symmetry gets restored at long distances, giving rise to the instantaneous causal structure. Most likely power law correlators (cf. Berezinsky-Kosterlitz-Thouless).

- At short distances instantaneous effects are small. One may be tricked into thinking that the causal structure is the standard one, unless precision measurements (or long enough observations) reveal "acausal" effects.

- To definitely confirm this picture and make sure everything makes sense non-perturbatively would be nice to have a solvable model...

$\beta_- = 0$

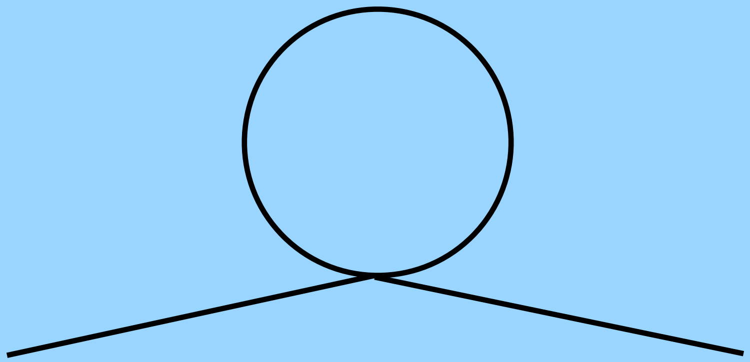
$$S = \int dx_+ dx_- \left\{ \pm \frac{1}{g^2} \partial_+ \psi \partial_- \psi + \frac{\beta}{2g^2} (\partial_+ \psi)^2 \mathrm{e}^{2\psi} \right\}$$

- β can be changed by the shift of ψ
- half+ of the conformal group

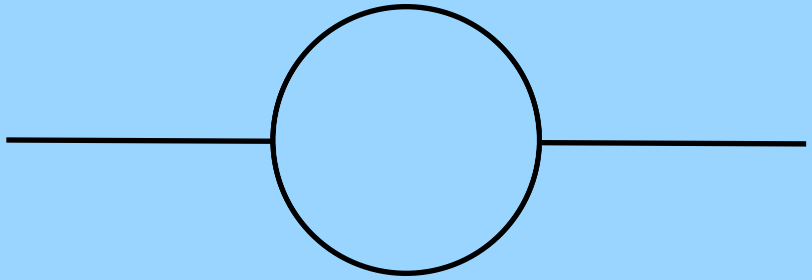
$$\psi(x^+, x^-) \to \psi(g(x^+), f(x^-)) + \frac{1}{2} \log f'(x^-) - \frac{1}{2} \log g'(x^+)$$

where f is arbitrary, $g = \dfrac{ax^+ + b}{cx^+ + d}$, $ad - bc = 1$

- all UV divergences can be removed by normal ordering



normal ordering removes ∞'s



normal ordering doesn't remove ∞'s

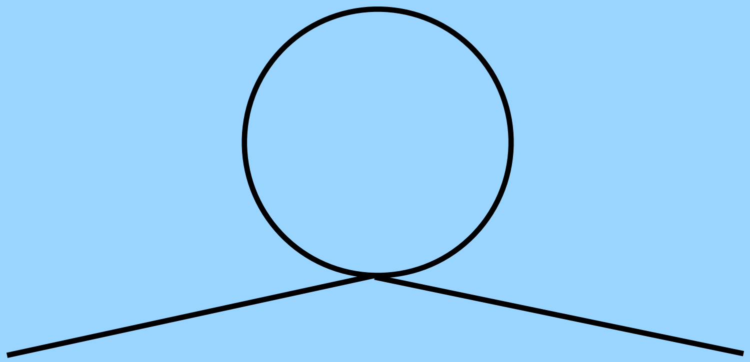
In (1+1)d:

$$\partial_+\psi\partial_-\psi - :U(\psi):$$

finite

$$\partial_+\psi\partial_-\psi - :U(\partial,\psi):$$

not finite



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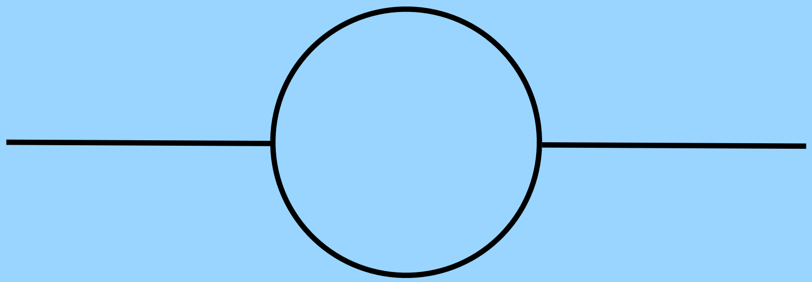

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In (1+1)d:

$$\partial_+\psi\partial_-\psi - :U(\psi):$$ finite

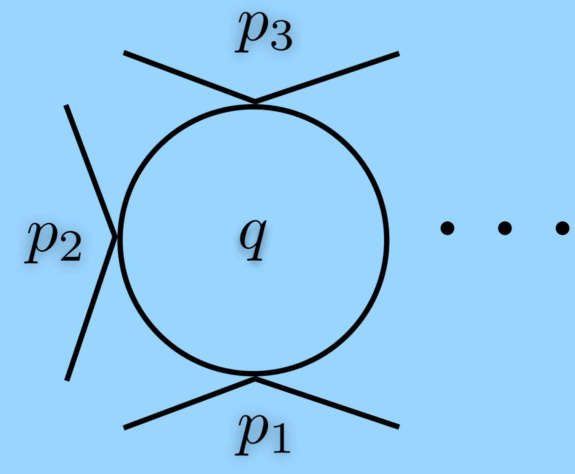
$$\partial_+\psi\partial_-\psi - :U(\partial,\psi):$$ not finite

$$\partial_+\psi\partial_-\psi - :U(\partial_+,\psi):$$ finite again!

e.g., a family of Lorentz invariant models:

$$\partial_+\psi\partial_-\psi - :U(\partial_+ \mathrm{e}^{\psi}):$$

one-loop example



$$\int \frac{d^2q\ q_+^n}{\prod_i \left((q+p_i)^2+\mu^2\right)} = \int_0^\Lambda dq\ q^n \int_0^{2\pi} \frac{d\phi\ \mathrm{e}^{in\phi}}{\prod_i \left(q^2+p_i^2+\mu^2+2qp\cos\left(\phi-\phi_i\right)\right)}$$

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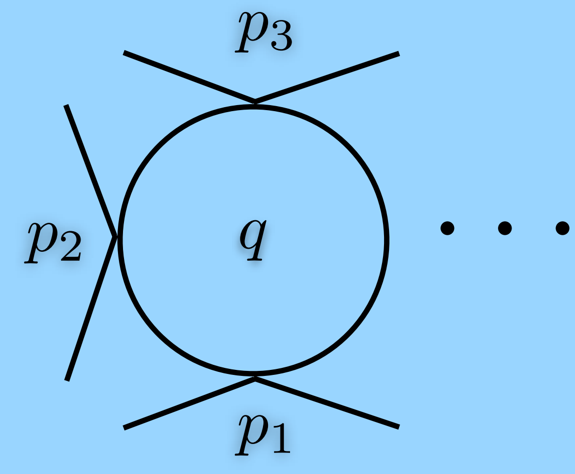

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one may hope to obtain non-perturbative information/solve the theory. work in progress...

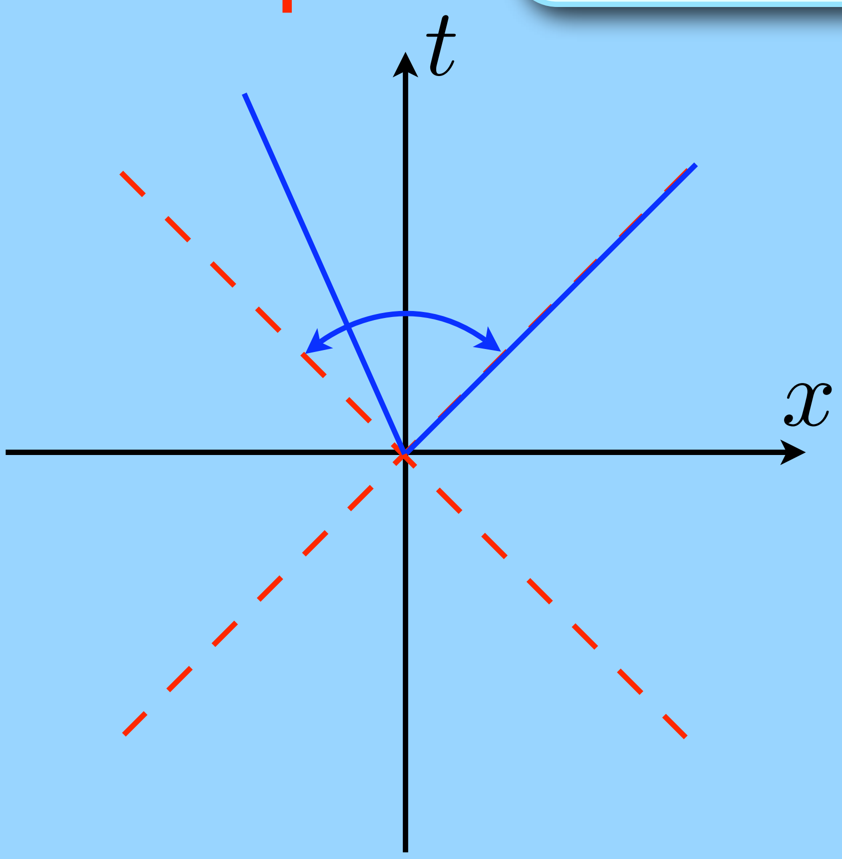
one consequence: $\pm\frac{1}{g^2}\partial_+\psi\partial_-\psi+\frac{\beta}{2g^2}(\partial_+\psi)^2\mathrm{e}^{2\psi}$

$g = const \qquad \beta = Z_\beta\tilde{\beta}$ --- to all orders

⇒ scale invariance is preserved at the quantum level

$$\pm\frac{1}{g^2}\partial_+\psi\partial_-\psi + \frac{\beta}{2g^2}(\partial_+\psi)^2 e^{2\psi}$$

“+”



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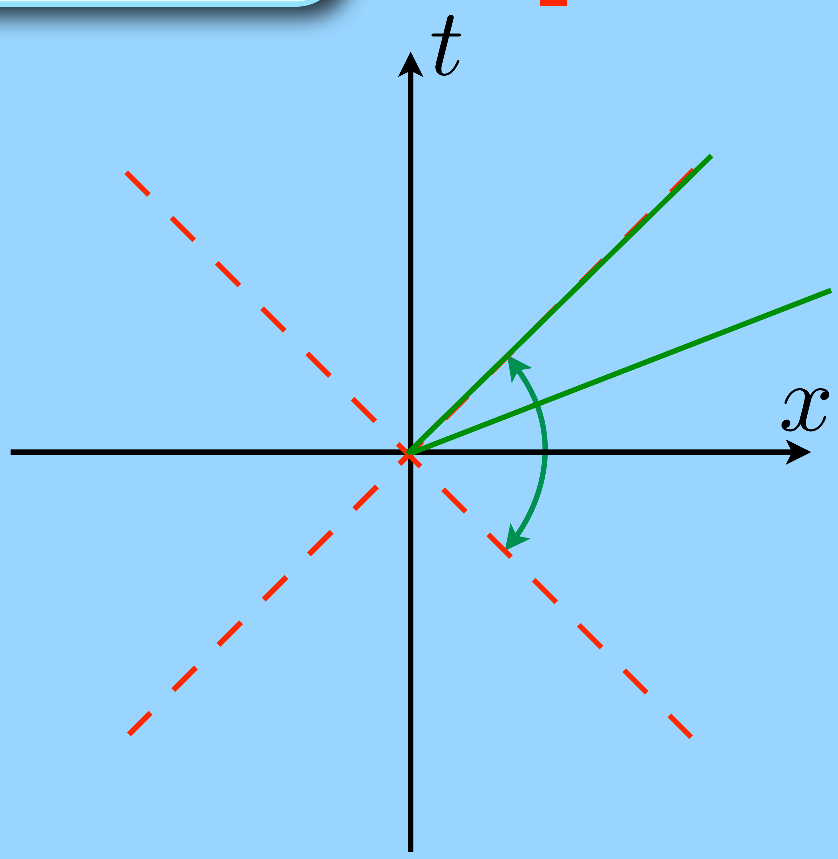
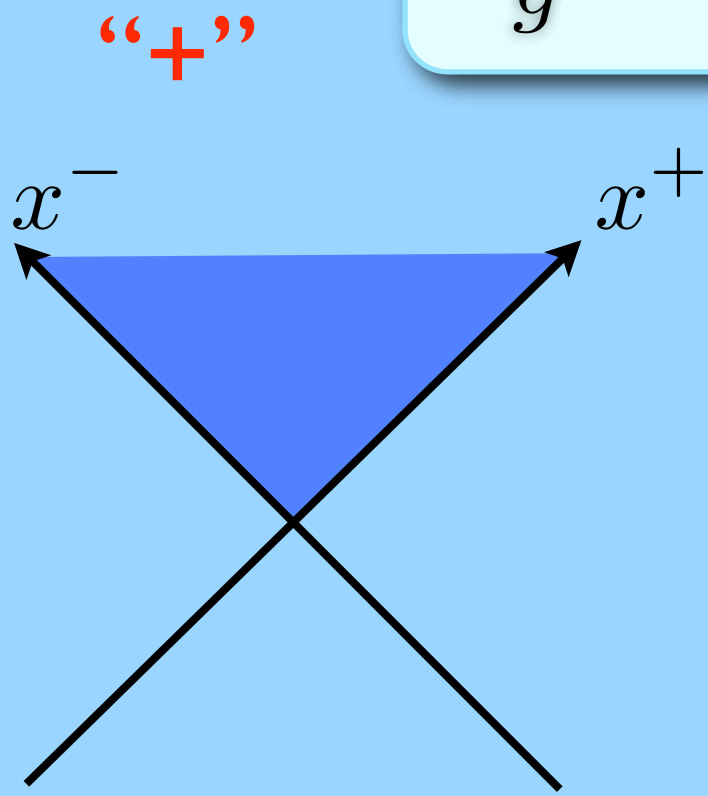
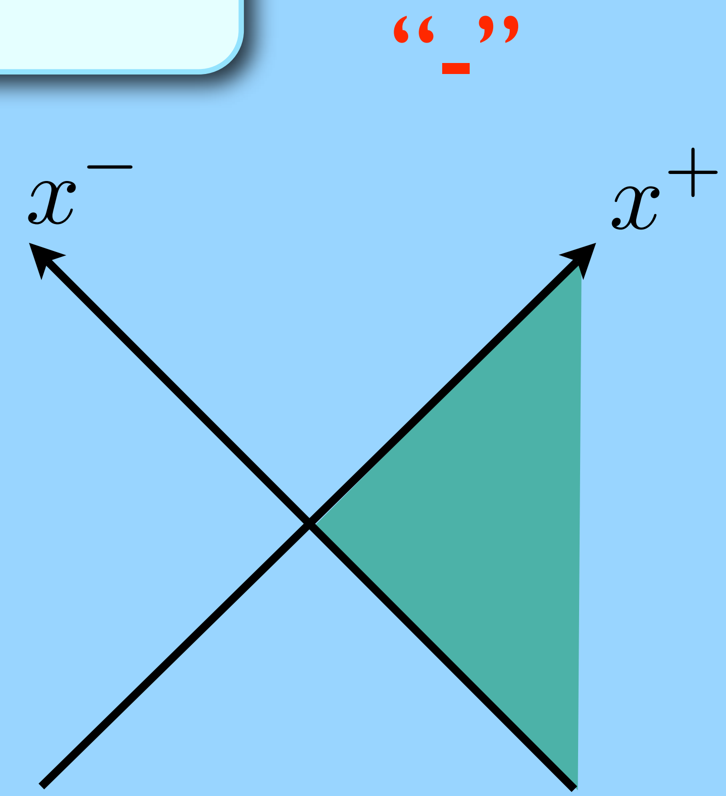


$$\pm\frac{1}{g^2}\partial_+\psi\partial_-\psi + \frac{\beta}{2g^2}(\partial_+\psi)^2 e^{2\psi}$$





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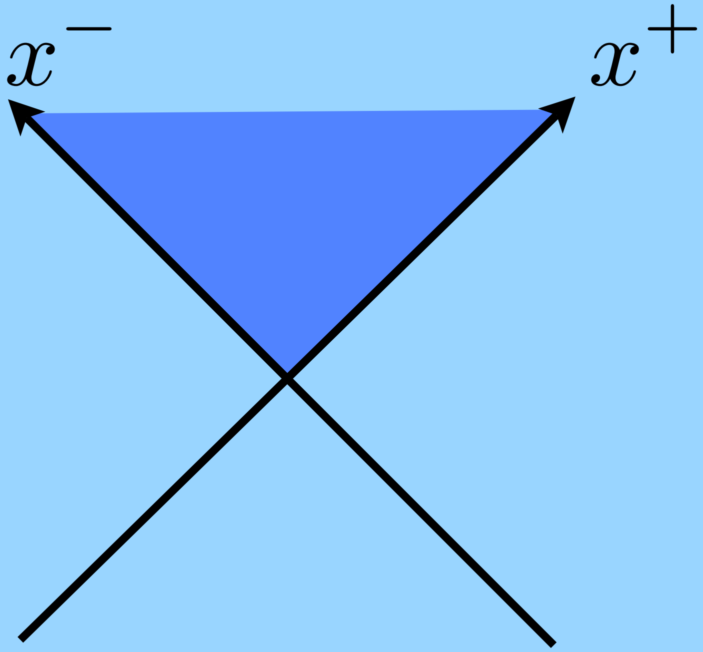
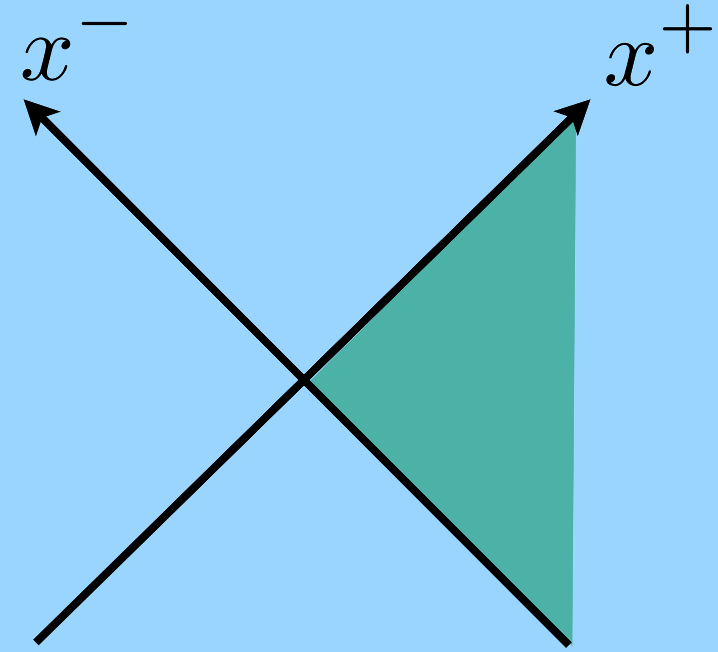
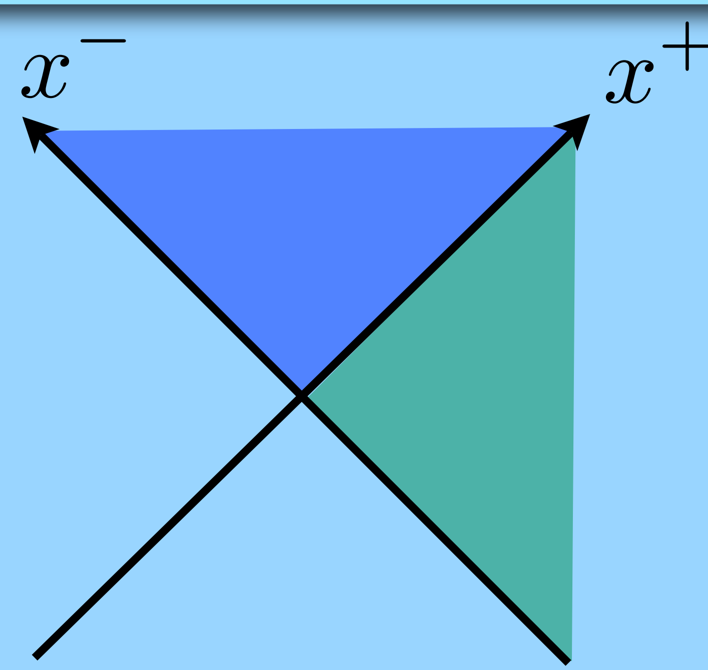
"+"



"-"



$$\frac{1}{g^2}(\partial_+\psi\partial_-\psi-\partial_+X\partial_-X)+\frac{\beta}{2g^2}(\partial_+\psi)^2\mathrm{e}^{2\psi}+\frac{\beta_X}{2g^2}(\partial_+X)^2\mathrm{e}^{2\psi}$$



Coupling to gravity: preliminary

$$S_{gr} = \frac{1}{2\pi\kappa}\int d^2x\sqrt{-g}R + S_{EA}(g_{\mu\nu}, V_\mu)$$

conformal gauge

$$g_{\mu\nu} = \mathrm{e}^{\phi}\eta_{\mu\nu}$$

$$V_- = \mathrm{e}^{\psi} \qquad V_+ = \mathrm{e}^{\psi-\phi}$$

$$\frac{1}{g^2}\partial_+\psi\partial_-(\psi-\phi) + \frac{\beta_+}{g^2}\mathrm{e}^{2\psi-\phi}(\partial_+\psi)^2 + \frac{\beta_-}{g^2}\mathrm{e}^{\phi-2\psi}\left(\partial_-(\psi-\phi)\right)^2 + S_{Liouv}(\phi)$$

What's next?

Two ways to look at these models:

(Sophisticated) toys for how non-local physics could look like. Coupling to gravity is clearly the next thing to study. Dilaton gravity in (1+1)d has black holes. An example of a *very radical* resolution of how information gets recovered from the black hole?

2d quantum gravity can be thought of as a world-sheet theory for a (non-critical) string. Is it possible to have a string theory (a theory of extended objects!) with instantaneous excitations on the world-sheet? If yes, will be a way to generate non-local theories in higher dimensions (UV complete Higgs phases of gravity?).

A sample action to start with:

$$S(\phi,\psi,X^i)=\int d^2x\left\{\frac{1}{g^2}\left[\partial_+\psi\partial_-(\psi-\phi)-\partial_+X^i\partial_-X^i\right]+\frac{\beta}{2g^2}\mathrm{e}^{2\psi-\phi}\left[(\partial_+\psi)^2+(\partial_+X^i)^2\right]\right\}+S_L(\phi)$$

Both ways, the next question to ask is

Are there 2d CFT's with instantaneous causal structure?

If unitary instantaneous CFT's are there, what happens on the AdS side??

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Are there 2d CFT's with instantaneous causal structure?

If unitary instantaneous CFT's are there, what happens on the AdS side??

Overall conclusion: there are lots of miracles in 2d and instantaneous theories are one of them. Being conservative, one can be rather confident that even if some inconsistency is to be found, it won't be a stupid one and we will learn more both about QFT and gravity.

THANK YOU!