# Cosmic Censorship for Phantom Energy

Eugeny Babichev[⋆,†] Sergei Chernov[⋆,‡]
Vyacheslav Dokuchaev[⋆] Yury Eroshenko[⋆]

[⋆]Institute for Nuclear Research of the Russian Academy of Sciences, Moscow

[†]Astroparticle Centre, Universite Paris 7, France

[‡]P. N. Lebedev Physical Institute of the Russian Academy of Sciences, Moscow

QUARKS — 2008

# Main message

**There is a threat of black hole transformation into the naked singularity by accretion of phantom energy!**

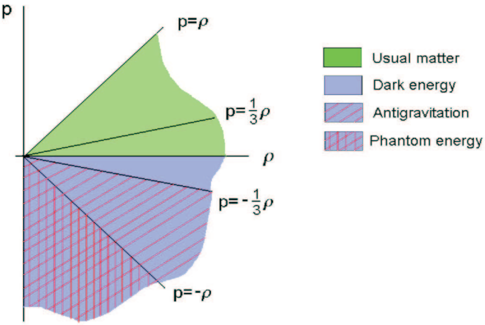
This idea was first proposed (but wrongly treated) by Madrid Jimenez and Gonzalez-Diaz arXiv:astro-ph/0510051

# Keywords and definitions

- Reissner-Nordsröm black hole $m, \;\; e$
- Kerr-Newman black hole $m, \;\; e, \;\; a = J/m$
- Event horizon $r_+ = m + \sqrt{m^2 - a^2 - e^2}$
- Cosmic censorship conjecture $a^2 + e^2 \leq m^2$

  *R. Penrose 1969*
- Dark energy $p < 0$
- Phantom energy $p + \rho < 0$

# Phase diagram




Equations of state:

| | |
|---|---|
| ultra-hard | $p = \rho$ |
| thermal photon gas | $p = \rho/3$ |
| dark energy | $p < 0$ |
| quintessence | $p < -\rho/3$ |
| vacuum | $p = -\rho$ |
| phantom energy | $p + \rho < 0$ |

We study the distribution of perfect fluid with an arbitrary equation of state $p = p(\rho)$ in the gradational field of black holes and naked singularities. In particular, we find the analytical solution for steady-state accretion of a test perfect fluid onto the Reissner-Nordström black hole. It is shown that electrically charged and/or rotating black holes evolve to a near extreme state by accretion of phantom fluid due to a gradual decrease of their mass. In contrast to the black hole case, a static atmosphere of fluid around electrically charged naked singularity is established with a zero accretion flux. In the case of a rotating naked singularity this atmosphere would be stationary due to the azimuthal frame-dragging. The specific feature of extreme black hole is a diverging of fluid energy density at the event horizon. This divergence is a manifestation of violation of perfect fluid approximation at the extreme case. We suppose that back reaction of fluid on the near extreme black hole prevents its transformation into the naked singularity in accordance with the cosmic censorship conjecture.

# Perfect fluid in the Reissner-Nordsröm metric

$$ds^2 = f\, dt^2 - \frac{1}{f}\, dr^2 - r^2(d\theta^2 + \sin^2\theta\, d\phi^2)$$

$$f \equiv 1 - \frac{2m}{r} + \frac{e^2}{r^2} = 0, \quad r = r_+ = m + \sqrt{m^2 - e^2}, \quad m^2 \geq e^2$$

$$T_{\mu\nu} = (\rho + p)u_\mu u_\nu - p g_{\mu\nu}, \quad u_\mu u^\mu = 1, \quad p = p(\rho)$$

For the Kerr-Newman case see report by S. Chernov at this Session

# Integrals of motion

- (I) Energy conservation (Bernoulli equation)
  $T^{\mu\nu}{}_{;\nu} = 0 \quad \Longrightarrow \quad (\rho + p)(f + u^2)^{1/2} r^2 u = C_1$
- (II) Energy flux conservation
  $u_\mu T^{\mu\nu}{}_{;\nu} = 0 \quad \Longrightarrow \quad u^\mu \rho_{,\mu} + (\rho + p) u^\mu_{;\mu} = 0$

$$ur^2 \exp\left[\int_{\rho_\infty}^{\rho} \frac{d\rho'}{\rho' + p(\rho')}\right] = -Am^2, \quad u = \frac{dr}{ds} < 0$$

- (I/II)

$$(\rho + p)(f + u^2)^{1/2} \exp\left[-\int_{\rho_\infty}^{\rho} \frac{d\rho'}{\rho' + p(\rho')}\right] = -\frac{C_1}{Am^2} = \rho_\infty + p(\rho_\infty)$$

- Energy flux (rate of black hole mass changing)

$$\dot{m} = -4\pi r^2 T_0{}^r = 4\pi Am^2[\rho_\infty + p(\rho_\infty)]$$

$$\dot{m} < 0 \quad \text{at} \quad \rho_\infty + p(\rho_\infty) < 0$$

# Fixation of integration constants

*Following to Michel 1972*

At the horizon $r = r_+$

$$A\,\frac{m^2}{r_+^2}\left[\frac{\rho_+ + p(\rho_+)}{\rho_\infty + p(\rho_\infty)}\right] = \exp\left[2\int_{\rho_\infty}^{\rho_+}\frac{d\rho'}{\rho' + p(\rho')}\right]$$

At the critical (sonic) point $r = r_*$

$$u_*^2 = \frac{mr_* - e^2}{2r_*^2}, \quad c_s^2(\rho_*) = \frac{mr_* - e^2}{2r_*^2 - 3mr_* + e^2}, \quad c_s(\rho)^2 = \frac{\partial p}{\partial \rho}$$

$$\frac{r_*}{m} = \frac{1+3c_{s*}^2}{4c_{s*}^2}\left\{1 \pm \left[1 - \frac{8c_{s*}^2(1+c_{s*}^2)}{(1+3c_{s*}^2)^2}\frac{e^2}{m^2}\right]^{1/2}\right\}, \quad c_{s*} = c_s(r_*)$$

Critical point exists if

$$\frac{e^2}{m^2} \le \frac{\left(1+3c_s^2\right)^2}{8c_s^2\left(1+c_s^2\right)}, \quad r_* \ge r_+, \quad e^2 \le m^2$$

# Solution for generalized linear equation of state

$p = \alpha(\rho - \rho_0)$, $\alpha = const > 0$, $\rho_0 = const$, $\rho \geq 0$, $p/\rho \equiv w \neq const$

Advantage for $p < 0$:

$$c_s^2 = \frac{\partial p}{\partial \rho} = \alpha > 0 \text{ – hydrodynamically stable}$$

$$c_s^2 = w = const < 0 \text{ – hydrodynamic instability}$$

$$A = \alpha^{1/2} \frac{r_*^2}{m^2} \left( \frac{2\alpha r_*^2}{m r_* - e^2} \right)^{\frac{1-\alpha}{2\alpha}}$$

$$f + u^2 = \left( -\frac{u x^2}{A} \right)^{2\alpha}, \quad \frac{\rho + p}{\rho_\infty + p(\rho_\infty)} = \left( -\frac{A}{u x^2} \right)^{1+\alpha}$$

Analytical solutions $u = u(r) < 0$, $\rho = \rho(r)$ and $p = p(r)$ at $\alpha = 1/4$, $1/2$, $2/3$, $1$ and $2$.

# Analytical example: $\alpha = 1/3$

**Thermal photon gas at $\rho_0 = 0$, phantom energy at $\rho_0 > 4\rho_\infty$**

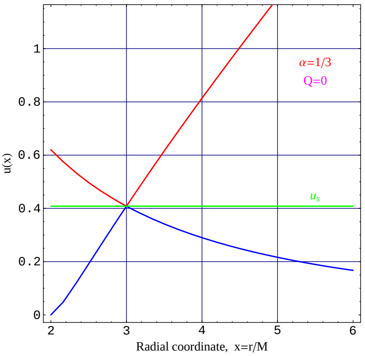
$$\rho = \frac{\rho_0}{4} + \left(\rho_\infty - \frac{\rho_0}{4}\right)\left(\frac{1+2z}{3f}\right)^2,$$

$$z = \begin{cases} \cos\dfrac{2\pi - \beta}{3}, & r_+ \le x \le r_*; \\[2ex] \cos\dfrac{\beta}{3}, & x > r_* \end{cases}$$

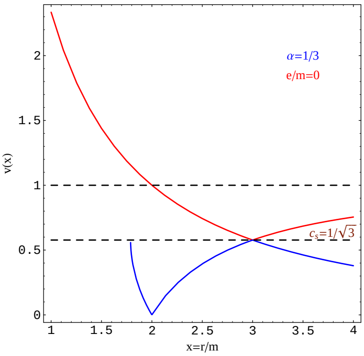
$$\beta = \arccos\left(1 - \frac{27}{2}A^2\frac{f^2}{x^4}\right), \quad x = r/m$$

# Two branches of hydrodynamic flow

## Radial 4-velocity $u(r)$

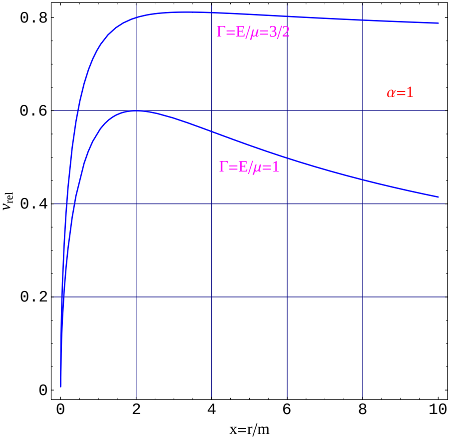



## Radial 3-velocity $v(r)$




Accretion flow of the thermal photon gas with $\alpha = c_s^2 = 1/3$, $r_* = 3m$ and $u_* = (1/6)^{1/2} \simeq 0.408$ onto the Schwarzschild black hole ($e = 0$).

# Relative radial velocity of accreting fluid




Relative radial 3-velocity of accreting ultra-hard fluid with respect to the local observer falling into black hole from infinity with initial Lorentz-factors $\Gamma = E/\mu = 1$ (i.e., at rest at $\infty$) and $\Gamma = E/\mu = 3$.

# Static atmosphere around naked singularity

## Linear equation of state

Reissner-Nordström naked singularity ($e^2 > m^2$)

$$\rho(r) = \frac{\alpha\rho_0}{1+\alpha} + \left(\rho_\infty - \frac{\alpha\rho_0}{1+\alpha}\right) f^{-\frac{1+2\alpha}{2\alpha}}.$$

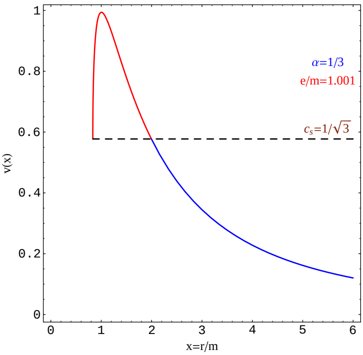
Stationary atmosphere around Kerr naked singularity ($a^2 > m^2, \;\; \alpha = 1$)

$$\left[\frac{\rho - \rho_0/2}{\rho_\infty - \rho_0/2}\right]^{1/2} = \frac{1}{r^2 + a^2\cos^2\theta}\left[\frac{(r^2+a^2)^2}{r^2 - 2mr + a^2} - a^2\sin^2\theta\right].$$

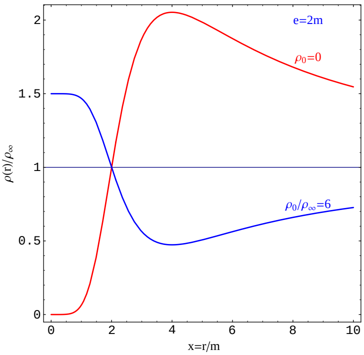
See for details report by S. Chernov at this Session

# Static atmosphere around naked singularity

## Terminal point



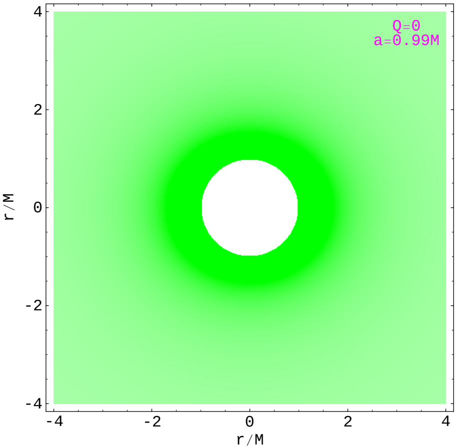

## Static atmosphere




Local radial 3- velocity $v(r)$ in the case of "stationary accretion" of thermal radiation ($\alpha = 1/3$) onto the Reissner-Nordström naked singularity with $e = 1.001m$. Solution of inflowing thermal radiation "terminates" at radius $r_{\rm b} = 0.832m$ (left panel).

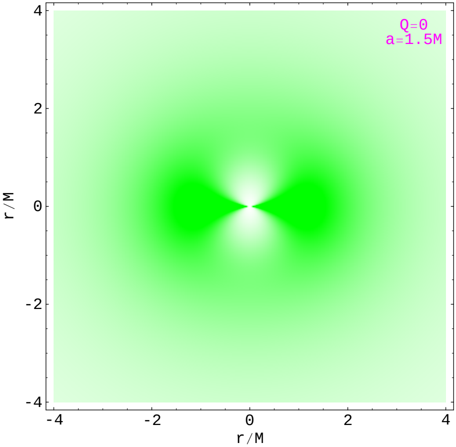
# Density distribution around Kerr black hole

*Petrich Shapiro Teukolsky 1988*



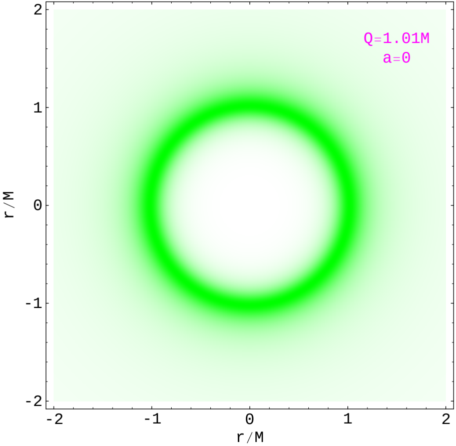

Stationary density distribution of ultra-hard fluid around the Kerr black hole (accretion flow)

# Density distribution around Kerr naked singularity



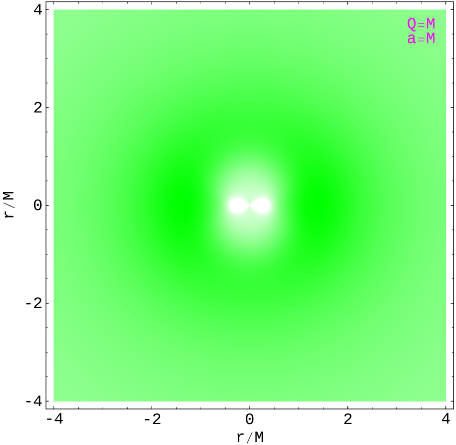

Stationary density distribution of ultra-hard fluid around the Kerr-Newman naked singularity (stationary atmosphere with a zero influx) The empty cavity around the naked singularity ($Q^2 + a^2 > M^2$) exists only in the case when both $Q \neq 0$ and $a \neq 0$.

## Density distribution around Reissner-Nordström naked singularuty



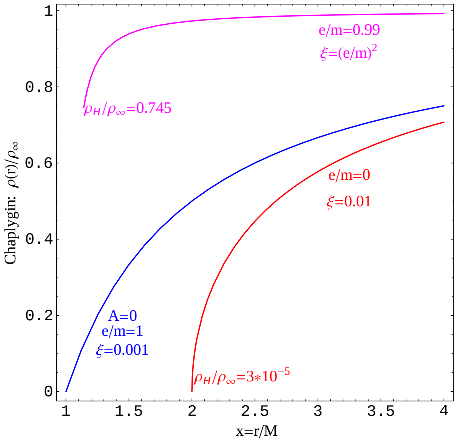

Stationary density distribution of ultra-hard fluid around the Kerr-Newman naked singularity (stationary atmosphere with a zero influx) The empty cavity around the naked singularity ($Q^2 + a^2 > M^2$) exists only in the case when both $Q \neq 0$ and $a \neq 0$.

## Density distribution around Kerr-Newman naked singularuty




Stationary density distribution of ultra-hard fluid around the Kerr-Newman naked singularity (stationary atmosphere with a zero influx) The empty cavity around the naked singularity ($Q^2 + a^2 > M^2$) exists only in the case when both $Q \neq 0$ and $a \neq 0$.

# Chaplygin gas around charged black hole




Energy density distribution

## Approaching to the extreme black hole state $a^2 + e^2 \to m^2$

**extreme parameter** $\epsilon \equiv (m^2 - e^2)/m^2 \leq 1$

$$e = const, \quad J = ma = const$$

Reissner-Nordström case

$$\int_0^{t_{\rm NS}} \dot{m}\, dt = e - m(0), \quad t_{\rm NS} = \frac{q^3 - 3q^2 + 2 - 2(1-q^2)^{3/2}}{3q^4}\,\tau$$

$$q = e/m(0), \quad \tau = -\{4\pi[\rho_\infty + p(\rho_\infty)]m(0)\}^{-1}$$

Kerr case

$$\int_0^{t_{\rm NS}} \dot{m}\, dt = \sqrt{J} - m(0), \; t_{\rm NS} = \frac{1}{6\tilde{a}^{1/2}}\left[1 - \frac{1-\sqrt{1-\tilde{a}^2}}{\tilde{a}^{3/2}} + 2F(\frac{1}{2}\arccos\tilde{a}, 2)\right]\tau$$

$\tilde{a} = J/m^2(0)$, $F(\phi, k)$ — elliptic integral of the first kind

Time $t_{\rm NS}$ is finite in test fluid approximation!

However, test fluid approximation may be violated at extreme case!

## Violation of the test fluid approximation at $a^2 + e^2 \to m^2$

For $p = \rho$

$$\left[\frac{\rho_+ - \rho_0/2}{\rho_\infty - \rho_0/2}\right]^{1/2} = \frac{1}{r_+^2 + a^2\cos^2\theta}\left(\frac{4r_+^2 m}{\sqrt{m^2 - a^2}} - a^2\sin^2\theta\right) \to \infty$$

at $m \to a$

$$\rho_+ = \rho(r_+) = \frac{\rho_0}{2} + \left(\rho_\infty - \frac{\rho_0}{2}\right)\frac{2r_+}{\sqrt{m^2 - e^2}} \to \pm\infty$$

at $m \to e$

Energy density is diverging in general at the extreme case!?

# Conclusions and questions

- Analytical solution for steady-state accretion of a test perfect fluid onto the Reissner-Nordström black hole
- Electrically charged and/or rotating black holes evolve to a near extreme state by accretion of phantom fluid
  $e^2 + a^2 \to m^2$
- Static atmosphere of fluid around electrically charged naked singularity $e^2 > m^2$
- Stationary atmosphere (frame-dragging) around a rotating naked singularity with a zero accretion flux $a^2 + e^2 > m^2$
- Divergence of fluid energy density at the event horizon of extreme black hole!? $a^2 + e^2 \to m^2, \;\; \rho(r_+) \to \infty$
- Violation of test fluid approximation at the extreme case!?
  $a^2 + e^2 = m^2$
- Back reaction of fluid on the near extreme black hole prevents its transformation into the naked singularity in accordance with the cosmic censorship conjecture!? $a^2 + e^2 \leq m^2$