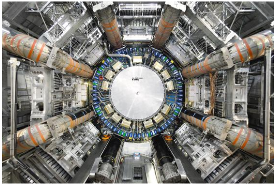
Non-minimal coupling in inflation and inflating with the Higgs boson



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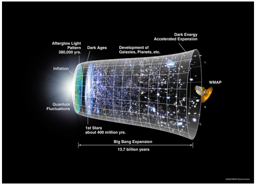

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based on F.B., M.Shaposhnikov, Phys. Lett. B **659**, 703 (2008)

Outline

1. Inflation—"standard" approach
   - Cosmological requirements
   - Large field chaotic inflation
2. Non-minimal coupling in $\lambda\phi^4$
   - The action
   - Conformal transformation
   - Large non-minimal coupling limit
   - Generic non-minimal coupling case
   - WMAP-5 allowed parameters
3. SM Higgs as the inflaton
   - Non-minimally coupled Standard Model
   - Radiative corrections—not (too) dangerous
   - Higgs mass
4. Conclusions

Outline

Cosmological implications

Problems in cosmology

- Flatness problem (at $T \sim M_P$ density was tuned $|\Omega - 1| \lesssim 10^{-59}$)
- Entropy of the Universe $S \sim 10^{87}$
- Size of the Universe (at $T \sim M_P$ size was $10^{29} M_P^{-1}$)
- Horizon problem

Solution

Inflation!

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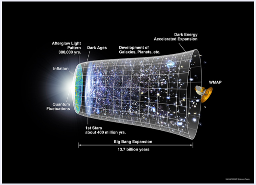
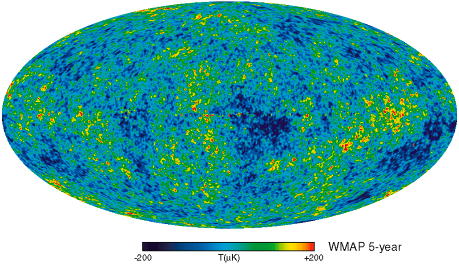
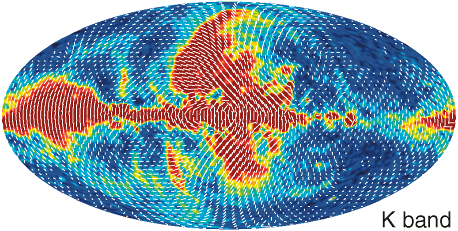


CMB

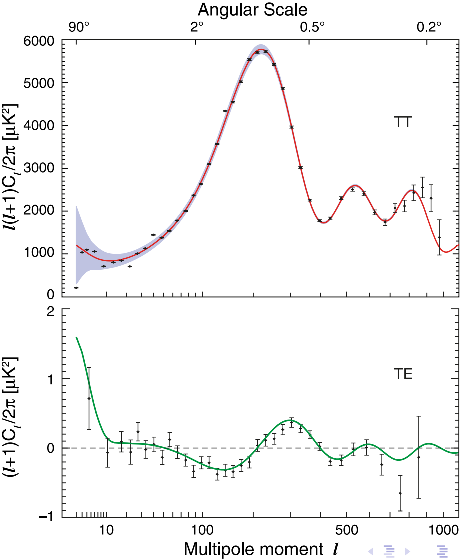
Temperature fluctuations



Polarization



CMB spectrum



$\lambda\phi^4$ inflation

One scalar field

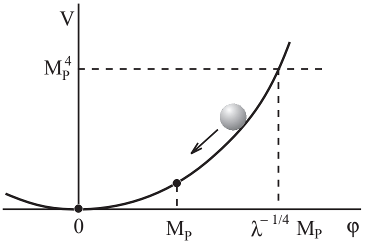
$$S = \int d^4x \left[\frac{\partial_\mu\phi\partial^\mu\phi}{2} - V(\phi)\right] , \qquad V(\phi) = \frac{\lambda}{4}\phi^4$$

Predicts primordial perturbation parameters

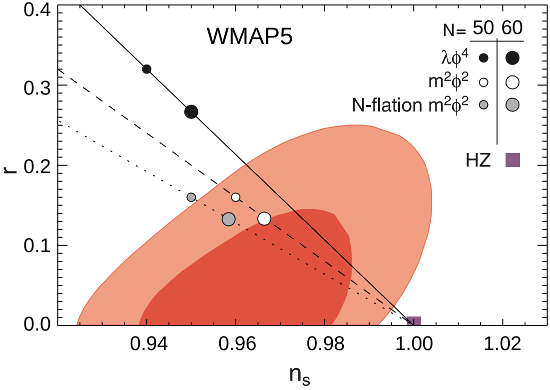
- COBE normalization
  $U/\varepsilon = (0.027 M_P)^4$

  $$\Rightarrow \quad \lambda \simeq 10^{-13}$$

- Spectral index $n_s = 0.95$
- Tensor/scalar ratio $r = 0.26$



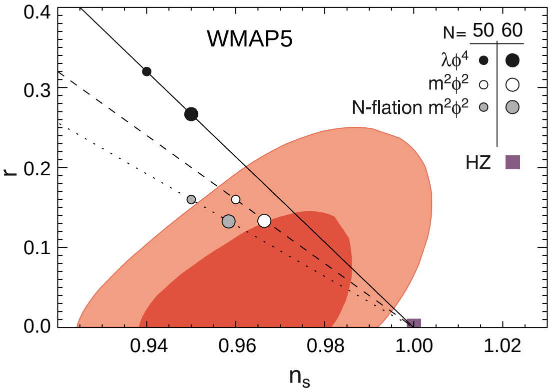
$\lambda\phi^4$ inflation predictions



Usual conclusion

$\lambda\phi^4$ is disfavoured

$\lambda\phi^4$ inflation predictions



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Possible operators in the model+gravity

- Dimension ≤ 4
- No new degrees of freedom (no higher derivatives)

$$S = \int d^4x \sqrt{-g} \left[-\frac{M_P^2}{2} R + \frac{\partial_\mu \phi \partial^\mu \phi}{2} - V(\phi) - \frac{\xi}{2}\phi^2 R + aR^2 + bR_{\mu\nu}R^{\mu\nu} + cR_{\mu\nu\lambda\rho}R^{\mu\nu\lambda\rho} + d\Box R \right]$$

- The non-minimally coupled term is in fact *required* by the renormalization properties of the theory in curved space-time background

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- The non-minimally coupled term is in fact *required* by the renormalization properties of the theory in curved space-time background

Non-minimally coupled scalar field—inflation

Quite an old idea

Add $\phi^2 R$ term to/instead of the usual $M_P R$ term in the gravitational action

- A.Zee'78, L.Smolin'79, B.Spokoiny'84
- D.Salopek J.Bond J.Bardeen'89

"Jordan frame" action

$$S_J = \int d^4x \sqrt{-g} \left\{ -\frac{M^2 + \xi\phi^2}{2} R + g_{\mu\nu} \frac{\partial^\mu \phi \partial^\nu \phi}{2} - \frac{\lambda}{4}\phi^4 \right\}$$

Conformal transformation

It is possible to get rid of the non-minimal coupling by the conformal transformation (field redefinition)

$$\hat{g}_{\mu\nu} = \Omega^2 g_{\mu\nu}\,, \quad \Omega^2 = 1 + \frac{\xi\phi^2}{M_P^2}$$

and also redefinition of the scalar field to make canonical kinetic term

$$\frac{d\hat{\phi}}{d\phi} = \sqrt{\frac{\Omega^2 + 6\xi^2\phi^2/M_P^2}{\Omega^4}} \implies \begin{cases} \phi \simeq \hat{\phi} & \text{for } \phi < M_P/\xi \\ 1 + \frac{\xi\phi^2}{M_P^2} \simeq \exp\left(\frac{2\hat{\phi}}{\sqrt{6}M_P}\right) & \text{for } \phi > M_P/\xi \end{cases}$$

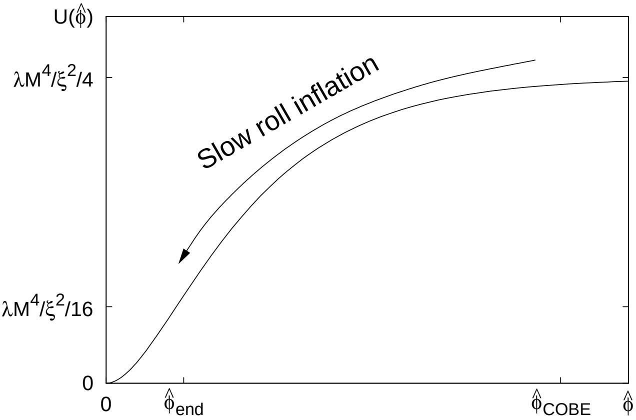
Resulting action (Einstein frame action)

$$S_E = \int d^4x \sqrt{-\hat{g}} \left\{ -\frac{M_P^2}{2}\hat{R} + \hat{g}_{\mu\nu}\frac{\partial^\mu\hat{\phi}\partial^\nu\hat{\phi}}{2} - \frac{1}{\Omega(\hat{\phi})^4}\frac{\lambda}{4}\phi(\hat{\phi})^4 \right\}$$

Case of large ξ

- Easy to analyse and is in fact the main case we will need for inflation in the Standard Model
- Generic ξ just interpolates between usual (minimal coupling) case and large ξ case.

Inflationary potential



For $\hat{\phi} \gtrsim M_P$:
$$U(\hat{\phi}) \simeq \frac{\lambda M_P^4}{4\xi^2}\left(1 - \exp\left(-\frac{2\hat{\phi}}{\sqrt{6}M_P}\right)\right)^2$$

Slow roll stage

$$\varepsilon = \frac{M_P^2}{2}\left(\frac{dU/d\hat{\phi}}{U}\right)^2 \simeq \frac{4M_P^4}{3\xi^2\phi^4} \simeq \frac{4}{3}e^{-\frac{4\hat{\phi}}{\sqrt{6}M_P}}$$

$$\eta = M_P^2\frac{d^2U/d\hat{\phi}^2}{U} \simeq \frac{4M_P^4}{3\xi^2\phi^4}\left(1-\frac{\xi\phi^2}{M_P^2}\right) \simeq \frac{4}{3}e^{-\frac{4\hat{\phi}}{\sqrt{6}M_P}}\left(1-e^{\frac{2\hat{\phi}}{\sqrt{6}M_P}}\right)$$

Slow roll ends at $\hat{\phi}_{\text{end}} \simeq M_P$ (or $\phi_{\text{end}} \simeq M_P/\sqrt{\xi}$)

Number of e-folds of inflation at the moment ϕ_N is $N \simeq \frac{6}{8}\frac{\phi_N^2-\phi_{\text{end}}^2}{M_P^2/\xi}$

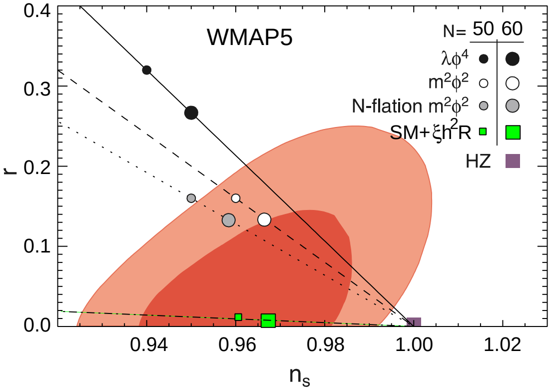
$$\hat{\phi}_{60} \simeq 5M_P$$

COBE normalization $U/\varepsilon = (0.027M_P)^4$ gives

$$\xi \simeq \sqrt{\frac{\lambda}{3}\frac{N_{\text{COBE}}}{0.027^2}} \simeq 49000\sqrt{\lambda}$$

Smallness of λ can be compensated by large ξ

CMB parameters—spectrum and tensor modes



$$n = 1 - 6\varepsilon + 2\eta \simeq 1 - \frac{8(4N+9)}{(4N+3)^2} \simeq 0.97$$

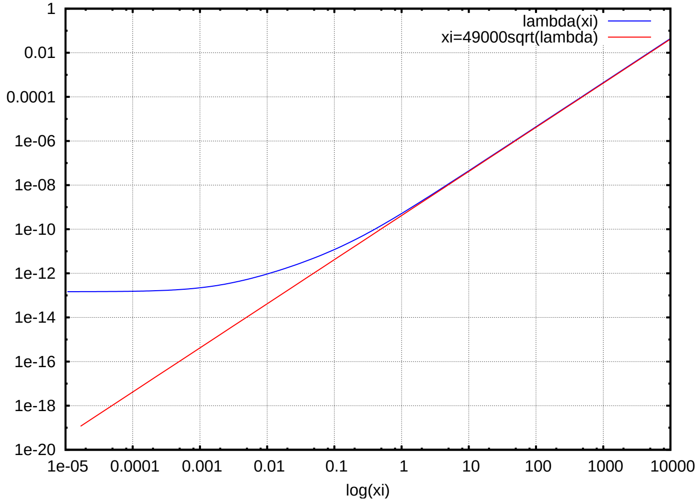
$$r = 16\varepsilon \simeq \frac{192}{(4N+3)^2} \simeq 0.0033$$

Before moving on to using the Higgs field as the inflaton, let us elaborate a bit on generic ξ case

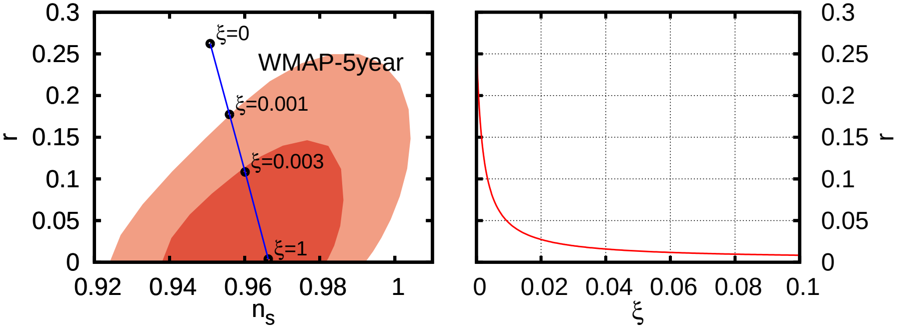
What minimal ξ is needed to reconcile $\lambda\phi^4$ inflation with CMB data?

S.Tsujikawa B.Gumjudpai'04

ξ dependence of λ



WMAP-5 bounds



Message

With non-minimal coupling it is very natural for $\lambda\phi^4$ inflation to be compatible with observations!

Outline

Non-minimaly coupled Higgs boson

$$S = \int d^4x \sqrt{-g} \left[\mathrm{Tr}(F_{\mu\nu}F^{\mu\nu}) + \frac{|D_\mu H|^2}{2} - V(H) + \bar{\Psi}\not{D}\Psi + YH\bar{\Psi}_L\Psi_R \right.$$
$$\left. - \frac{M_P^2}{2}R - \xi H^\dagger H R \right]$$

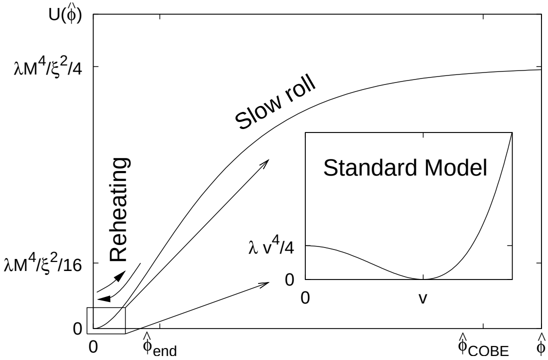
COBE normalization $U/\varepsilon = (0.027M_P)^4$ now determines ξ

$$\xi \simeq \sqrt{\frac{\lambda}{3}} \frac{N_{\mathrm{COBE}}}{0.027^2} \simeq 49000\sqrt{\lambda} = 49000 \frac{m_H}{\sqrt{2}v}$$

Connection of the parameter ξ and the Higgs mass!

Note: $\xi v^2 \lll M_P^2$, so all inflationary analysis can be made just with quartic potential

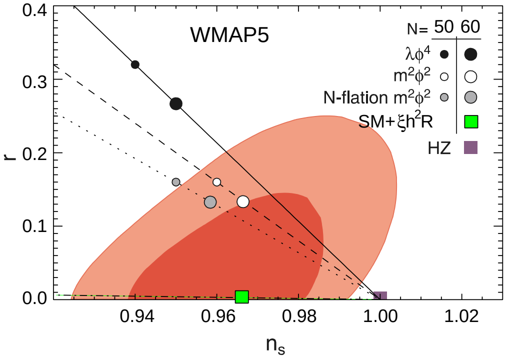
After inflation—back to the SM



$\frac{M_P}{\xi} < \hat{\phi} < M_P$: $U \simeq \frac{\lambda M_P^2}{6\xi^2}\hat{\phi}^2$, $\Omega \simeq 1$, $\hat{\phi} \simeq \sqrt{\frac{3}{2}}\frac{\xi h^2}{M_P}$, $T_{reh} \gtrsim 10^{13}\,\text{GeV}$

For $\hat{\phi} \lesssim M_P/\xi$: the Standard Model

CMB parameters—spectrum and tensor modes



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Not the end of the story — see next talk

Radiative corrections

Could be a problem

In the ordinary situation effective potential is generated

$$\Delta U(\phi) \sim \frac{m^4(\phi)}{64\pi^2} \log \frac{m^2(\phi)}{\mu^2} + A\Lambda^2 + B\Lambda^4$$

We suppose that quadratic divergences are dealt with (eg. in dimensional regularization)

Radiative corrections

Could be a problem

In the ordinary situation effective potential is generated

$$\Delta U(\phi) \sim \frac{m^4(\phi)}{64\pi^2} \log \frac{m^2(\phi)}{\mu^2}$$

standard Yukawa interaction $m = y \cdot h$

$$\Delta U \propto -y^4 \phi^4 \log \frac{\phi^2}{\mu^2}$$

Spoils flatness of the potential (for top quark $y \sim 1$!)

Radiative corrections

This is also cured by non-minimal coupling!

Effective potential is still generated

$$\Delta U(\hat{\phi}) \sim \frac{m^4(\hat{\phi})}{64\pi^2} \log \frac{m^2(\hat{\phi})}{\mu^2}$$

Conformal transformation: fermions

$$S_J = \int d^4x \sqrt{-g} \left\{ \bar{\psi} \partial\!\!\!/ \psi + y \phi \bar{\psi} \psi \right\}$$

$$\hat{\psi} = \Omega^{-3/2} \psi$$

$$S_E = \int d^4x \sqrt{-\hat{g}} \left\{ \bar{\hat{\psi}} \partial\!\!\!/ \hat{\psi} + y \frac{\phi(\hat{\phi})}{\Omega(\hat{\phi})} \bar{\hat{\psi}} \hat{\psi} \right\}$$

Radiative corrections

This is also cured by non-minimal coupling!

Effective potential is still generated

$$\Delta U(\hat{\phi}) \sim \frac{m^4(\hat{\phi})}{64\pi^2} \log \frac{m^2(\hat{\phi})}{\mu^2}$$

The interactions are suppressed now!

$$m(\hat{\phi}) = y \frac{\phi(\hat{\phi})}{\Omega(\hat{\phi})} \xrightarrow{\hat{\phi}\to\infty} \text{const}$$

(where $\Omega(\hat{\phi}) \propto \phi(\hat{\phi})$ for large $\hat{\phi}$)

$$\Longrightarrow \qquad \Delta U(\hat{\phi}) \to y^4 \frac{M_P^4}{\xi^2} \left(1 - e^{-\frac{2\hat{\phi}}{\sqrt{6}M_P}}\right)^2 \log\left(\frac{m^2(\hat{\phi})}{\mu^2}\right) \to \text{const}$$

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Effective potential is still generated

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The same for self interactions

$$m^2(\hat{\phi}) = U''(\hat{\phi}) = \frac{\lambda M_P^2}{3\xi^2} \left(2e^{-\frac{2\hat{\phi}}{\sqrt{6}M_P}} - 1 \right) e^{-\frac{2\hat{\phi}}{\sqrt{6}M_P}} \xrightarrow{\hat{\phi}\to\infty} 0$$

$$\Longrightarrow \qquad \Delta U(\hat{\phi}) \to 0$$

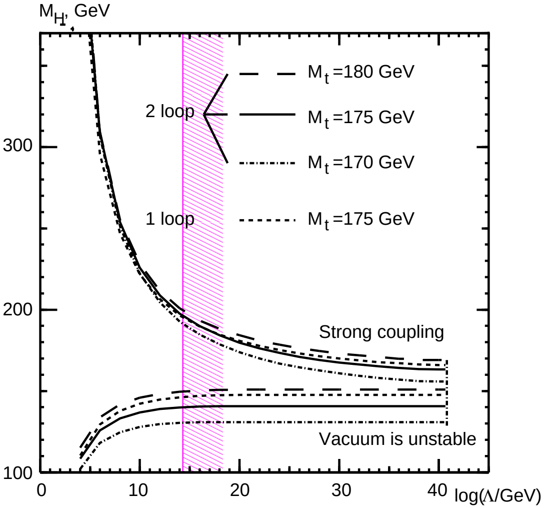
Expected window for the Higgs mass

Standard Model should remain applicable up to

$$M_P/\xi \simeq 10^{14}\,\text{GeV}$$

We expect the Higgs mass

$$130\,\text{GeV} < M_H < 190\,\text{GeV}$$



Yu.Pirogov O.Zenin'98

Outline

Conclusions

Main conclusion

Non-minimal gravity coupling in inflationary models changes predictions a lot and in a very interesting way!

- Adding non-minimal coupling $\frac{\xi\phi^2}{2}R$ with small $\xi > 10^{-3}$ makes $\lambda\phi^4$ chaotic inflation agree with WMAP data.
- These type of models generally gives a very small amount of tensor perturbations after inflation
- Adding non-minimal coupling $\xi H^\dagger HR$ of the Higgs field to the gravity makes inflation possible without introduction of new fields
  - The new parameter of the model, non-minimal coupling ξ, relates the normalization of CMB fluctuations and the Higgs mass $\xi \simeq 49000 m_H/\sqrt{2}v$
  - spectral index $n_s \simeq 0.97$
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