Classical Scalar Field Potential in SM

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Talk at Quarks'08 Seminar, Sergiev Posad, Russia, 23-29 May, 2008.

Outline

- Motivation
- Vector boson mass generation mechanism
- Conditions on the potential form and parameters
- Special choices of the potential
- Initial data mechanism of symmetry breaking
- Problems and outlook

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- The SM Higgs potential looks to be an artificial product of the correspondence principle to the Landau-Ginzburg potential in the superconductivity theory
- Can we take a different form for the potential keeping the SM EW sector unchanged?

$U(1)$ case (I)

Let's consider interaction of a $U(1)$ abelian field A with a complex two-component scalar field Φ:

$$\mathcal{L}=\partial_\mu\Phi^\dagger\partial^\mu\Phi-V(\Phi)-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}+ig\left(\Phi^\dagger\partial_\mu\Phi-(\partial_\mu\Phi^\dagger)\Phi\right)A^\mu+g^2\Phi^\dagger\Phi A_\mu A^\mu$$

The Lagrangian is $U(1)$–invariant if

$$V(\Phi)\equiv V(|\Phi|) \qquad \not\Leftarrow \qquad V(\Phi)\equiv V(\Phi^\dagger\Phi)$$

In polar variables reflecting the gauge symmetry,

$$\Phi(x)\equiv\sigma(x)e^{i\theta(x)}, \qquad \Phi^\dagger(x)\equiv\sigma(x)e^{-i\theta(x)},$$

the Lagrangian takes the form

$$\mathcal{L}=\partial_\mu\sigma\partial^\mu\sigma-V(\Phi)+g^2\sigma^2(A_\mu+\frac{1}{g}\partial_\mu\theta)(A^\mu+\frac{1}{g}\partial^\mu\theta)-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

$U(1)$ case (II)

To generate a mass term for the gauge boson we assume that there is a non-zero vacuum expectation value

$$\langle 0|\sigma|0\rangle \equiv \sigma_0 = \frac{\eta}{\sqrt{2}} \neq 0 \qquad \text{so that} \qquad \sigma(x) = \frac{\eta + h(x)}{\sqrt{2}}$$

σ_0 is the zeroth harmonic of the radial degree of freedom of the scalar field:

$$\frac{1}{V_0}\int_{V_0} \mathrm{d}^3x\, \sigma(x) = \sigma_0, \qquad \frac{1}{V_0}\int_{V_0} \mathrm{d}^3x\, h(x) = 0$$

Fixing the gauge of the vector field by

$$A_\mu(x) \to B_\mu(x) = A_\mu(x) + \frac{1}{g}\partial_\mu\theta(x), \qquad \text{we get}$$

$$\begin{aligned}\mathcal{L} &= \frac{1}{2}\partial_\mu h \partial^\mu h - V(\sigma(x)) + \frac{1}{2}g^2\eta^2 B_\mu B^\mu + g^2\eta h B_\mu B^\mu \\ &+ \frac{1}{2}g^2 h^2 B_\mu B^\mu - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}\end{aligned}$$

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- **$SU(2) \times U(1)$ case is evaluated in polar variables in the same manner, see e.g. [1]**
- **the potential is not yet given explicitly, and certain additional conditions on it to be applied**

Conditions on the potential

For the symmetry reason

$$V(\Phi) = V(|\Phi|) = V(\sigma) \tag{1}$$

Universe energy density

$$V(\sigma_0) + V_{\rm eff}(\sigma_0) = 0 \longrightarrow V(\sigma_0) = 0, \qquad V_{\rm eff}(\sigma_0) = 0 \tag{2}$$

Minimum of the classical potential

$$\left.\frac{\mathrm{d}V(\sigma)}{\mathrm{d}\sigma}\right|_{\sigma=\sigma_0} = 0, \qquad \left.\frac{\mathrm{d}^2V(\sigma)}{\mathrm{d}\sigma^2}\right|_{\sigma=\sigma_0} \leq 0 \tag{3}$$

Renormalizablity, stability, minimality etc.

$$V(\sigma) = c_0 + c_1\sigma + c_2\sigma^2 + c_3\sigma^3 + c_4\sigma^4, \qquad c_4 \geq 0 \tag{4}$$

Potential types (I)

- The standard choice

$$V_{\mathrm{std}}(\Phi) = \lambda\left(\sigma^2 - \frac{\eta^2}{2}\right)^2 = \lambda\eta^2 h^2 + \lambda\eta h^3 + \frac{\lambda}{4} h^4$$

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- Sub-class of concave potentials with a single global minimum

$$V_I(\sigma) = m_h^2(\sigma - \sigma_0)^2 + \lambda(\sigma - \sigma_0)^4 = \frac{m_h^2}{2}h^2 + \frac{\lambda}{4}h^4$$

with two free parameters m_h and $\lambda > 0$; no triple Higgs couplings at the tree level. EW sector, including interactions of Higgs with vector bosons and fermions is exactly the same as in SM

Potential types (II)

- **Let's drop the Higgs self-interaction:**

$$V_{II}(\sigma) = m_h^2(\sigma - \sigma_0)^2 = \frac{m_h^2}{2} h^2$$

one free parameter m_h. Only gauge and Yukawa interactions in the theory. Remind that Φ^4 theory to remain perturbative at all scales should be trivial, i.e. $\lambda = 0$

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- Neither m_h nor λ is required to get the EW boson masses

$$V_{IV}(\sigma) \equiv 0$$

  it is the minimal choice

Symmetry breaking by ID (I)

- **Initial Data (ID) should be considered as an ingredient of a physical model. For a model with a local gauge symmetry ID is a must because of the second Nöther theorem.**

- **Physical vacuum comes to our theoretical constructions from aside as a kind of ID**

- **All scalar objects (Higgs, condensates etc.) in the theory can mix with vacuum. And can means do**

- **In order to construct a *physical* theory, i.e. the one describing *observables*, we have to extract all zeroth modes and put into the *physical* vacuum. This statement is nothing else but Einstein's cosmological principle (1917).**

Symmetry breaking by ID (II)

Let's consider a part of a conformal symmetric Lagrangian describing scalar field σ and its interactions with fermions (s) and vector bosons (v)

$$\mathcal{L}_{\rm Higgs} = \partial_\mu\sigma\partial^\mu\sigma - \sigma\sum_s f_s\bar{s}s + \frac{\sigma^2}{4}\sum_{\rm v} g_{\rm v}^2 V^2 - \lambda\sigma^4$$

Following Kirzhnits and Linde we can *try* to apply the cosmological principle and average our scalar components over a large enough volume V_0:

$$\langle\sigma\rangle \equiv \frac{1}{V_0}\int {\rm d}^3x\sigma, \quad \langle\mathbf{A}\rangle \equiv \frac{1}{V_0}\int {\rm d}^3x\sum_s f_s\bar{s}s, \quad \langle\mathbf{B}\rangle \equiv \frac{1}{V_0}\int {\rm d}^3x\frac{1}{4}\sum_{\rm v} g_{\rm v}^2 V^2$$

Again we shift the scalar field $\sigma = \langle\sigma\rangle + h/\sqrt{2}$

Symmetry breaking by ID (III)

The Lagrangian takes the form

$$\mathcal{L}_{\rm Higgs} = \partial_0\langle\sigma\rangle\partial_0\langle\sigma\rangle - \mathbf{V}_0(\langle\sigma\rangle) + \frac{1}{2}\partial_\mu h\partial^\mu h - h^2\frac{m_h^2}{2} - h^3\lambda\sqrt{2}\langle\sigma\rangle + \ldots$$

where

$$\begin{aligned} \mathbf{V}_0(\langle\sigma\rangle) &= \langle\sigma\rangle\langle\mathbf{A}\rangle - \langle\sigma\rangle^2\langle\mathbf{B}\rangle + \lambda\langle\sigma\rangle^4 \\ m_h^2 &= 6\lambda\langle\sigma\rangle^2 - \langle\mathbf{B}\rangle \end{aligned}$$

Assuming that $\mathbf{V}_0(\langle\sigma\rangle) = 0$ and $\partial\mathbf{V}_0(\langle\sigma\rangle)/\partial\langle\sigma\rangle = 0$ we get a set of linear equations for the condensates giving

$$m_h^2 = \langle\mathbf{B}\rangle = 3\lambda\langle\sigma\rangle^2, \qquad \langle\mathbf{A}\rangle = 2\lambda\langle\sigma\rangle^3$$

Non-zero vacuum expectation values can be interpreted as non-trivial initial data.

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- If the classical potential is not unique, we need a steering principle to choose one (conformal symmetry, minimality, whatever)
- In any case, if Higgs gave masses to all other particles, who gave m_h? In other words, is it possible to look for the origin of dimensional parameters within the SM?
- Choosing one of the alternative classical potentials discussed above does not automatically solve difficulties of the SM