

Spontaneous P-parity breaking in dense baryon matter

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We guess **P- violation** to occur **at nearly zero** temperature but **large** baryon number density due to **condensation** of parity-odd mesons (pions, kaons,... and their heavy twins)

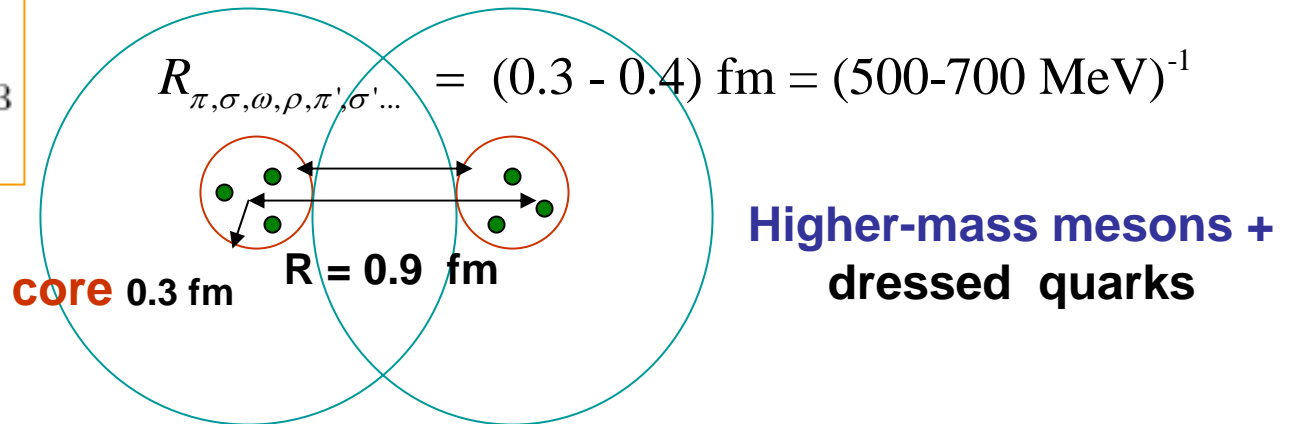
$$\rho_B \gg \rho_N \simeq 0.17 \text{ fm}^{-3} = (1.8 \text{ fm})^{-3}$$

How large?

Beyond the range of validity of pion-nucleon effective Lagrangian but **not large enough** for quark percolation, $\rho_B \sim (3 \div 8) \rho_N$
i.e. in the hadronic phase with **heavy meson** excitations playing an essential role in dense nuclear matter where quark-nuclear matter duality can be effectively used.

Example

$$\rho_B = 8\rho_N \simeq (0.9 \text{ fm})^{-3}$$

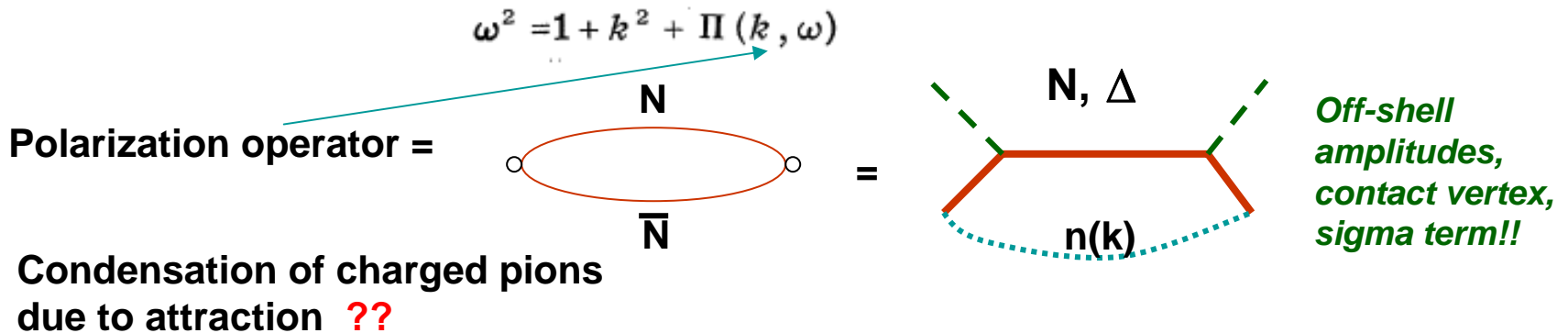


A. Migdal, 1971

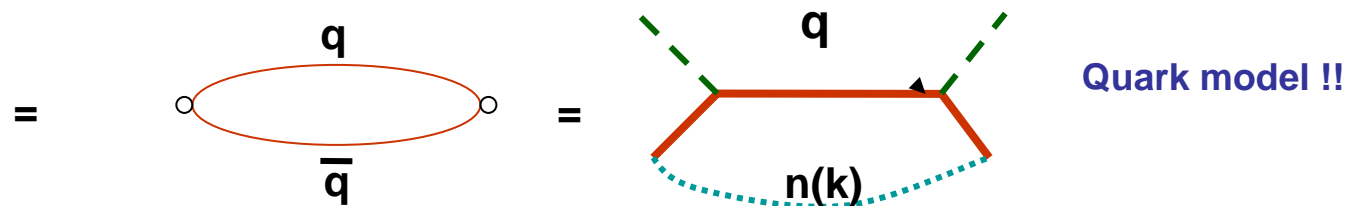
$$\omega^2 = 1 + k^2 - 4\pi n F(k) \quad \longleftarrow \quad \hbar = c = m_\pi = 1$$

where n is the nucleon density and $F(k)$ is the forward pion-nucleon scattering amplitude

An “exact” calculation includes the particle-hole excitations of the nuclear medium



or for large densities the particle-hole excitations of the quark medium



Confinement ??

Effective lagrangian approach

Low energies \Longrightarrow Chiral lagrangian for pions:

$$L_\pi = \frac{1}{4} F_\pi^2 \operatorname{tr} \left(D_\mu U D^\mu U^\dagger + m_\pi^2 (U + U^\dagger) \right)$$

$$U = \exp \left(i \frac{\pi^a \tau^a}{F_\pi} \right)$$

$$D_\mu U \equiv \partial_\mu U + [V_\mu, U]$$

Vector field

Density vs. chemical potential

(symmetric nuclear matter)

$$\langle N^\dagger N(x) \rangle = \rho_B \quad \Leftrightarrow \quad \int dx \mu_B (\bar{N} \gamma_0 N(x) - \rho_B)$$

Corresponds to singlet vector current!

$$V_0 = \mu_B$$

Disappears from pion lagrangian?

No! It is hidden in structural constants and masses.

$$F_\pi^2(\mu_B) \quad m_\pi^2(\mu_B)$$

We need a model of formation for these parameters:

QCD ?

At least an extension to QCD motivated Linear sigma models and/or Quark models !

From the hadron side heavy meson states are in game!

Extended linear sigma model

with two multiplets of scalar and pseudoscalar mesons
(with matching as close to QCD as possible –spectral sum rules)

$$H_j = \sigma_j \mathbf{I} + i \hat{\pi}_j, \quad j = 1, 2; \quad H_j H_j^\dagger = (\sigma_j^2 + (\pi_j^a)^2) \mathbf{I}, \quad \hat{\pi}_j \equiv \pi_j^a \tau^a$$

Chiral limit \longrightarrow $SU_L(2) \times SU_R(2)$ symmetry

$$V_{\text{eff}} = \frac{1}{2} \text{tr} \left\{ - \sum_{j,k=1}^2 H_j^\dagger \Delta_{jk} H_k \right. \\ \left. + \lambda_1 (H_1^\dagger H_1)^2 + \lambda_2 (H_2^\dagger H_2)^2 + \lambda_3 H_1^\dagger H_1 H_2^\dagger H_2 \right. \\ \left. + \frac{1}{2} \lambda_4 (H_1^\dagger H_2 H_1^\dagger H_2 + H_2^\dagger H_1 H_2^\dagger H_1) + \frac{1}{2} \lambda_5 (H_1^\dagger H_2 + H_2^\dagger H_1) H_1^\dagger H_1 \right. \\ \left. + \frac{1}{2} \lambda_6 (H_1^\dagger H_2 + H_2^\dagger H_1) H_2^\dagger H_2 \right\} + \mathcal{O}\left(\frac{|H|^6}{\Lambda^2}\right),$$

9 real constants

$$\Delta_{jk} \sim \lambda_A \sim N_c$$

**Chiral expansion in $1/\Lambda$
in hadron phase of QCD**

$$\Lambda \simeq 4\pi F_\pi \simeq M_{\text{dyn}}$$

Chirally symmetric parameterization

$$H_1(x) = \sigma_1(x)U(x) = \sigma_1(x)\xi^2(x); \quad \langle H_1 \rangle = \langle \sigma_1 \rangle > 0$$

$$H_2(x) = \xi(x)(\sigma_2(x) + i\hat{\pi}_2(x))\xi(x) = \sigma_2(x)U(x) + i\xi(x)\hat{\pi}_2(x)\xi(x)$$

Effective potential

$$\begin{aligned} V_{\text{eff}} = & - \sum_{j,k=1}^2 \sigma_j \Delta_{jk} \sigma_k - \Delta_{22} (\pi_2^a)^2 \\ & + \lambda_1 \sigma_1^4 + \lambda_2 \sigma_2^4 + (\lambda_3 + \lambda_4) \sigma_1^2 \sigma_2^2 + \lambda_5 \sigma_1^3 \sigma_2 + \lambda_6 \sigma_1 \sigma_2^3 \\ & + (\pi_2^a)^2 \left((\lambda_3 - \lambda_4) \sigma_1^2 + \lambda_6 \sigma_1 \sigma_2 + 2\lambda_2 \sigma_2^2 \right) + \lambda_2 \left((\pi_2^a)^2 \right)^2 \end{aligned}$$

Vacuum states

Neutral pseudoscalar condensate breaking P-parity?

$$\pi_2^a = \delta^{a0} \rho$$

No! (Vafa-Witten theorem) $\rho = 0$ in QCD at zero quark density

Eqs. for vacuum states

$$2(\Delta_{11}\sigma_1 + \Delta_{12}\sigma_2) = 4\lambda_1\sigma_1^3 + 3\lambda_5\sigma_1^2\sigma_2 + 2(\lambda_3 + \lambda_4)\sigma_1\sigma_2^2 + \lambda_6\sigma_2^3 \\ + \rho^2(2(\lambda_3 - \lambda_4)\sigma_1 + \lambda_6\sigma_2),$$

$$2(\Delta_{12}\sigma_1 + \Delta_{22}\sigma_2) = \lambda_5\sigma_1^3 + 2(\lambda_3 + \lambda_4)\sigma_1^2\sigma_2 + 3\lambda_6\sigma_1\sigma_2^2 + 4\lambda_2\sigma_2^3 \\ + \rho^2(\lambda_6\sigma_1 + 4\lambda_2\sigma_2),$$

$$0 = 2\pi_2^a \left(-\Delta_{22} + (\lambda_3 - \lambda_4)\sigma_1^2 + \lambda_6\sigma_1\sigma_2 + 2\lambda_2\sigma_2^2 + 2\lambda_2\rho^2 \right)$$

Necessary and sufficient condition to avoid P-parity breaking
in normal QCD vacuum ($\mu = 0$)

$$(\lambda_3 - \lambda_4)\sigma_1^2 + \lambda_6\sigma_1\sigma_2 + 2\lambda_2\sigma_2^2 > \Delta_{22} \quad \lambda_2 > 0$$

Second variation for $\rho = 0$

$$\frac{1}{2}V_{11}^{(2)} = -\Delta_{11} + 6\lambda_1\sigma_1^2 + 3\lambda_5\sigma_1\sigma_2 + (\lambda_3 + \lambda_4)\sigma_2^2, \quad > 0$$

$$V_{12}^{(2)} = -2\Delta_{12} + 3\lambda_5\sigma_1^2 + 4(\lambda_3 + \lambda_4)\sigma_1\sigma_2 + 3\lambda_6\sigma_2^2,$$

$$\frac{1}{2}V_{22}^{(2)} = -\Delta_{22} + (\lambda_3 + \lambda_4)\sigma_1^2 + 3\lambda_6\sigma_1\sigma_2 + 6\lambda_2\sigma_2^2 \quad > 0$$

$$\frac{1}{2}V_{ab}^{(2)\pi} = \delta_{ab} \left(-\Delta_{22} + (\lambda_3 - \lambda_4)\sigma_1^2 + \lambda_6\sigma_1\sigma_2 + 2\lambda_2\sigma_2^2 \right) > 0$$



$\pi(1300)$

Embedding a chemical potential $\int dx \mu (\bar{q} \gamma_0 q(x) - \rho_B)$

Local coupling to quarks

$$\mathcal{L}_{int} = -(\bar{q}_R \textcircled{H_1} q_L + \bar{q}_L H_1^\dagger q_R)$$

Superposition of physical meson states

From a quark model

$$\Delta V_{\text{eff}}(\mu) = \frac{\mathcal{N}}{2} \Theta(\mu - |H_1|) \left[\mu |H_1|^2 \sqrt{\mu^2 - |H_1|^2} - \frac{2\mu}{3} (\mu^2 - |H_1|^2)^{3/2} - |H_1|^4 \ln \frac{\mu + \sqrt{\mu^2 - |H_1|^2}}{|H_1|} \right] \left(1 + O\left(\frac{\mu^2}{\Lambda^2}\right) \right)$$

$\mathcal{N} \equiv \frac{N_c N_f}{4\pi^2}$

Monotonous function

Density and Fermi momentum

$$\varrho_B = -\frac{1}{3} \partial_\mu \Delta V_{\text{eff}}(\mu) = \frac{N_c N_f}{9\pi^2} p_F^3 = \frac{N_c N_f}{9\pi^2} (\mu^2 - |\langle H_1 \rangle|^2)^{3/2} > 0$$

Dense matter drivers of minimum

Only the first Eq. for stationary points is modified

$$2(\Delta_{11}\sigma_1 + \Delta_{12}\sigma_2) = 4\lambda_1\sigma_1^3 + 3\lambda_5\sigma_1^2\sigma_2 + 2(\lambda_3 + \lambda_4)\sigma_1\sigma_2^2 + \lambda_6\sigma_2^3 \\ + \rho^2 \left(2(\lambda_3 - \lambda_4)\sigma_1 + \lambda_6\sigma_2 \right) + 2\mathcal{N}\Theta(\mu - \sigma_1) \left[\mu\sigma_1\sqrt{\mu^2 - \sigma_1^2} - \sigma_1^3 \ln \frac{\mu + \sqrt{\mu^2 - \sigma_1^2}}{\sigma_1} \right]$$

Monotonous function!

Necessary conditions to approach to P-violation phase

$$\sigma_{1,2} = \sigma_{1,2}(\mu)$$

$$\partial_\mu \left[(\lambda_3 - \lambda_4)\sigma_1^2 + \lambda_6\sigma_1\sigma_2 + 2\lambda_2\sigma_2^2 \right] < 0$$

One condition for 9 parameters:

P-violation is not exceptional but rather typical!

P-parity violation phase

Critical points μ_c **when** $\rho(\mu_c) = 0$

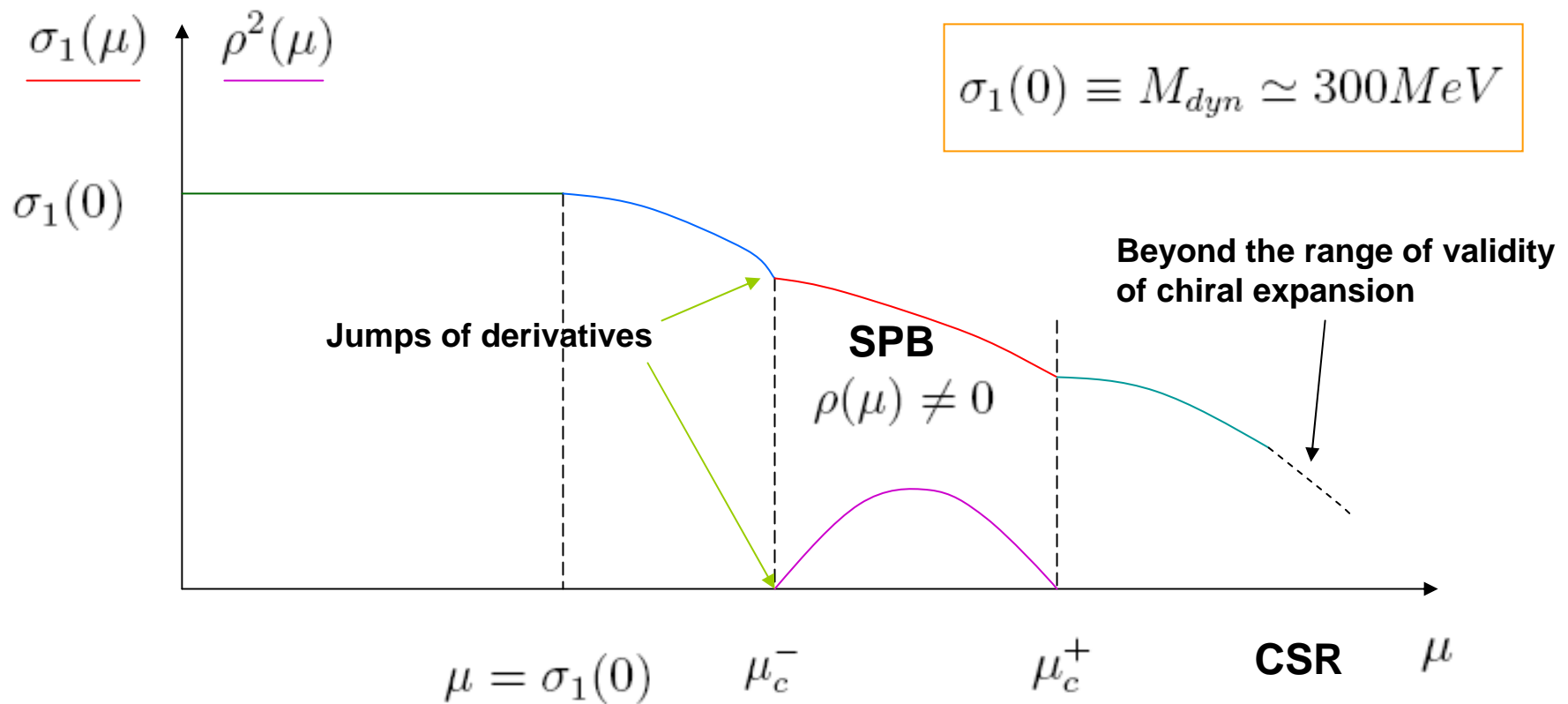
$$(4\lambda_2\Delta_{12} - \lambda_6\Delta_{22})r^2 + (2\lambda_6\Delta_{12} - 4\lambda_4\Delta_{22})r + 2(\lambda_3 - \lambda_4)\Delta_{12} - \lambda_5\Delta_{22} = 0$$

For $r \equiv \frac{\sigma_2}{\sigma_1}$

There are in general two solutions for two $\mu_c^- < \mu_c^+$

But for $4\lambda_2\Delta_{12} = \lambda_6\Delta_{22}$ **only one solution and P-violation phase may be left via 1st order phase transition**

Spontaneous P-parity breaking (IId order phase transition)



With increasing μ one enters SPB phase and leaves it before (?) encountering any new phase (CSR, CFL ...)

Kinetic terms

symmetric under $SU_L(2) \times SU_R(2)$

$$\mathcal{L}_{kin} = \frac{1}{4} \sum_{j,k=1}^2 A_{jk} \text{tr} \left\{ \partial_\mu H_j^\dagger \partial^\mu H_k \right\}$$

Three more real constants A_{jk} but one, A_{22} is redundant

Kinetic part quadratic in meson fields

$$\begin{aligned} \mathcal{L}_{kin}^{(2)} = & \frac{1}{2} \sum_{j,k=1}^2 A_{jk} \partial_\mu \Sigma_j \partial^\mu \Sigma_k + \frac{\rho^2}{2F_0^2} A_{22} \partial_\mu \pi^0 \partial^\mu \pi^0 - \frac{\rho}{F_0} \sum_{j=1}^2 A_{j2} \partial_\mu \Sigma_j \partial^\mu \pi^0 \\ & + \sum_{j,k=1}^2 \left[\frac{1}{2F_0^2} A_{jk} \bar{\sigma}_j \bar{\sigma}_k \partial_\mu \pi^a \partial^\mu \pi^a + \frac{1}{F_0} A_{j2} \bar{\sigma}_j \partial_\mu \pi^a \partial^\mu \Pi^a \right] + \frac{1}{2} A_{22} \partial_\mu \Pi^a \partial^\mu \Pi^a \end{aligned}$$

**Normalizations
and notations**

$$F_0^2 = \sum_{j,k=1}^2 A_{jk} \bar{\sigma}_j \bar{\sigma}_k \simeq (90\text{MeV})^2, \quad \zeta \equiv \frac{1}{F_0} \sum_{j=1}^2 A_{j2} \bar{\sigma}_j$$

Pion weak decay constant

P-breaking phase

Partially diagonalized kinetic term

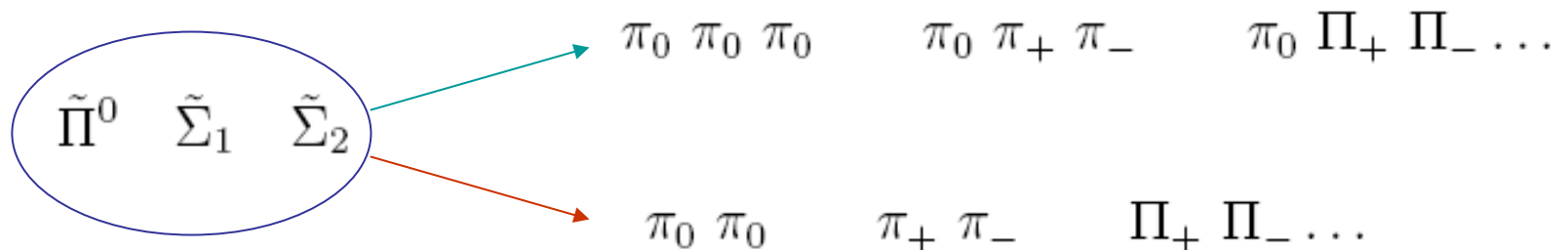
$$\begin{aligned} \mathcal{L}_{kin}^{(2)} = & \partial_\mu \tilde{\pi}^\pm \partial^\mu \tilde{\pi}^\mp + (A_{22} - \zeta^2) \partial_\mu \Pi^\pm \partial^\mu \Pi^\mp + \frac{1}{2} \left(1 + \frac{A_{22} \rho^2}{F_0^2} \right) \partial_\mu \tilde{\pi}^0 \partial^\mu \tilde{\pi}^0 \\ & + \frac{1}{2} \left(A_{22} - \frac{F_0^2}{F_0^2 + A_{22} \rho^2} \zeta^2 \right) \partial_\mu \Pi^0 \partial^\mu \Pi^0 + \frac{1}{2} \sum_{j,k=1}^2 \frac{A_{jk} F_0^2 + \rho^2 \det A \delta_{1j} \delta_{1k}}{F_0^2 + A_{22} \rho^2} \partial_\mu \Sigma_j \partial^\mu \Sigma_k \\ & - \frac{F_0 \rho}{F_0^2 + A_{22} \rho^2} \zeta \partial_\mu \Pi^0 \sum_{j=1}^2 A_{j2} \partial^\mu \Sigma_j \end{aligned}$$

Isospin breaking $SU_V(2) \rightarrow U(1)$

Further diagonalization $\Pi^0 \quad \Sigma_1 \quad \Sigma_2 \quad \Longrightarrow \quad \tilde{\Pi}^0 \quad \tilde{\Sigma}_1 \quad \tilde{\Sigma}_2$

mixes neutral pseudoscalar and scalar states

**Therefore genuine mass states don't possess
a definite parity in decays**



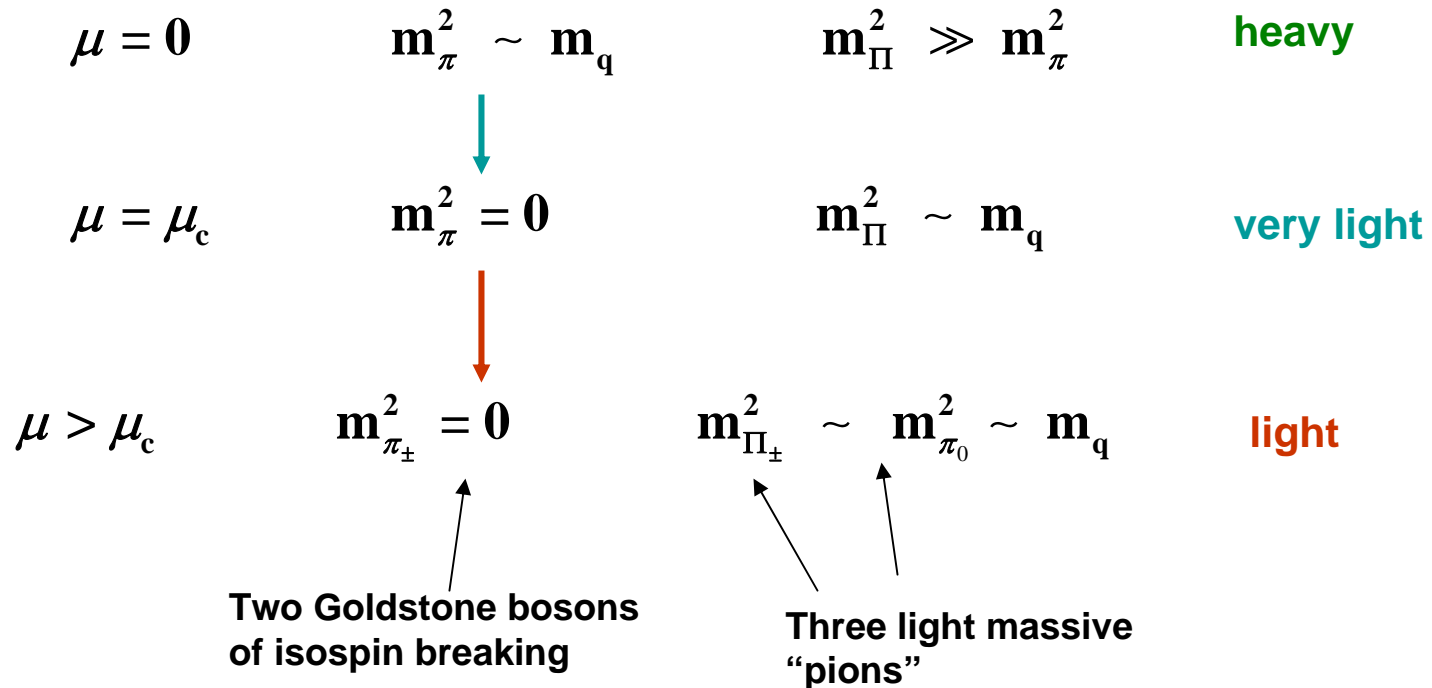
Beyond the chiral limit: $m_q \neq 0$

Two new lowest-dimensional operators

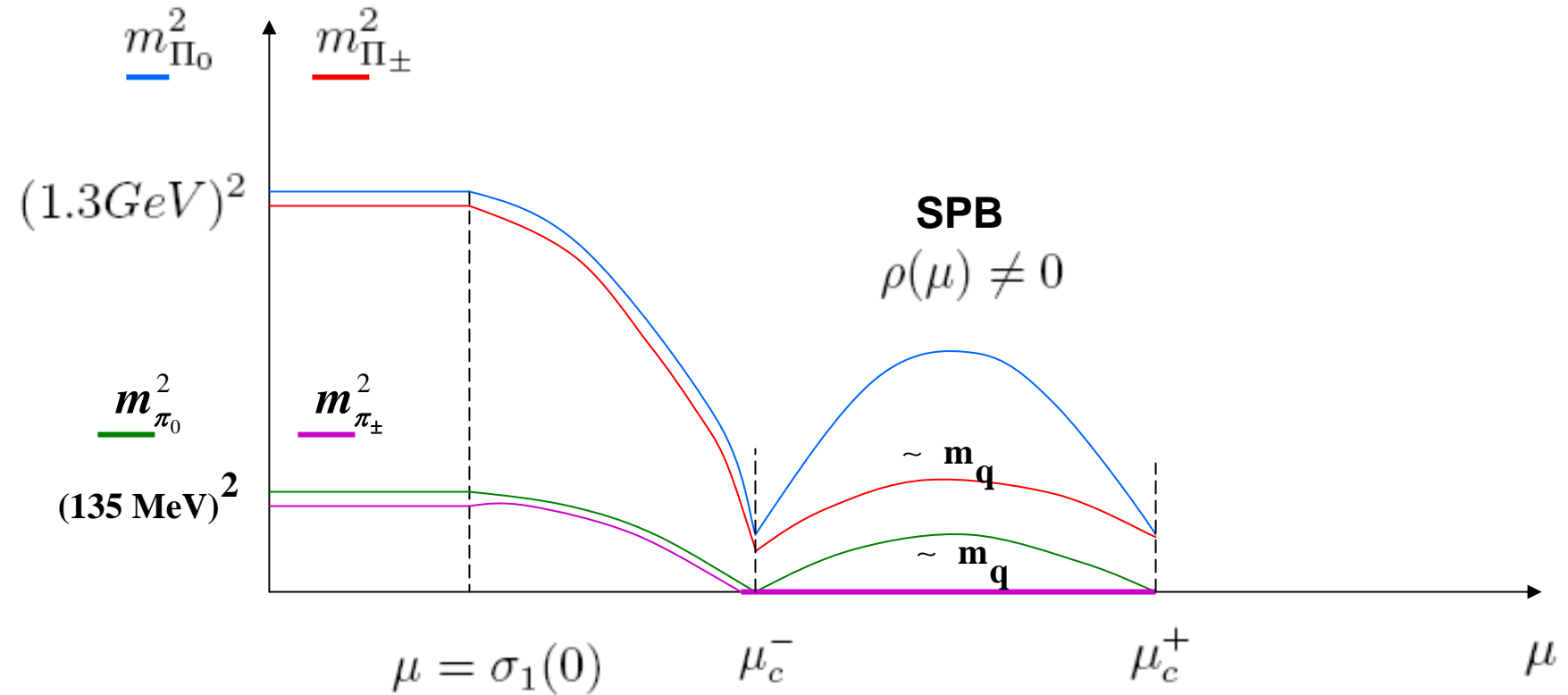
$$\frac{1}{2}m_q d_1 \text{tr}(H_1 + H_1^\dagger)$$

$$\frac{1}{2}m_q d_2 \text{tr}(H_2 + H_2^\dagger)$$

The spectrum in dense matter



Mass spectrum of “pseudoscalar” states (massive quarks)



In P-breaking phase there are 2 massless pseudoscalar mesons

$$\tilde{\pi}_{\pm}$$

Minimal model admitting SPB

$$\lambda_5 = \lambda_6 = 0 \text{ entails } \Delta_{12} = 0;$$

from consistency $\sigma_2 = 0$

If $A_{12} = 0$ such an effective Lagrangian is symmetric under $Z_2 \times Z_2$

$$H_1 \rightarrow -H_1 \text{ or } H_2 \rightarrow -H_2$$

Fit on hadron phenomenology

$$\left\{ \begin{array}{l} A_{11} = \frac{1}{9} = A_{22}, \quad F_0 = 100 \text{ MeV}, \quad \sigma_1 = 300 \text{ MeV} = 3F_0, \\ m_{s,1} = 0.7 \text{ GeV} = 7F_0, \quad m_p = 1.3 \text{ GeV} = 13F_0, \quad m_{s,2} = 1.5 \text{ GeV} = 15F_0, \end{array} \right.$$

$$\Delta_{11} \simeq 2.7F_0^2, \quad \lambda_1 \simeq 0.15, \quad \lambda_4 \simeq 0.35.$$

For

$$\varrho_{B,crit} \simeq 3\varrho_{B,nuclear}$$



$$\mu_c \simeq 4.3F_0$$



$$\sigma_{1,crit} \simeq 1.8F_0$$

$$\lambda_3 \simeq 3.6$$

$$\Delta_{22} \simeq 11F_0^2$$

$$m_{s,1} \simeq 1.7F_0, \quad m_{s,2} \simeq 4.5F_0$$

λ_2 remains undetermined

Where to see and how to check SPV ?

- a) Approaching to SPV phase is indicated by a rapid decrease in heavy resonance masses . Below phase transition point one finds an abnormally *light and long-living pseudoscalar resonances!*
- b) At the very point of the P-breaking phase transition one has three massless pion-like state.
- c) After phase transition two massless charged pseudoscalars remain as Goldstone bosons enhancing charged pion production .

Hunting for new light pseudoscalars!

- d) F_Π and extended PCAC: it is modified for massless charged pions giving an enhancement of electroweak decays of heavy pions.
 - e) Additional isospin breaking: $f_{\pi_0} \neq f_{\pi_\pm}$
-

Perspectives

One can search for enhancement of long-range correlations in the pseudoscalar channel in lattice simulations? $T \neq 0$; $\text{Im } \mu \neq 0$

Program for GSI SIS 200 ?

Experiment -- Compressed Baryon Matter (CBM)?

Polarization operator in the Migdal's approach

$$\omega^2 = \mathbf{k}^2 + \Pi(\omega, \mathbf{k}, \mu)$$

for two pseudoscalar states π, π'

Take masses $m_\pi^2(\mu) = 0$ $m_{\pi'}^2(\mu)|_{\mu \rightarrow \mu_{\text{crit}}} \rightarrow 0$

and wave function normalizations

$$Z_\pi \approx Z_{\pi'} \approx 1$$

$$\Pi(\omega, \mathbf{k}, \mu) = - \frac{(\omega^2 - \mathbf{k}^2)^2}{m_{\pi'}^2(\mu) - 2(\omega^2 - \mathbf{k}^2)}$$

Has a pole in the narrow resonance approach
and changes sign for high energies

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Our inspiration from

Polarization operator in details

$$\Pi^{(1)\pi^-}(\omega, k) = -2 \int \frac{d^3p}{(2\pi)^3} [D_{\pi^-n}(\omega, k)n_n(p) + D_{\pi^-p}(\omega, k)n_p(p)],$$

where D_{π^-n} and D_{π^-p} are the spin-averaged forward scattering amplitudes, $n_n(p)$ and $n_p(p)$ are the occupation functions of the neutrons and the protons respectively,

$$\Pi^{(1)\pi^-}(\omega, k; \rho) = \Pi_N^{(1)\pi^-} + \Pi_\Delta^{(1)\pi^-} + \Pi_D^{(1)\pi^-} + \Pi_\sigma^{(1)\pi^-}$$

Off-shell amplitudes from
an effective lagrangian

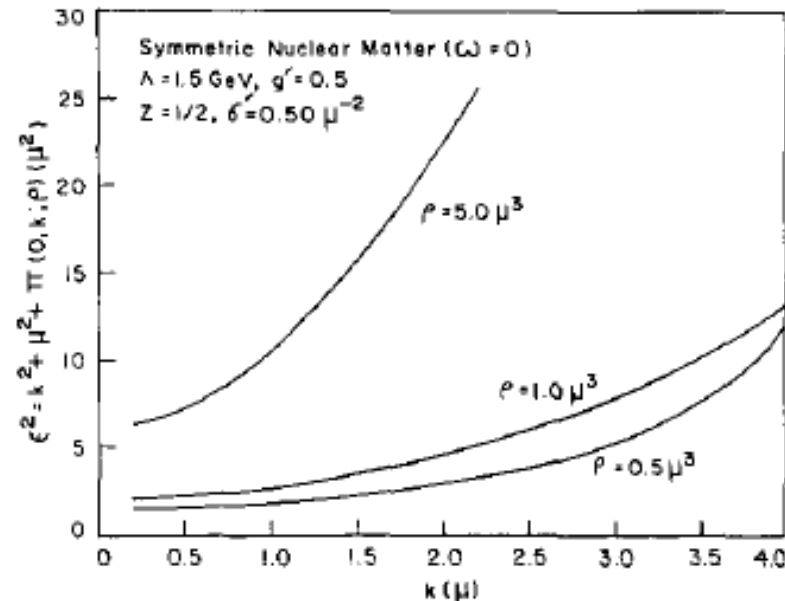
where the subscripts N , Δ , D and σ refer to contributions from nucleon exchange, delta exchange, direct pion-nucleon scattering and the pion-nucleon σ term,

Most pessimistic!

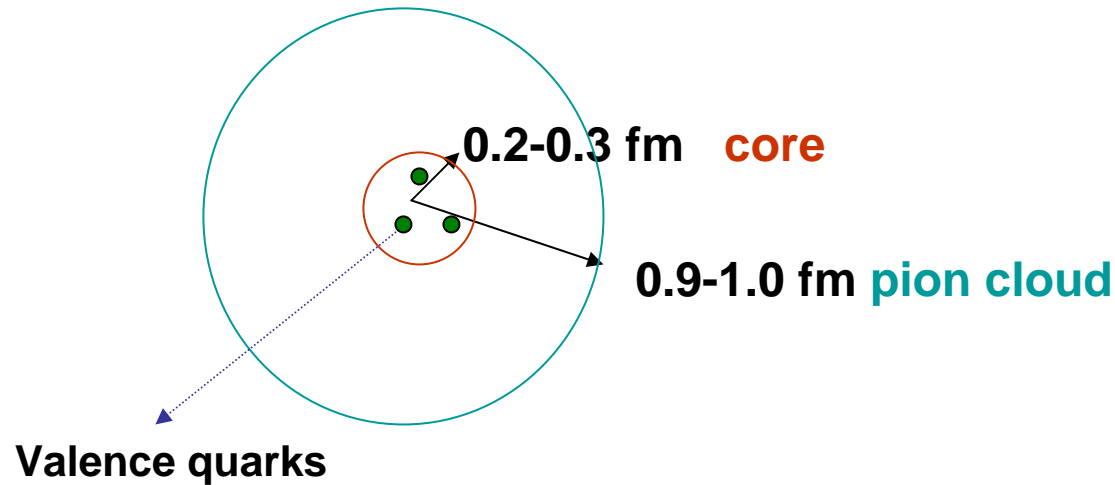
T Shamsunnahar et al.

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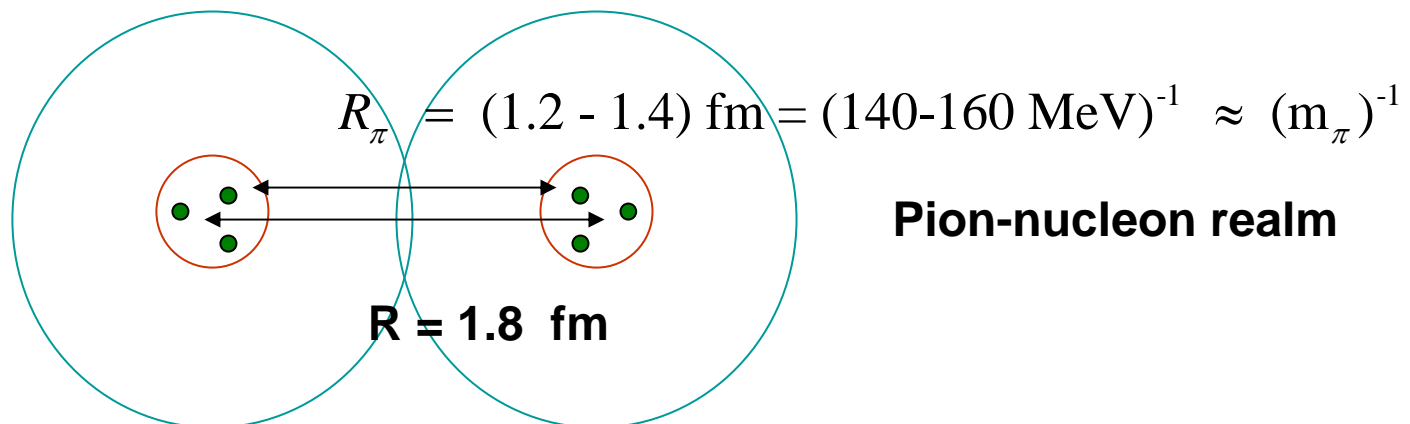
No pion condensate ?



Nucleon in consensus



Normal nuclear matter

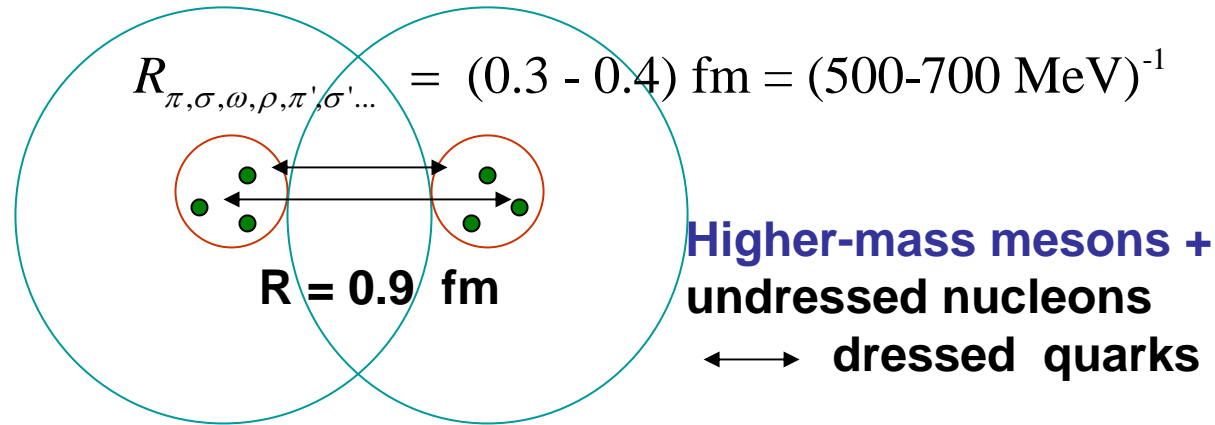


Pion-nucleon realm

Dense nuclear matter

$$\rho_B = 8\rho_N \simeq (0.9 \text{ fm})^{-3}$$

Neutron stars

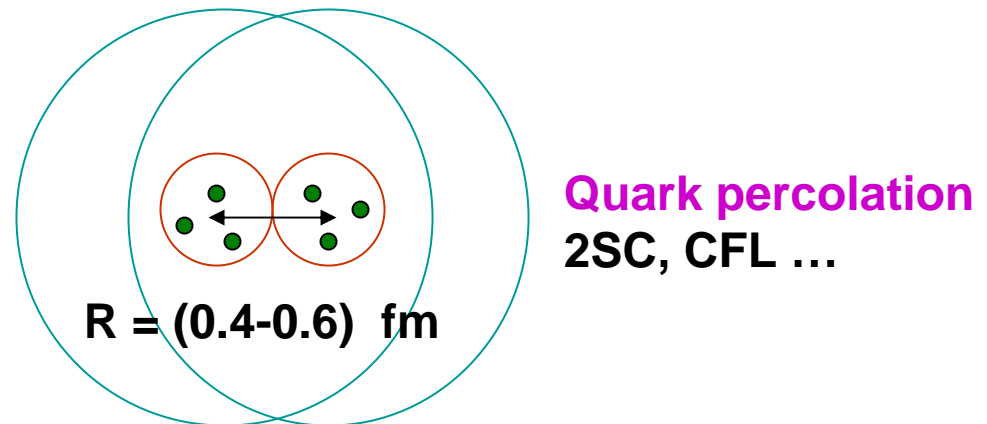


Superdense nuclear matter

$$\rho_B = (25 \div 100) \rho_N$$

$$\simeq (0.4 \div 0.6 \text{ fm})^{-3}$$

Quark stars?



It represents a consensus of several models:
 nuclear potentials, meson-nucleon effective Lagrangians,
 extended Skyrme models, chiral bag models ...
 still the NJL ones give larger core sizes due to lack of confinement

Meson spectrum in SPB phase

Neutral pi-prime condensate breaks vector SU(2) to U(1)
and **two charged pi-prime mesons become massless**

$$\frac{1}{2}V_{11}^{(2)\sigma} = 4\lambda_1\sigma_1^2 + 2\lambda_5\sigma_1\sigma_2 + 2\lambda_4\sigma_2^2 - 2\mathcal{N}\sigma_1^2 \ln \frac{\mu + \sqrt{\mu^2 - \sigma_1^2}}{\sigma_1}$$

$$V_{12}^{(2)\sigma} = 2\lambda_5\sigma_1^2 + 4\lambda_3\sigma_1\sigma_2 + 2\lambda_6\sigma_2^2$$

$$\frac{1}{2}V_{22}^{(2)\sigma} = 2\lambda_4\sigma_1^2 + 2\lambda_6\sigma_1\sigma_2 + 4\lambda_2\sigma_2^2$$

$$\left. \begin{aligned} V_{10}^{(2)\sigma\pi} &= \left(4(\lambda_3 - \lambda_4)\sigma_1 + 2\lambda_6\sigma_2\right)\rho \\ V_{20}^{(2)\sigma\pi} &= \left(2\lambda_6\sigma_1 + 8\lambda_2\sigma_2\right)\rho \end{aligned} \right\} \text{Mixture of massive scalar and neutral pseudoscalar states}$$

$$\frac{1}{2}V_{00}^{(2)\pi} = 4\lambda_2\rho^2 \quad \boxed{\frac{1}{2}V_{\pm\mp}^{(2)\pi} = 0}$$

Estimations of coupling constants in Quasilocal Quark Model

$$\mathcal{L}_{\text{QQM}} = \bar{q}(i\not{\partial})q + \sum_{k,l=1}^2 a_{kl} [\bar{q}f_k(s)q\bar{q}f_l(s)q - \bar{q}f_k(s)\tau^a\gamma_5 q\bar{q}f_l(s)\tau^a\gamma_5 q]. \quad (3)$$

Here a_{kl} represents a symmetric matrix of real coupling constants and $f_k(s)$, $s \equiv -\partial^2/\Lambda^2$ are the polynomial form factors specifying the quasilocal (in momentum space) interaction. The form factors are orthogonal on the unit interval and the results of calculations do not depend on a concrete choice of form factors in the large-log approximation. A convenient choice is $f_1(s) = 2 - 3s$, $f_2(s) = -\sqrt{3}s$. The values of couplings λ_i in Eq. (2) are then fixed for $i = 2, \dots, 6$: $\lambda_2 = \frac{9N_c}{32\pi^2}$, $\lambda_3 = \frac{3N_c}{8\pi^2}$, $\lambda_4 = \frac{3N_c}{16\pi^2}$, $\lambda_5 = -\frac{5\sqrt{3}N_c}{8\pi^2}$, $\lambda_6 = \frac{\sqrt{3}N_c}{8\pi^2}$.

λ_1 is rather arbitrary

P-parity violation is possible!