New results on gauge-invariant TMD PDFs in QCD

I. Cherednikov, N. Stefanis

(BLTP, JINR, Dubna and TP2, Ruhr-U., Bochum)

QUARKS'2008

Renormalization properties are used as a principal tool of analysis of the completely gauge-invariant definition of the transverse-momentum dependent parton distribution functions. The UV anomalous dimension is calculated in the one-loop order in the light-cone gauge and the consistent treatment of the additional singularities, which produce undesirable contributions in the anomalous dimensions, is discussed. The generalized renormalization of TMD PDF based on the renormalization procedure for the Wilson exponentials with obstructions (cusps or intersections) is proposed. The reduction of the re-defined TMD PDF to the integrated PDF's is also considered.

- **Integrated PDF's:** definition; gauge invariance; RG properties

- **Unintegrated (TMD) PDF's**: straightforward definition; shortcomings (complete gauge invariance, undesirable divergences)

- **Looking for a solution:** saving gauge invariance; factorization; reduction to the integrated case

- Towards the **"completely correct"** TMD PDF's: anomalous dimensions sum rule as a starting point; calculation of AD in the light-cone gauge; generalized renormalization and cancelation of extra divergences

- Conclusions and outlook

Integrated PDF's: definition; gauge invariance; RG properties

$$q_{i/h}(x) = \frac{1}{2}\int \frac{d\xi^-}{2\pi}\, \mathrm{e}^{-ik^+\xi^-} \langle h(P)|\bar{\psi}_i(\xi^-,\mathbf{0}_\perp)\gamma^+\psi_i(0^-,\mathbf{0}_\perp)|h(P)\rangle$$

Gauge invariance is saved by the insertion of the **gauge link**:

$$[y,x|\Gamma] = \mathcal{P}\exp\left[-ig\int_{x[\Gamma]}^{y} dz_\mu A_a^\mu(z)t_a\right]$$

so that

$$\hat{q}_{i/h}(x) = \frac{1}{2}\int \frac{d\xi^-}{2\pi}\, \mathrm{e}^{-ik^+\xi^-} \langle h(P)|\bar{\psi}_i(\xi^-,\mathbf{0}_\perp)[\xi^-,0^-]\gamma^+\psi_i(0^-,\mathbf{0}_\perp)|h(P)\rangle$$

Renormalization properties are described by the DGLAP equation.

Unintegrated (TMD) PDF's

"Naive" definition:

$$f_i(x, \boldsymbol{k}_\perp) = \frac{1}{2} \int \frac{d\xi^- d^2\xi_\perp}{2\pi(2\pi)^2} \; \mathrm{e}^{-ik^+\xi^- + ik_\perp \cdot \xi_\perp} .$$

$$\cdot \langle p | \bar{\psi}_i(\xi^-, \xi_\perp)[\xi^-, \boldsymbol{\xi}_\perp; \infty^-, \boldsymbol{\xi}_\perp;]^\dagger \gamma^+ [\infty^-, \mathbf{0}_\perp; 0^-, \mathbf{0}_\perp] \psi_i(0^-, 0_\perp) | p \rangle \; |_{\xi^+=0}$$

Formally:

$$\int d^2k_\perp f_i(x, \boldsymbol{k}_\perp) = q_i(x)$$

However: this definition suffers from several shortcomings.

- **extra divergences** associated with light-cone gauge, or light-like Wilson lines (*in the integrated case, these divergences cancel*)
- **gauge invariance** is not complete: in the light-cone gauge, dependence on the pole prescription in the gluon propagator still takes place
- reduction to the integrated case: formal integration doesn't produce correct result

Looking for the solution:

- **gauge invariance** is completely restored by means of the additional transverse Wilson path integral at the light-cone infinity (`Belitsky, Ji, Yuan`). This gauge link contribute only in the light-cone gauge, and cancel pole-prescription dependence

- **extra divergences** can be avoided by using non-light-like gauge connectors in the covariant gauges (`Collins, Soper`), or non-light-cone axial gauge. Thus, additional variable introduced (rapidity cutoff); calculations become more complicated; problems with factorization.

- **generalized renormalization** for the light-like Wilson lines (`Collins, Hautmann`): extra divergences cancel by the additional "soft" factor, defined by the vacuum average of special Wilson lines (demonstrated explicitly in the covariant gauge, in the 1-loop order)

Towards the “completely correct” definition:

- starting from the requirement of the **gauge invariance**, one formulates the anomalous dimensions sum rule:

- calculate the **anomalous dimension** of TMD PDF in the light-cone gauge and identify extra divergences in terms of the *defect* of anomalous dimension

- perform the **generalized renormalization** of TMD PDF, in analogue to the renormalization of the Wilson contours with cusps or self-intersection

In the **tree** approximation, the TMD PDF reads

$$f^{(0)}(x, k_\perp) =$$

$$= \frac{1}{2} \int \frac{d\xi^- d^2\xi_\perp}{2\pi(2\pi)^2} \mathrm{e}^{-ik^+\xi^- + ik_\perp \cdot \xi_\perp} \langle p | \bar{\psi}(\xi^-, \xi_\perp) \gamma^+ \psi(0^-, 0_\perp) | p \rangle =$$

$$= \delta(1 - x) \delta^{(2)}(\boldsymbol{k}_\perp)$$

The one-gluon exchanges, contributing to the UV-divergences, are described by the diagrams:

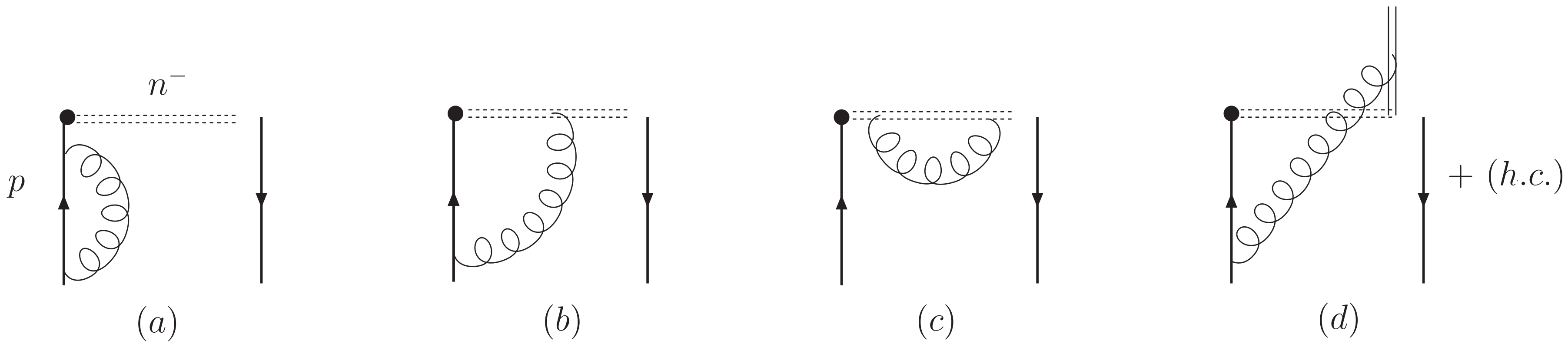

Figure 1: One-gluon exchanges for the TMD PDF: diagrams producing UV divergences. Only (a) and (d) contribute in the light-cone gauge

Source of the uncertainties and extra divergences: pole in the gluon propagator

$$D^{\mu\nu}_{\rm LC}(q) = \frac{1}{q^2}\left[g^{\mu\nu} - \frac{q^\mu n^{-\nu}}{[q^+]} - \frac{q^\nu n^{-\mu}}{[q^+]}\right]$$

Possible pole prescriptions:

$$d^{\mu\nu}_{\rm PV}(q) = -(q^\mu n^{-\nu} + q^\nu n^{-\mu})\frac{1}{2}\left(\frac{1}{q^+ + i\eta} + \frac{1}{q^+ - i\eta}\right)$$

$$d^{\mu\nu}_{\rm Adv/Ret}(q) = -(q^\mu n^{-\nu} + q^\nu n^{-\mu})\frac{1}{q^+ \mp i\eta}$$

In what follows, we keep η small, but finite. Dimensional regularization is used to control UV singularities.

The **UV divergent** part read:

$$\Sigma_{\rm left}^{UV}(p,\alpha_s;\epsilon) =$$

$$= -\frac{\alpha_s}{\pi} c_F \, \frac{1}{\epsilon} \left[-\frac{3}{4} - \ln \frac{\eta}{p^+} + \frac{i\pi}{2} + i\pi \; C_\infty \right] + \alpha_s c_F \; \frac{1}{\epsilon} \left[i C_\infty \right]$$

prescription dependence is canceled

$$C_\infty = \begin{cases} \quad 0 \ , \ \text{Advanced} : \frac{1}{[q^+]} = \frac{1}{q^+ - i\eta} \\ -1 \ , \ \text{Retarded} : \frac{1}{[q^+]} = \frac{1}{q^+ + i\eta} \\ -\frac{1}{2} \ , \ \text{PrincipalValue} : \ \frac{1}{[q^+]} = \frac{1}{2} \left(\frac{1}{q^+ - i\eta} + \frac{1}{q^+ + i\eta} \right) \end{cases}$$

Taking into account $(h.c.)$ contributions, one gets total real UV divergent part:

$$\Sigma_{\rm tot}(p, \alpha_s(\mu); \epsilon) = \Sigma_{\rm left} + \Sigma_{\rm right} =$$
$$= -\frac{\alpha_s}{4\pi} c_F \, \frac{2}{\epsilon} \left(-3 - 4 \ln \frac{\eta}{p^+} \right)$$

Anomalous dimension is defined as

$$\gamma = \frac{1}{2} \frac{1}{Z^{(1)}} \mu \frac{\partial \alpha_s(\mu)}{\partial \mu} \frac{\partial Z^{(1)}(\mu, \alpha_s(\mu); \epsilon)}{\partial \alpha_s}$$

and reads

$$\gamma_{\mathrm{LC}} = \gamma_{\mathrm{smooth}} - \delta\gamma \quad , \quad \gamma_{\mathrm{smooth}} = \frac{3}{4} \frac{\alpha_s}{\pi} c_F + O(\alpha_s^2)$$

defect of anomalous dimension

$$\delta\gamma = -\frac{\alpha_s}{\pi} c_F \, \ln \frac{\eta}{p^+}$$

contains undesirable p^+-dependent term which should be removed by a consistent procedure.

Note, that $\delta\gamma$ is nothing else, but the cusp anomalous dimension:

$$p^+ = (p \cdot n^-)$$

Renormalization of the Wilson operators with obstructions (cusps, self-intersections) requires additional renormalization factor depending on the cusp angle

$$Z_{p^+} = \left[\langle 0 | \mathcal{P} \exp \left[ig \int_{\Gamma_\pm} d\zeta^\mu \ \hat{A}^a_\mu(\zeta) \right] |0\rangle \right]^{-1}$$

Choose the integration path as follows

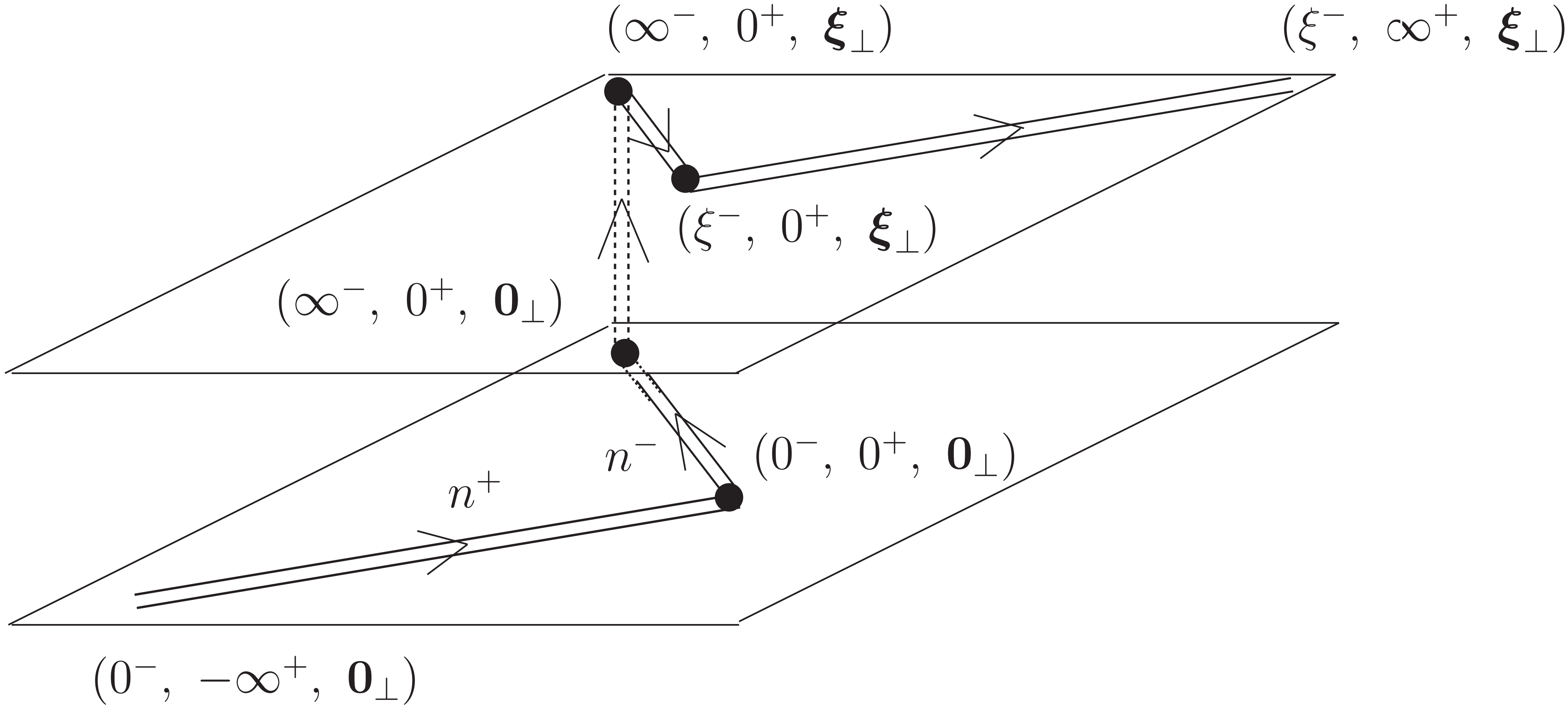

Figure 2: Space-time picture: integration trajectory for the additional cusp-dependent renormalization factor

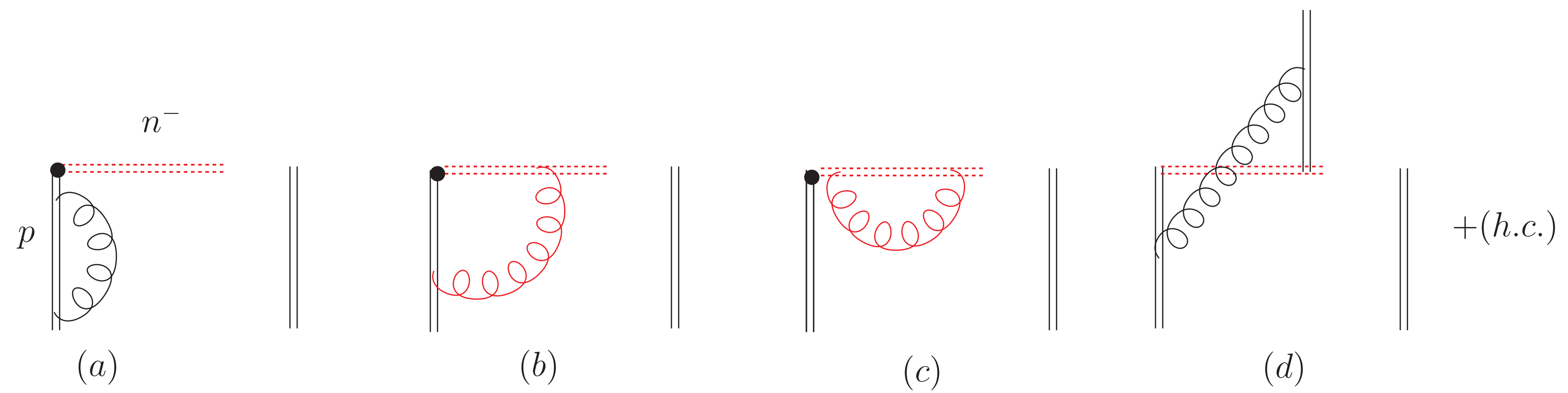

Figure 3: One-gluon exchanges for the generalized multiplicative renormalization factor

The generalized **renormalization constant** reads

$$\hat{Z}_{\rm GEN} = 1 + \frac{\alpha_s}{4\pi} c_F \, \frac{2}{\epsilon} \left(-3 - 4 \ln \frac{\eta}{p^+} + 4 \ln \frac{\eta}{p^+} \right) = 1 - \frac{3\alpha_s}{4\pi} c_F \, \frac{2}{\epsilon}$$

so that

$$\frac{1}{2} \mu \frac{d}{d\mu} \ln \hat{Z}_f(\mu, \alpha_s, p^+) = \frac{3}{4} \frac{\alpha_s}{\pi} c_F + O(\alpha_s^2)$$

i.e., equal to the anomalous dimension of the corresponding operator with the **smooth gauge connector**, according to the anomalous dimensions sum rule.

ADSR can be formulated in the following form

$$\mathbf{AD}\ \frac{1}{2}\int \frac{d\xi^- d^2\xi_\perp}{2\pi(2\pi)^2} \mathrm{e}^{-ik^+\xi^- + ik_\perp\cdot\xi_\perp}\ \langle p|\bar{\psi}(\xi)\gamma^+[\xi,0]_{\mathrm{direct\ link}}\psi(0)|p\rangle =$$

$$= \mathbf{AD}\ \frac{1}{2}\int \frac{d\xi^- d^2\xi_\perp}{2\pi(2\pi)^2} \mathrm{e}^{-ik^+\xi^- + ik_\perp\cdot\xi_\perp}$$

$$\cdot\langle p|\bar{\Psi}(\xi|\infty)\gamma^+\Psi(0|\infty)|p\rangle\cdot\Phi(p^+,n^-|0^-,\mathbf{0}_\perp)\Phi^\dagger(p^+,n^-|\xi^-,\boldsymbol{\xi}_\perp)$$

Generalized definition of TMD PDF:

$$
\begin{aligned}
& f_{q/q}^{\mathrm{mod}}\left(x, \boldsymbol{k}_\perp; \mu, \eta\right) = \\
& \frac{1}{2} \int \frac{d\xi^- d^2\boldsymbol{\xi}_\perp}{2\pi(2\pi)^2} \mathrm{e}^{-ik^+\xi^- + ik_\perp \cdot \xi_\perp} \left\langle q(p) \right| \bar{\psi}(\xi^-, \boldsymbol{k}_\perp) [\xi^-, \boldsymbol{\xi}_\perp; \infty^-, \boldsymbol{\xi}_\perp]^\dagger \\
& \times [\infty^-, \boldsymbol{\xi}_\perp; \infty^-, \boldsymbol{\infty}_\perp]^\dagger \gamma^+ [\infty^-, \boldsymbol{\infty}_\perp; \infty^-, \mathbf{0}_\perp][\infty^-, \mathbf{0}_\perp; 0^-, \mathbf{0}_\perp] \\
& \times \psi(0^-, \mathbf{0}_\perp) \left| q(p) \right\rangle \left[\Phi(p^+, n^- | 0^-, \mathbf{0}_\perp) \Phi^\dagger(p^+, n^- | \xi^-, \boldsymbol{\xi}_\perp) \right]
\end{aligned}
$$

Soft factor:

$$
\Phi(p^+, n^- | 0) = \left\langle 0 \middle| \mathcal{P} \exp \left[ig \int_{\mathcal{C}_{\mathrm{cusp}}} d\zeta^\mu \, t^a A_\mu^a(\zeta) \right] \middle| 0 \right\rangle
$$

$$
\Phi^\dagger(p^+, n^- | \xi) = \left\langle 0 \middle| \mathcal{P} \exp \left[-ig \int_{\mathcal{C}_{\mathrm{cusp}}} d\zeta^\mu \, t^a A_\mu^a(\xi + \zeta) \right] \middle| 0 \right\rangle
$$

Evolution equation:

$$\eta \frac{d}{d\eta} f_{q/q}^{\rm mod}(x, \boldsymbol{k}_\perp; \mu, \eta) = [\mathcal{K}(\mu) + \mathcal{G}(\mu)] \otimes f_{q/q}^{\rm mod}(x, \boldsymbol{k}_\perp; \mu, \eta)$$

$$\mathcal{K}(\mu) + \mathcal{G}(\mu) =$$

$$= \frac{\alpha_s}{\pi} C_{\rm F}\, \delta(1-x) \left[\delta^{(2)}(\boldsymbol{k}_\perp) \left(\ln \frac{\mu^2}{p^2} - \ln \frac{\mu^2}{\lambda^2} \right) - \frac{1}{\pi} \left(\frac{1}{\boldsymbol{k}_\perp^2 + x\lambda^2 - x(1-x)p^2} + \frac{1}{\boldsymbol{k}_\perp^2 + \lambda^2} \right) \right]$$

The renormalization-group behavior of the functions $\mathcal{K}(\mu)$ and $\mathcal{G}(\mu)$ is determined by the universal cusp anomalous dimension

$$\frac{1}{2} \mu \frac{d}{d\mu} \ln \mathcal{K}(\mu) = -\frac{1}{2} \mu \frac{d}{d\mu} \ln \mathcal{G}(\mu) = \gamma_{\rm cusp} = \frac{\alpha_s}{\pi} C_{\rm F} + O(\alpha_s^2)$$

Dependence on the dimensional regularization scale μ of the re-defined TMD PDF:

$$\frac{1}{2}\mu\frac{d}{d\mu}\ln f_{q/q}^{\mathrm{mod}}(x,\boldsymbol{k}_\perp;\mu,\eta) = \frac{3}{4}\frac{\alpha_s}{\pi}C_{\mathrm{F}} + O(\alpha_s^2)$$

Consistency equation:

$$\mu\frac{d}{d\mu}\left[\eta\frac{d}{d\eta}f_{q/q}^{\mathrm{mod}}(x,\boldsymbol{k}_\perp;\mu,\eta)\right] = 0$$

Reduction to the integrated PDF:

$$\int d^{\omega-2}\boldsymbol{k}_\perp f^{\rm mod}_{i/a}(x,\boldsymbol{k}_\perp;\mu,\eta) = f_{i/a}(x,\mu)$$

DGLAP equation

$$\mu\frac{d}{d\mu}f_{i/a}(x,\mu) = \sum_j \int_x^1 \frac{dz}{z}\, P_{ij}\left(\frac{x}{z}\right) f_{j/a}(z,\mu)$$

$$P_{ij}(x) = \frac{\alpha_s}{\pi}C_{\rm F}\ \left[\frac{3}{2}\delta(1-x) + \frac{1+x^2}{(1-x)_+}\right] + O(\alpha_s^2)$$

Conclusions

- The anomalous dimension of the TMD PDF in the **light-cone gauge** is calculated in 1-loop order.
- The generalized completely gauge invariant **definition of TMD PDF** is proposed.
- The **evolution equation** is derived.
- The direct connection to the **integrated PDF** is established.

I.Ch., N. Stefanis:

`Phys. Rev. D77 (2008) 094001;`

`Nucl. Phys. B (2008), in print`