LIGHT SCALARS IN FIELD THEORY

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OUTLINE

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Evidence for four-quark components of light scalars is given.

The priority of Quantum Field Theory in revealing the light scalar mystery is emphasized.

Introduction

The scalar channels in the region up to 1 GeV became a stumbling block of QCD. The point is that both perturbation theory and sum rules do not work in these channels because there are not solitary resonances in this region.

At the same time the question on the nature of the light scalar mesons is major for understanding the mechanism of the chiral symmetry realization, arising from the confinement, and hence for understanding the confinement itself.

Kategorischer Imperativ

To discuss actually the nature of the putative nonet of the light scalar mesons: the putative $f_0(600)$ (or $\sigma(600)$) and $\kappa(700-900)$ mesons and the well-established $f_0(980)$ and $a_0(980)$ mesons, one should explain not only their mass spectrum, particularly the mass degeneracy of the $f_0(980)$ and $a_0(980)$ states, but answer the next real challenges.

1. **The copious $\phi \to \gamma f_0(980)$ decay and especially the copious $\phi \to \gamma a_0(980)$ decay, which looks as the decay plainly forbidden by the Okubo-Zweig-Iizuka (OZI) rule in the quark-antiquark model $a_0(980) = (u\bar{u} - d\bar{d})/\sqrt{2}$.**

Kategorischer Imperativ

2. Absence of $J/\psi \to a_0(980)\rho$ and $J/\psi \to f_0(980)\omega$ in contrast with copious $J/\psi \to a_2(1320)\rho$, $J/\psi \to f_2(1270)\omega$ if $a_0(980)$ and $f_0(980)$ are P-wave states of $q\bar{q}$ like $a_2(1320)$ and $f_2(1270)$ respectively.
3. Absence of $J/\psi \to \gamma f_0(980)$ in contrast with copious $J/\psi \to \gamma f_2(1270)$ and $J/\psi \to \gamma f_2'(1525)\phi$ if $f_0(980)$ is P-wave state of $q\bar{q}$ like $f_2(1270)$ or $f_2'(1525)$.
4. Suppression of $a_0(980) \to \gamma\gamma$ and $f_0(980) \to \gamma\gamma$ in contrast with copious $a_2(1320) \to \gamma\gamma$, $f_2(1270) \to \gamma\gamma$ if $a_0(980)$ and $f_0(980)$ are P-wave state of $q\bar{q}$ like $a_2(1320)$and $f_2(1270)$ respectively.

Place in QCD, Chiral Limit

$$L = -(1/2)Tr\,(G_{\mu\nu}(x)G^{\mu\nu}(x)) + \bar{q}(x)(i\hat{D} - M)q(x),$$

$$M = \begin{pmatrix} m_u & 0 & 0 \\ 0 & m_d & 0 \\ 0 & 0 & m_s \end{pmatrix}, \quad q^j(x) = \begin{pmatrix} u^j(x) \\ d^j(x) \\ s^j(x) \end{pmatrix}, j = 1, 2, 3,$$

$$G_{\mu\nu}(x) = \partial_\mu G_\nu(x) - \partial_\nu G_\mu(x) + ig_0[G_\mu(x), G_\nu(x)],$$

$$G_\mu(x) = \frac{1}{2}\lambda_a G_{a\mu}(x), \quad a = 1, ..., 8, \quad Tr(\lambda_a \lambda_b) = 2\delta_{ab},$$

$$\hat{D} = \gamma^\mu D_\mu, \quad D_\mu = \partial_\mu + ig_0 G_\mu(x), \quad q(\bar{x})q(x) = q(\bar{x})^j q(x)^j.$$

Place in QCD, Chiral Limit

M mixes Left and Right Spaces $q_L(x) = (1/2)(1+\gamma_5)q(x)$ and $q_R(x) = (1/2)(1-\gamma_5)q(x)$. But in chiral limit $M_{f'f} \to 0$ these spaces separate realizing $U_L(3) \times U_R(3)$ flavour symmetry,

$$\mathrm{q}'_{\mathrm{L,R}}(\mathrm{x}) = \mathrm{u}_{\mathrm{L,R}} \cdot \mathrm{q}_{\mathrm{L,R}}(\mathrm{x}),$$

which, however, is broken by the gluonic anomaly up to

$$\mathrm{U_{vec}}(1) \times \mathrm{SU_L}(3) \times \mathrm{SU_R}(3).$$

As Experiment suggests, Confinement forms colourless observable hadronic fields and spontaneous breaking of chiral symmetry with massless pseudoscalar fields. There are two possible scenarios for QCD at low energy.

Confinement, σ-Models

1. **Non-linear σ-model.**

$$\mathbf{L} = \left(f_\pi^2/4\right) \mathrm{Tr} \left(\partial_\mu V(x) \partial^\mu V^+(x)\right) + \ldots ,$$

$$V(x) = \exp\left\{i\sqrt{2}\pi(x)/f_\pi\right\}, \quad V'(x) = U_L V(x) U_R ,$$

$$\pi(x) = \pi(x)_A t_A , \quad f_\pi = 92.4 \text{ MeV} ,$$

$$t_A = \frac{1}{\sqrt{3}} I \text{ at } A = 0, \; t_A = \frac{1}{\sqrt{2}} \lambda_A \text{ at } A = 1, 2, 3, 4, 5, 6, 7, 8 .$$

Confinement, σ-Models

2. Linear σ-model.

$$\mathrm{L} = (1/2)\,\mathrm{Tr}\left(\partial_\mu \mathsf{V}(\mathrm{x})\partial^\mu \mathsf{V}^+(\mathrm{x})\right) - \mathrm{W}\left(\mathsf{V}(\mathrm{x})\mathsf{V}^+(\mathrm{x})\right)\ ,$$

$$\mathsf{V}(\mathrm{x}) = \sigma(\mathrm{x}) + \mathrm{i}\pi(\mathrm{x}) = (\sigma_{\mathrm{A}}(\mathrm{x}) + \mathrm{i}\pi_{\mathrm{A}}(\mathrm{x}))\,\mathrm{t}_{\mathrm{A}}\ ,$$

$$\mathsf{V}'(\mathrm{x}) = \mathsf{U}_{\mathrm{L}}\mathsf{V}(\mathrm{x})\mathsf{U}_{\mathrm{R}}\ .$$

The experimental nonet of the light scalar mesons, the putative $f_0(600)$ (or $\sigma(600)$) and $\kappa(700-900)$ mesons and the well-established $f_0(980)$ and $a_0(980)$ mesons suggests the $U_L(3)\times U_R(3)$ linear σ-model.

Time

Hunting the light σ and κ mesons had begun in the sixties already and a preliminary information on the light scalar mesons in Particle Data Group (PDG) Reviews had appeared at that time. But long-standing unsuccessful attempts to prove their existence in a conclusive way entailed general disappointment and an information on these states disappeared from PDG Reviews. One of principal reasons against the σ and κ mesons was the fact that both $\pi\pi$ and πK scattering phase shifts do not pass over 90^0 at putative resonance masses.

$SU_L(2) \times SU_R(2)$ linear σ model

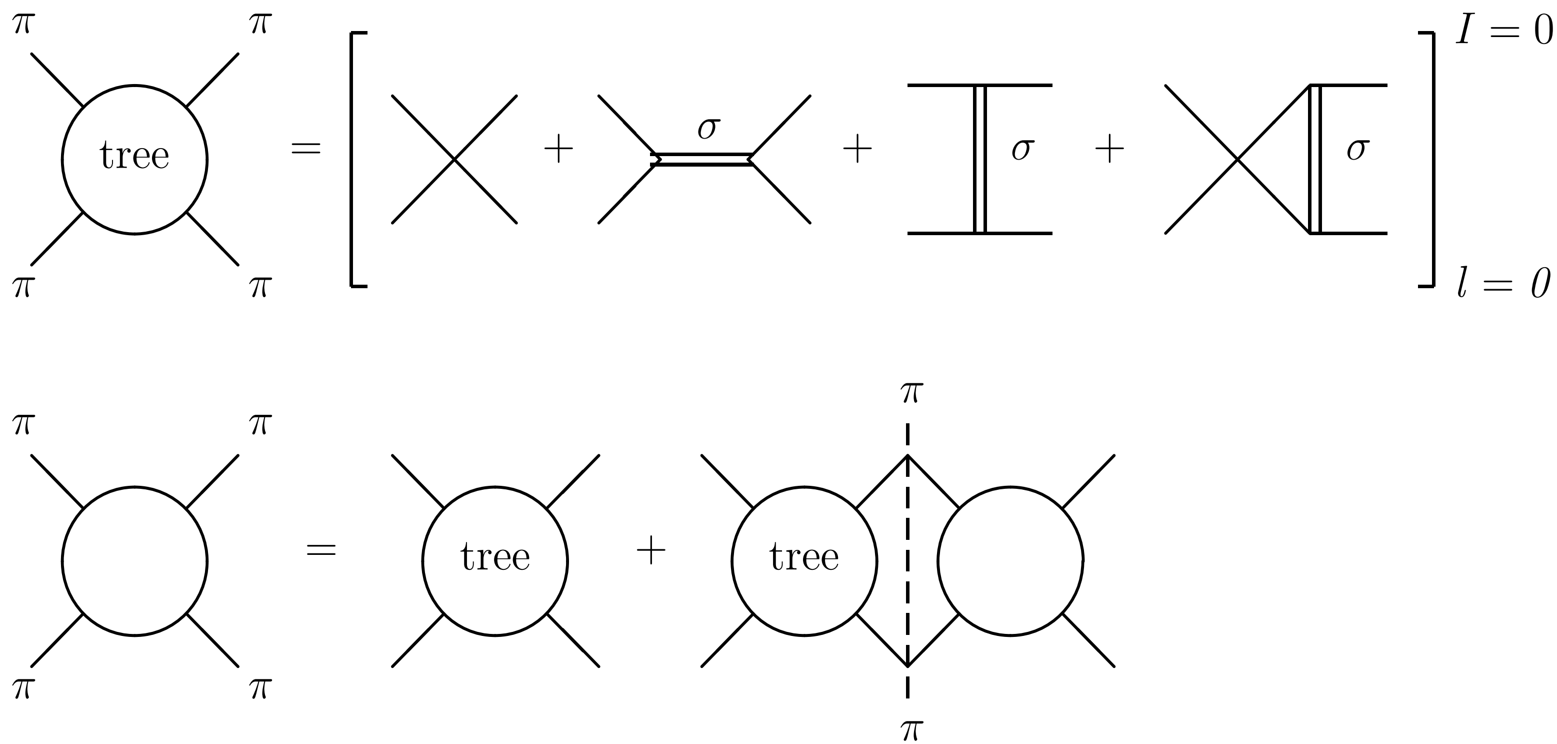
Situation changes when we showed that in the linear σ-model

$$\mathbf{L} = \frac{1}{2}\left[(\partial_\mu \sigma)^2 + (\partial_\mu \vec{\pi})^2\right] - \frac{m_\sigma^2}{2}\sigma^2 - \frac{m_\pi^2}{2}\vec{\pi}^2$$

$$-\frac{m_\sigma^2 - m_\pi^2}{8f_\pi^2}\left[\left(\sigma^2 + \vec{\pi}^2\right)^2 + 4f_\pi \sigma\left(\sigma^2 + \vec{\pi}^2\right)\right]^2$$

there is a negative background phase which hides the σ meson (1993, 1994). It has been made clear that shielding wide lightest scalar mesons in chiral dynamics is very natural. This idea was picked up and triggered new wave of theoretical and experimental searches for the σ and κ mesons.

Our approximation



Our approximation

$$T_0^{0(tree)} = \frac{m_\pi^2 - m_\sigma^2}{32\pi f_\pi^2}\left[5 - 3\frac{m_\sigma^2 - m_\pi^2}{m_\sigma^2 - s}\right.$$

$$\left. -2\frac{m_\sigma^2 - m_\pi^2}{s - 4m_\pi^2}\ln\left(1 + \frac{s - 4m_\pi^2}{m_\sigma^2}\right)\right],$$

$$\mathrm{T}_0^0 = \frac{\mathrm{T}_0^{0(\mathrm{tree})}}{1 - \mathrm{i}\rho_{\pi\pi}\mathrm{T}_0^{0(\mathrm{tree})}} = \frac{\mathrm{e}^{2\mathrm{i}(\delta_{\mathrm{bg}}+\delta_{\mathrm{res}})} - 1}{2\mathrm{i}\rho_{\pi\pi}}$$

$$= \frac{1}{\rho_{\pi\pi}}\left(\frac{\mathrm{e}^{2\mathrm{i}\delta_{\mathrm{bg}}} - 1}{2\mathrm{i}}\right) + \mathrm{e}^{2\mathrm{i}\delta_{\mathrm{bg}}}\mathrm{T}_{\mathrm{res}},$$

$$\rho_\pi \equiv \rho_{\pi\pi} \equiv \rho_{\pi\pi}(m) = \sqrt{1 - 4m_\pi^2/m^2}.$$

Our approximation

$$T_{res} = \frac{\sqrt{s}\Gamma_{res}(s)}{M_{res}^2 - s + \mathrm{Re}\Pi_{res}(M_{res}^2) - \Pi_{res}(s)} = \frac{e^{2i\delta_{res}} - 1}{2i\rho_{\pi\pi}},$$

$$T_{bg} = \frac{e^{2i\delta_{bg}} - 1}{2i\rho_{\pi\pi}} = \frac{\lambda(s)}{1 - i\rho_{\pi\pi}\lambda(s)}, \quad \lambda(s) = \frac{m_\pi^2 - m_\sigma^2}{32\pi f_\pi^2}\left[5 - \right.$$

$$\left. -2\frac{m_\sigma^2 - m_\pi^2}{s - 4m_\pi^2}\ln\left(1 + \frac{s - 4m_\pi^2}{m_\sigma^2}\right)\right], \quad g_{\sigma\pi^+\pi^-} = -\frac{m_\sigma^2 - m_\pi^2}{f_\pi},$$

$$\mathrm{Im}\Pi_{res}(s) = \frac{g_{res}^2(s)}{16\pi}\rho_{\pi\pi}\,, \; \mathrm{Re}\Pi_{res}(s) = -\frac{g_{res}^2(s)}{16\pi}\lambda(s)\rho_{\pi\pi}^2\,,$$

$$g_{res}(s) = \frac{g_{\sigma\pi\pi}}{|1 - i\rho_{\pi\pi}\lambda(s)|}\,, \; M_{res}^2 = m_\sigma^2 - \mathrm{Re}\Pi_{res}(M_{res}^2)\,.$$

Results in our approximation

$$T_0^2 = \frac{T_0^{2(tree)}}{1 - \imath\rho_{\pi\pi} T_2^{0(tree)}} = \frac{e^{2i\delta_0^2} - 1}{2i\rho_{\pi\pi}}, \quad \mathrm{g}_{\sigma\pi\pi} = \sqrt{\frac{3}{2}}\,\mathrm{g}_{\sigma\pi^+\pi^-},$$

$$T_0^{2(tree)} = \frac{m_\pi^2 - m_\sigma^2}{32\pi f_\pi^2}\left[2 - 2\frac{m_\sigma^2 - m_\pi^2}{s - 4m_\pi^2}\ln\left(1 + \frac{s - 4m_\pi^2}{m_\sigma^2}\right)\right].$$

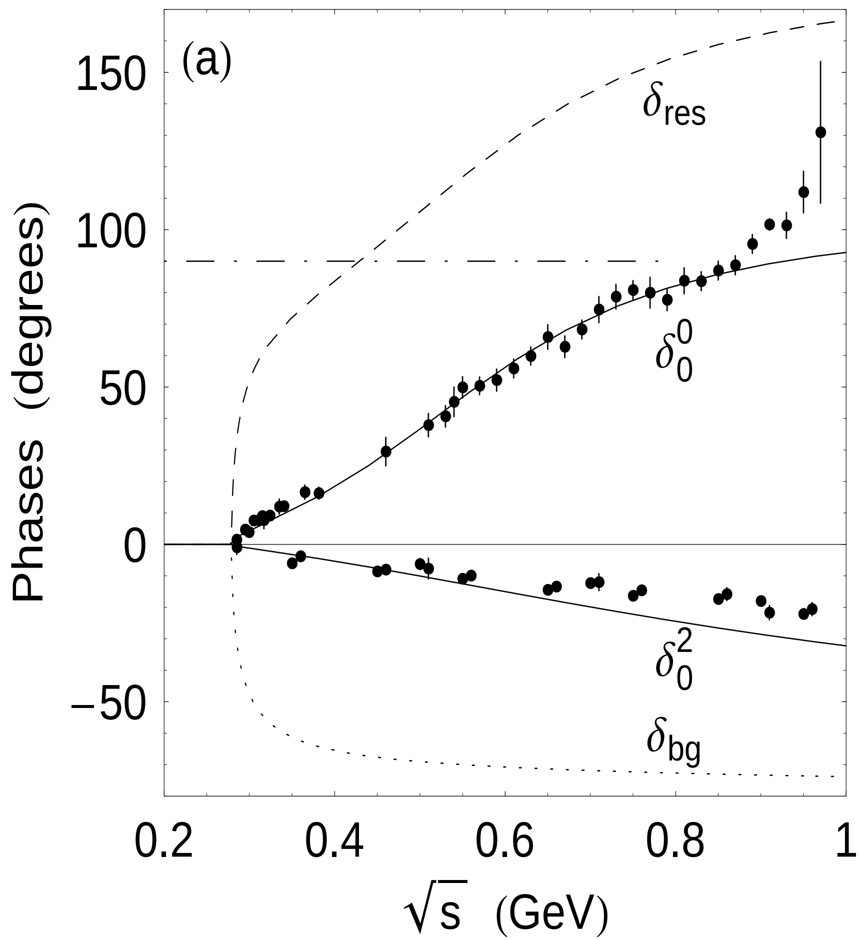
$$\mathrm{M_{res}} = 0.43\,\mathrm{GeV},\ \Gamma_{\mathrm{res}}(\mathrm{M_{res}^2}) = 0.67\,\mathrm{GeV},\ \mathrm{m}_\sigma = 0.93\,\mathrm{GeV},$$

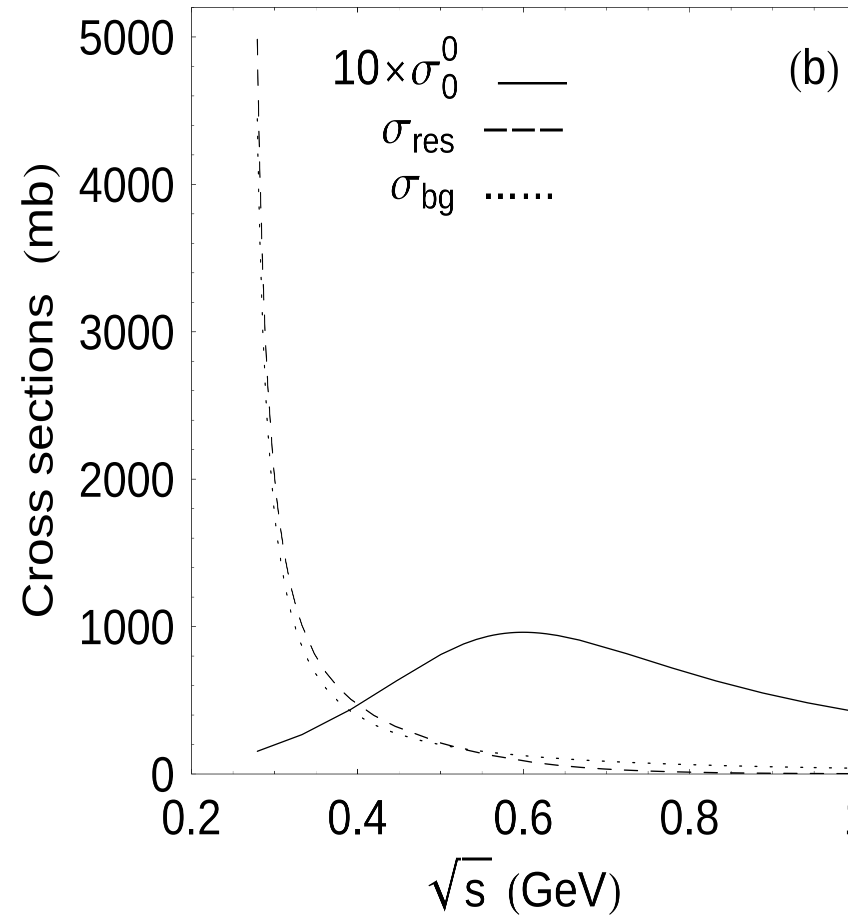
$$\Gamma_{\mathrm{res}}^{\mathrm{renorm}}(\mathrm{M_{res}^2}) = \frac{\Gamma_{\mathrm{res}}(\mathrm{M_{res}^2})}{\left(1 + \mathrm{dRe}\Pi_{\mathrm{res}}(\mathrm{s})/\mathrm{ds}|_{\mathrm{s=M_{res}^2}}\right)} = 0.53\,\mathrm{GeV},$$

$$\Gamma_{\mathrm{res}}(\mathrm{s}) = \frac{\mathrm{g_{res}^2(s)}}{16\pi\sqrt{\mathrm{s}}}\rho_{\pi\pi}, \quad a_0^0 = 0.18\,m_\pi^{-1}, \quad a_0^2 = -0.04\,m_\pi^{-1},$$

$$g_{res}(M_{res}^2)/g_{\sigma\pi\pi} = 0.33,\ (\mathrm{s_A})_0^0 = 0.45\,\mathrm{m}_\pi^2,\ (\mathrm{s_A})_0^2 = 2.02\,\mathrm{m}_\pi^2.$$

Chiral Shielding in $\pi\pi \to \pi\pi$





The σ model. Our approximation. $\delta = \delta_{res} + \delta_{bg}$.

The σ pole in $\pi\pi \to \pi\pi$

$$T_0^0 \to \frac{g_\pi^2}{s - s_R}, \qquad \mathrm{T_{res}} \to \frac{\left(\mathrm{g}_\pi^{\mathrm{res}}\right)^2}{\mathrm{s} - \mathrm{s_R}},$$

$$\mathrm{g}_\pi^2 = (0.12 + \mathrm{i}0.21)\mathrm{GeV}^2, \quad \left(\mathrm{g}_\pi^{\mathrm{res}}\right)^2 = -(0.25 + \mathrm{i}0.11)\mathrm{GeV}^2,$$

$$s_R = (0.21 - i0.26)\mathrm{GeV}^2,$$

$$\sqrt{\mathrm{s_R}} = \mathrm{M_R} - \mathrm{i}\frac{\Gamma_\mathrm{R}}{2} = (0.52 - \mathrm{i}0.25)\mathrm{GeV}.$$

Considering the residue of the σ pole in T_0^0 as the square of its coupling constant to the $\pi\pi$ channel is not a clear guide to understand the σ meson nature for its great obscure imaginary part.

The σ propagator

$$\frac{1}{D_\sigma(s)} = \frac{1}{M_{res}^2 - s + \mathrm{Re}\Pi_{res}(M_{res}^2) - \Pi_{res}(s)}.$$

The σ meson self-energy $\Pi_{res}(s)$ is caused by the intermediate $\pi\pi$ states, that is, by the four-quark intermediate states if we keep in mind that the $SU_L(2) \times SU_R(2)$ linear σ model could be the low energy realization of the two-flavour QCD. This contribution shifts the Breit-Wigner (BW) mass greatly $m_\sigma - M_{res} = 0.50$ GeV. So, half the BW mass is determined by the four-quark contribution at least. The imaginary part dominates the propagator modulus in the region 300 MeV $< \sqrt{s} <$ 600 MeV. So, the σ field is described by its four-quark component at least in this energy (virtuality) region.

Chiral shielding in $\gamma\gamma \to \pi^+\pi^-$

$$T_S(\gamma\gamma \to \pi^+\pi^-) = T_S^{Born}(\gamma\gamma \to \pi^+\pi^-)$$

$$+8\alpha I_{\pi^+\pi^-} T_S(\pi^+\pi^- \to \pi^+\pi^-)$$

$$= T_S^{Born}(\gamma\gamma \to \pi^+\pi^-) + 8\alpha I_{\pi^+\pi^-} \left(\frac{2}{3} T_0^0 + \frac{1}{3} T_0^2\right)$$

$$= \frac{2}{3} e^{i\delta_0^0} \left\{ T_S^{Born}(\gamma\gamma \to \pi^+\pi^-) \cos\delta_0^0 + 8\frac{\alpha}{\rho_{\pi\pi}} \mathrm{Re} I_{\pi^+\pi^-} \sin\delta_0^0 \right\}$$

$$+\frac{1}{3} e^{i\delta_0^2} \left\{ T_S^{Born}(\gamma\gamma \to \pi^+\pi^-) \cos\delta_0^2 + 8\frac{\alpha}{\rho_{\pi\pi}} \mathrm{Re} I_{\pi^+\pi^-} \sin\delta_0^2 \right\}$$

Chiral shielding in $\gamma\gamma \to \pi^0\pi^0$

$$T_S(\gamma\gamma \to \pi^0\pi^0) = 8\alpha I_{\pi^+\pi^-} T_S(\pi^+\pi^- \to \pi^0\pi^0)$$

$$= 8\alpha I_{\pi^+\pi^-} \left(\frac{2}{3}T_0^0 - \frac{2}{3}T_0^2\right)$$

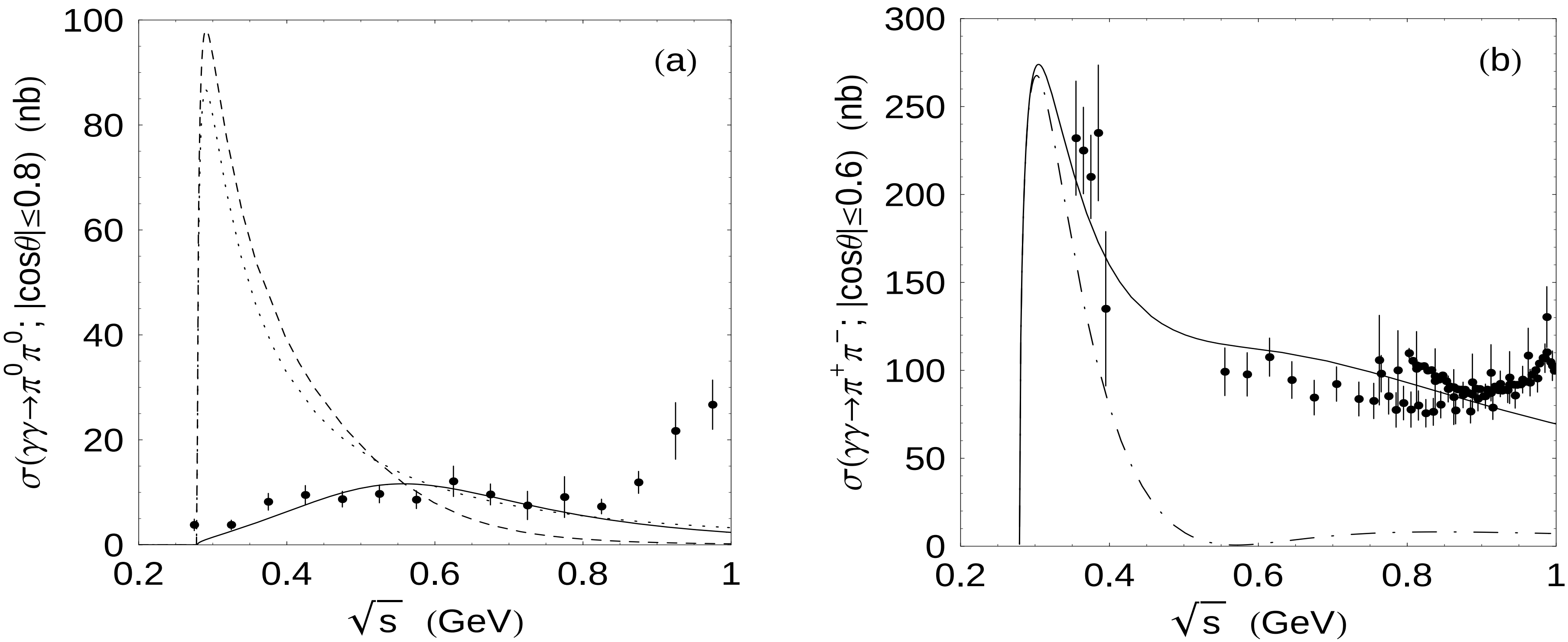
$$= \frac{2}{3}e^{i\delta_0^0}\left\{T_S^{Born}(\gamma\gamma \to \pi^+\pi^-)\cos\delta_0^0 + 8\frac{\alpha}{\rho_{\pi\pi}}\text{Re}I_{\pi^+\pi^-}\sin\delta_0^0\right\}$$

$$-\frac{2}{3}e^{i\delta_0^2}\left\{T_S^{Born}(\gamma\gamma \to \pi^+\pi^-)\cos\delta_0^2 + 8\frac{\alpha}{\rho_{\pi\pi}}\text{Re}I_{\pi^+\pi^-}\sin\delta_0^2\right\}$$

$$I_{\pi^+\pi^-} = \frac{m_\pi^2}{s}\left(\pi + i\ln\frac{1+\rho_{\pi\pi}}{1-\rho_{\pi\pi}}\right)^2 - 1\,, \quad s \geq 4m_\pi^2\,,$$

$$T_S^{Born}(\gamma\gamma \to \pi^+\pi^-) = \frac{8\alpha}{\rho_{\pi^+\pi^-}}\text{Im}I_{\pi^+\pi^-}\,.$$

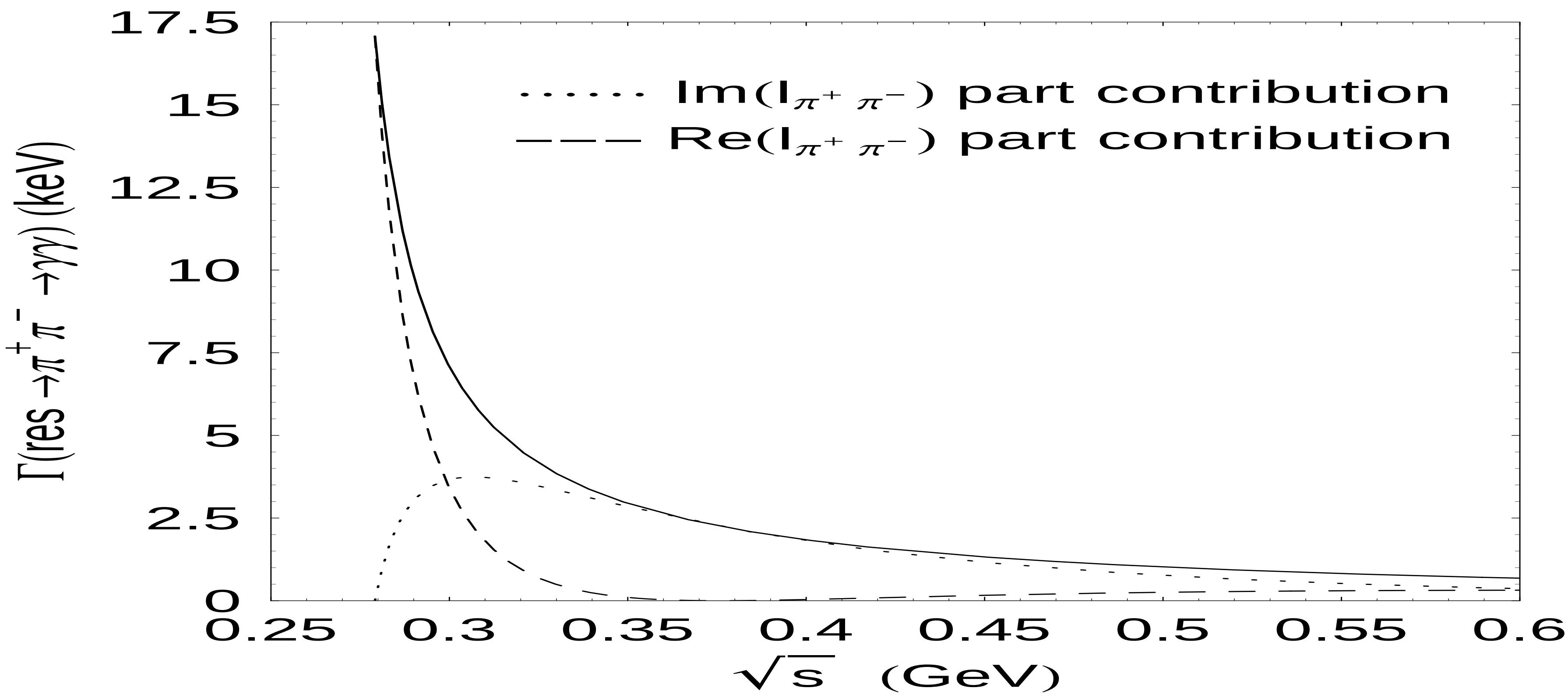
Chiral Shielding in $\gamma\gamma \to \pi\pi$



(a) The solid, dashed, and dotted lines are $\sigma_S(\gamma\gamma \to \pi^0\pi^0)$, $\sigma_{res}(\gamma\gamma \to \pi^0\pi^0)$, and $\sigma_{bg}(\gamma\gamma \to \pi^0\pi^0)$.

(b) The dashed-dotted line is $\sigma_S(\gamma\gamma \to \pi^+\pi^-)$. The solid line includes the higher waves from $T^{Born}(\gamma\gamma \to \pi^+\pi^-)$.

The $\sigma \to \gamma\gamma$ decay.



$$\mathbf{g}(\sigma \to \pi^+\pi^- \to \gamma\gamma,\, \mathbf{s}) = (\alpha/2\pi)\mathbf{I}_{\pi^+\pi^-} \times \mathbf{g}_{\mathbf{res}\,\pi^+\pi^-}(\mathbf{s}),$$

$$\Gamma(\sigma \to \pi^+\pi^- \to \gamma\gamma, \mathbf{s}) = \frac{1}{16\pi\sqrt{\mathbf{s}}} \left|\mathbf{g}(\sigma \to \pi^+\pi^- \to \gamma\gamma, \mathbf{s})\right|^2$$

Four-quark transition $\sigma \to \gamma\gamma$

So, the the $\sigma \to \gamma\gamma$ decay is described by the triangle $\pi^+\pi^-$-loop diagram $res \to \pi^+\pi^- \to \gamma\gamma$ ($I_{\pi^+\pi^-}$). Consequently, it is due to the four-quark transition because we imply a low energy realization of the two-flavour QCD by means of the the $SU_L(2) \times SU_R(2)$ linear σ model. As the previous Fig. suggests, the real intermediate $\pi^+\pi^-$ state dominates in $g(res \to \pi^+\pi^- \to \gamma\gamma)$ in the σ region $\sqrt{s} < 0.6$ GeV.

Thus the picture in the physical region is clear and informative. But, what about the pole in the complex s-plane? Does the pole residue reveal the σ indeed?

The σ pole in $\gamma\gamma \to \pi\pi$

$$\frac{1}{16\pi}\sqrt{\frac{3}{2}}\,T_S(\gamma\gamma \to \pi^0\pi^0) \to \frac{g_\gamma g_\pi}{s - s_R},$$

$$\frac{1}{16\pi}\sqrt{\frac{3}{2}}\,T_{res}(\gamma\gamma \to \pi^0\pi^0) \to \frac{g_\gamma^{res} g_\pi^{res}}{s - s_R},$$

$$g_\gamma g_\pi = (-0.45 - i0.19) \times 10^{-3}\,\mathrm{GeV}^2,$$

$$g_\gamma = (-0.985 + i0.12) \times 10^{-3}\,\mathrm{GeV}^2,$$

$$g_\gamma^{res} g_\pi^{res} = (0.53 - i0.13) \times 10^{-3}\,\mathrm{GeV}^2,$$

$$g_\gamma^{res} = (-0.45 - i0.95) \times 10^{-3}\,\mathrm{GeV}^2,$$

$$g_\gamma / g_\pi = g_\gamma^{res} / g_\pi^{res} = (-1.61 + i1.21) \times 10^{-3},$$

$$\Gamma(\sigma \to \gamma\gamma) = \frac{|g_\gamma|^2}{M_R} \approx \Gamma_{res}(\sigma \to \gamma\gamma) = \frac{|g_\gamma^{res}|^2}{M_R} \approx 2\,\mathrm{keV}.$$

The σ pole in $\gamma\gamma \to \pi\pi$

It is interesting to compare ratios $g_\gamma/g_\pi = g_\gamma^{res}/g_\pi^{res}$ with the ratio

$$\frac{\mathbf{g}\left(\mathbf{res} \to \pi^+\pi^- \to \gamma\gamma\,,\ \mathbf{M}^2_{\mathbf{res}}\right)}{\mathbf{g}_{\mathbf{res}}\left(\mathbf{M}^2_{\mathbf{res}}\right)} = (-0.35 + i1.25) \times 10^{-3}\,,$$

which are independent on the different normalization in themselves.

It is hard to believe that anybody could learn the complex but physically clear dynamics of the $\sigma \to \gamma\gamma$ decay described above from the residues of the σ pole.

First Discussion and Conclusion

Heiri Leutwyler and collaborators obtained

$$\sqrt{\mathrm{s_R}} = \mathrm{M_R} - \mathrm{i}\Gamma_\mathrm{R}/2 = \left(441^{+16}_{-8} - \mathrm{i}272^{+12.5}_{-9}\right) \times \text{MeV}$$

with the help of the Roy equation.

Our result agrees with the above only qualitatively.

$$\sqrt{\mathrm{s_R}} = \mathrm{M_R} - \mathrm{i}\Gamma_\mathrm{R}/2 = (518 - \mathrm{i}250) \times \text{MeV}.$$

This is natural, because our approximation gives only a semiquantitative description of the data at $\sqrt{s} < 0.4$ GeV. We do not regard also for effects of the $K\bar{K}$ channel, the $f_0(980)$ meson, and so on, that is, do not consider the $SU_L(3) \times SU_R(3)$ linear σ model.

First Discussion and Conclusion

Could the above scenario incorporates the primary lightest scalar **Bob Jaffe four-quark state**? Certainly the direct coupling of this state to $\gamma\gamma$ via neutral vector pairs ($\rho^0\rho^0$ and $\omega\omega$), contained in its wave function, is negligible

$\Gamma(q^2\bar{q}^2 \to \rho^0\rho^0 + \omega\omega \to \gamma\gamma) \approx 10^{-3}$ keV

as we showed in 1982. But its coupling to $\pi\pi$ is strong and leads to $\Gamma(q^2\bar{q}^2 \to \pi^+\pi^- \to \gamma\gamma)$ similar to $\Gamma(res \to \pi^+\pi^- \to \gamma\gamma)$ in the above Fig..

Let us add to $T_S(\gamma\gamma \to \pi^0\pi^0)$ the amplitude for the the direct coupling of σ to $\gamma\gamma$ conserving unitarity

$$T_{direct}(\gamma\gamma \to \pi^0\pi^0) = s g^{(0)}_{\sigma\gamma\gamma} g_{res}(s) e^{i\delta_{bg}} \Big/ D_{res}(s)\,,$$

where $g^{(0)}_{\sigma\gamma\gamma}$ is the direct coupling constant of σ to $\gamma\gamma$, the factor s is caused by gauge invariance.

First Discussion and Conclusion

Fitting the $\gamma\gamma \to \pi^0\pi^0$ data gives a negligible value of $g^{(0)}_{\sigma\gamma\gamma}$,

$$\Gamma^{(0)}_{\sigma\gamma\gamma} = \left| M^2_{res} g^{(0)}_{\sigma\gamma\gamma} \right|^2 \Big/ (16\pi M_{res}) \approx 0.0034 \text{ keV},$$

in astonishing agreement with our prediction (1982).

The majority of current investigations of the mass spectra in scalar channels does not study particle production mechanisms. That is why such investigations are only preprocessing experiments, and the derivable information is very relative.

The only progress in understanding the particle production mechanisms could essentially further us in revealing the light scalar meson nature, as is evident from the foregoing.

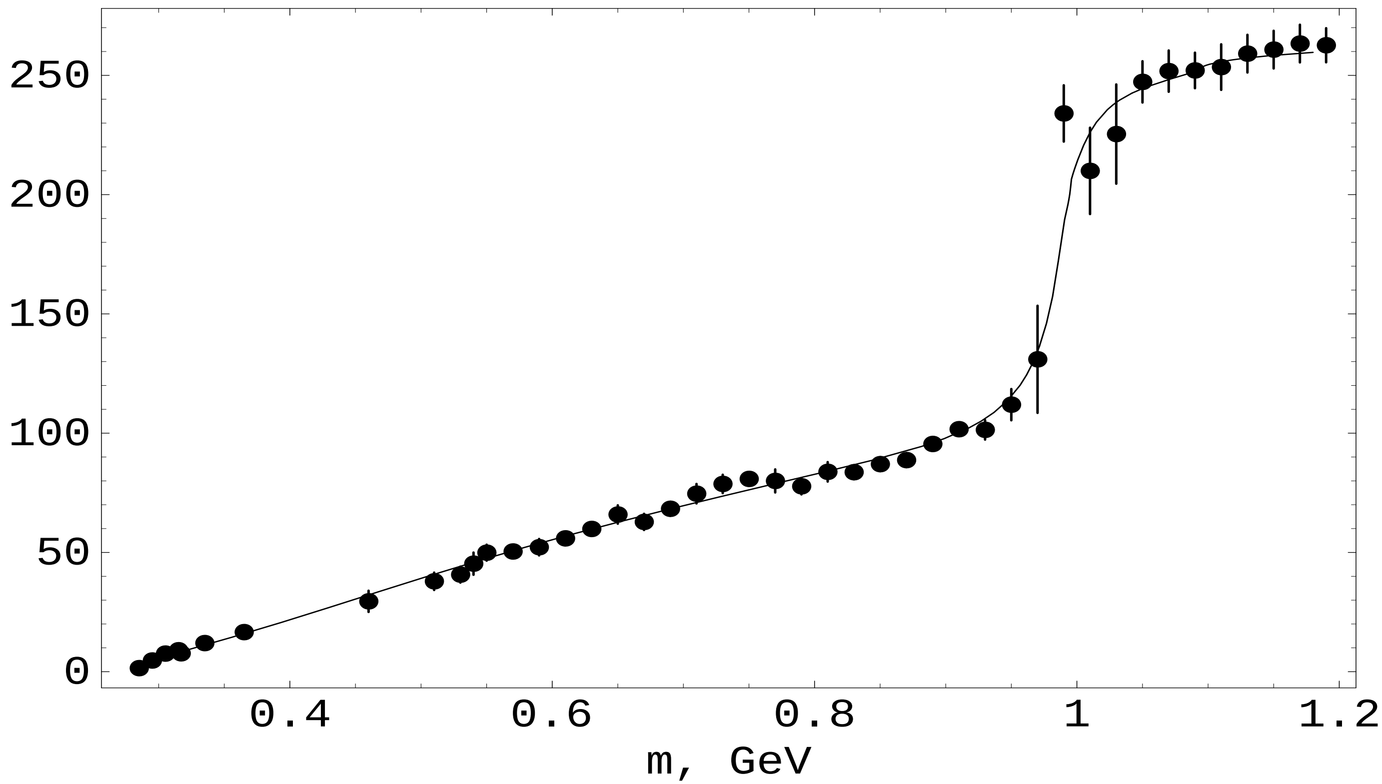
Troubles and Expectancies

In theory the principal problem is impossibility to use the linear σ-model in the tree level approximation inserting widths into σ meson propagators because such an approach breaks the both unitarity and Adler self-consistency conditions. Strictly speaking, the comparison with the experiment requires the non-perturbative calculation of the process amplitudes. Nevertheless, now there are the possibilities to estimate odds of the $U_L(3) \times U_R(3)$ linear σ-model to underlie physics of light scalar mesons in phenomenology. Really, even now there is a huge body of information about the S- waves of different two-particle pseudoscalar states and what is more

Troubles and Expectancies

the relevant information go to press almost continuously from BES, BNL, CERN, CESR, DAΦNE, FNAL, KEK, SLAC and others. As for theory, we know quite a lot about the scenario under discussion: the nine scalar mesons, the putative chiral shielding of the $\sigma(600)$ and $\kappa(700-900)$ mesons, the unitarity, analiticity and Adler self-consistency conditions. In addition, there is the light scalar meson treatment motivated by field theory. The foundations of this approach were formulated in our papers . In particular, in this approach were introduced propagators of scalar mesons (1979-1984), satisfying the Källen – Lehmann representation (2004).

Phen. Chiral Shielding, $\delta_0^0 = \delta_B^{\pi\pi} + \delta_{res}$



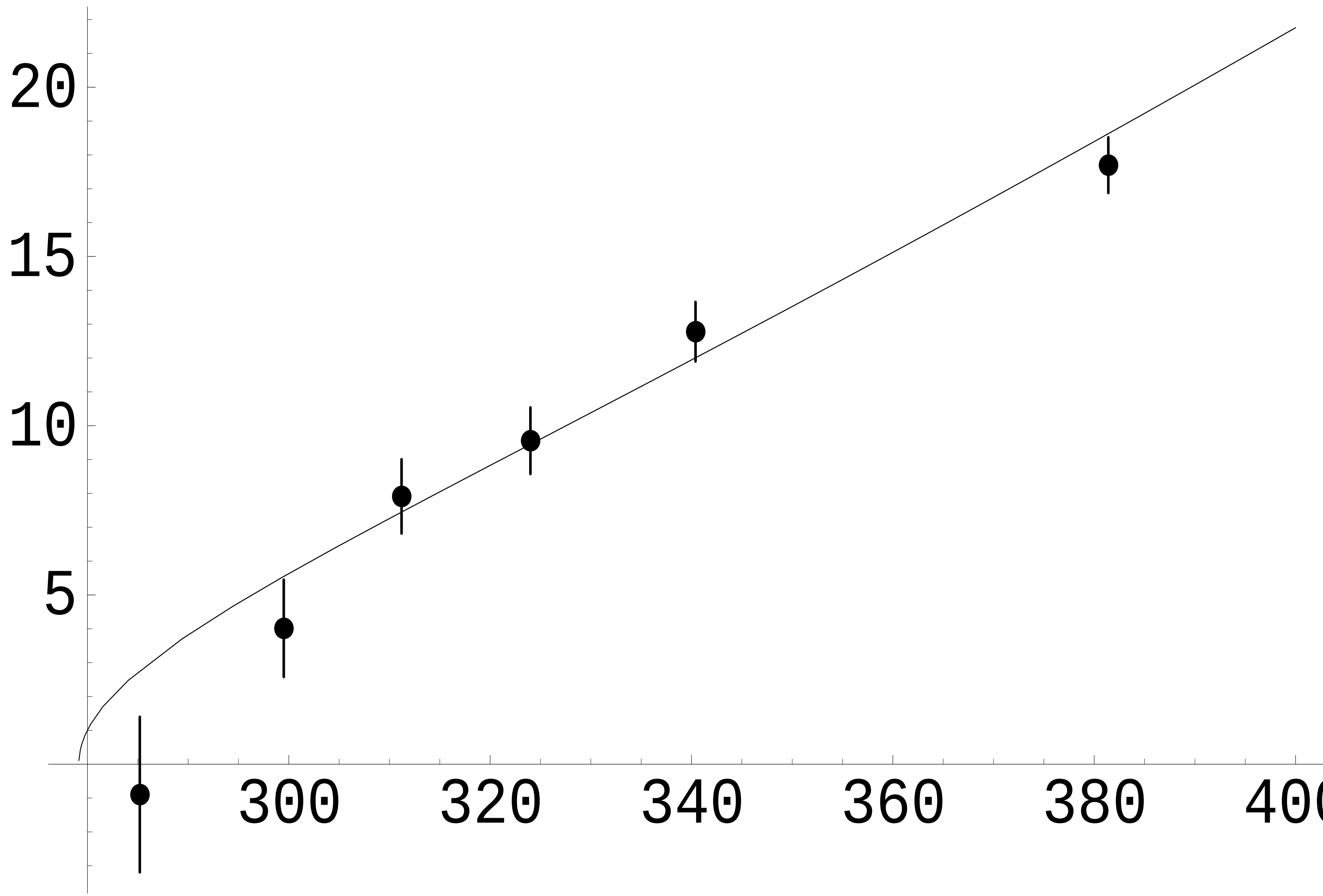
$$g^2_{\sigma\pi^+\pi^-}/4\pi = 0.99\,\text{GeV}^2, \quad g^2_{\sigma K^+K^-}/4\pi = 2\cdot 10^{-4}\,\text{GeV}^2$$

$$g^2_{f_0\pi^+\pi^-}/4\pi = 0.12\,\text{GeV}^2, \quad g^2_{f_0 K^+K^-}/4\pi = 1.04\,\text{GeV}^2$$

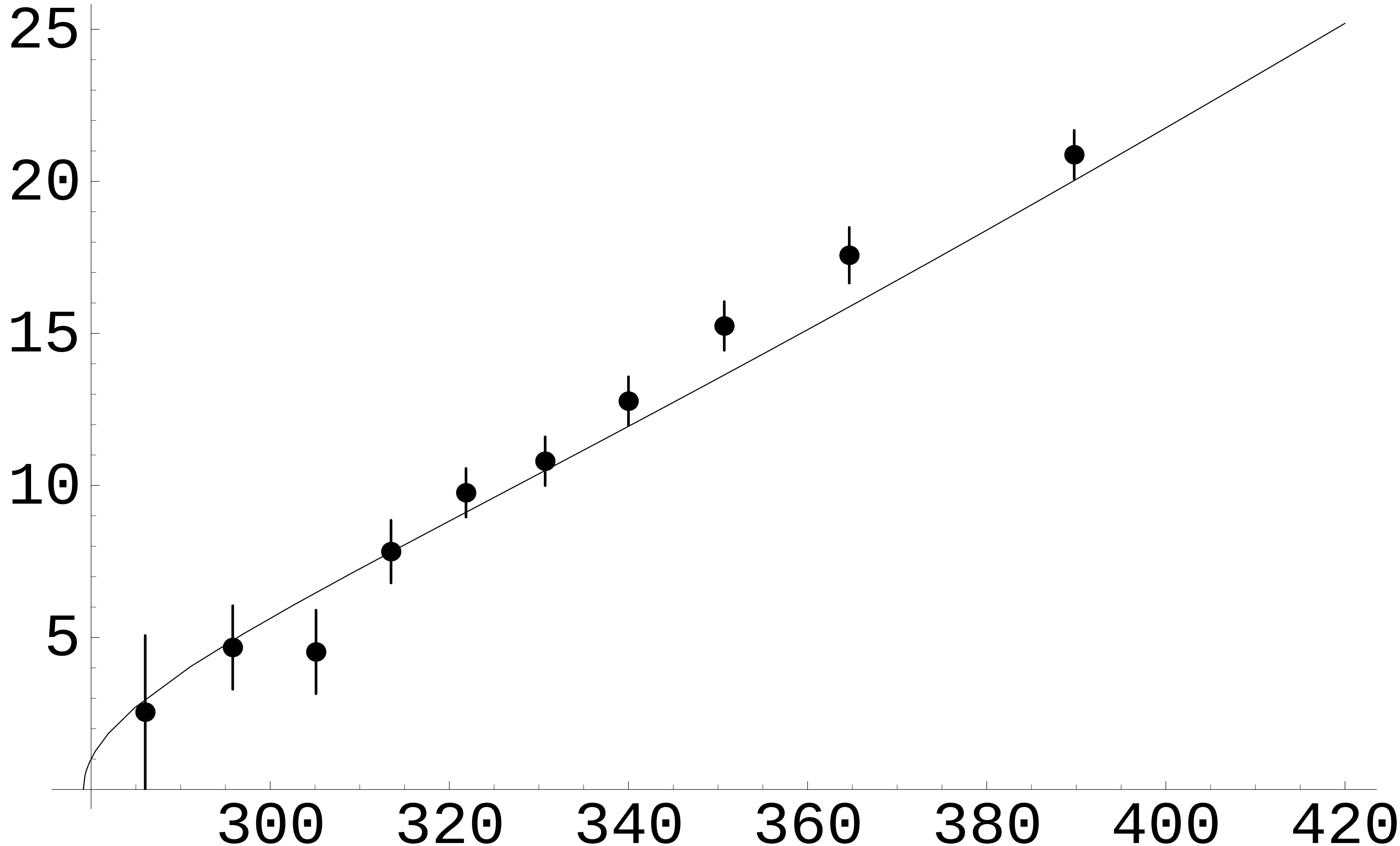
$$m_\sigma = 679\,\text{MeV}\,,\ \Gamma_\sigma = 498\,\text{MeV}\,,\ m_{f_0} = 989\,\text{MeV}\,,$$

the $l = I = 0$ $\pi\pi$ scattering length $a_0^0 = 0.223\, m_{\pi^+}^{-1}$

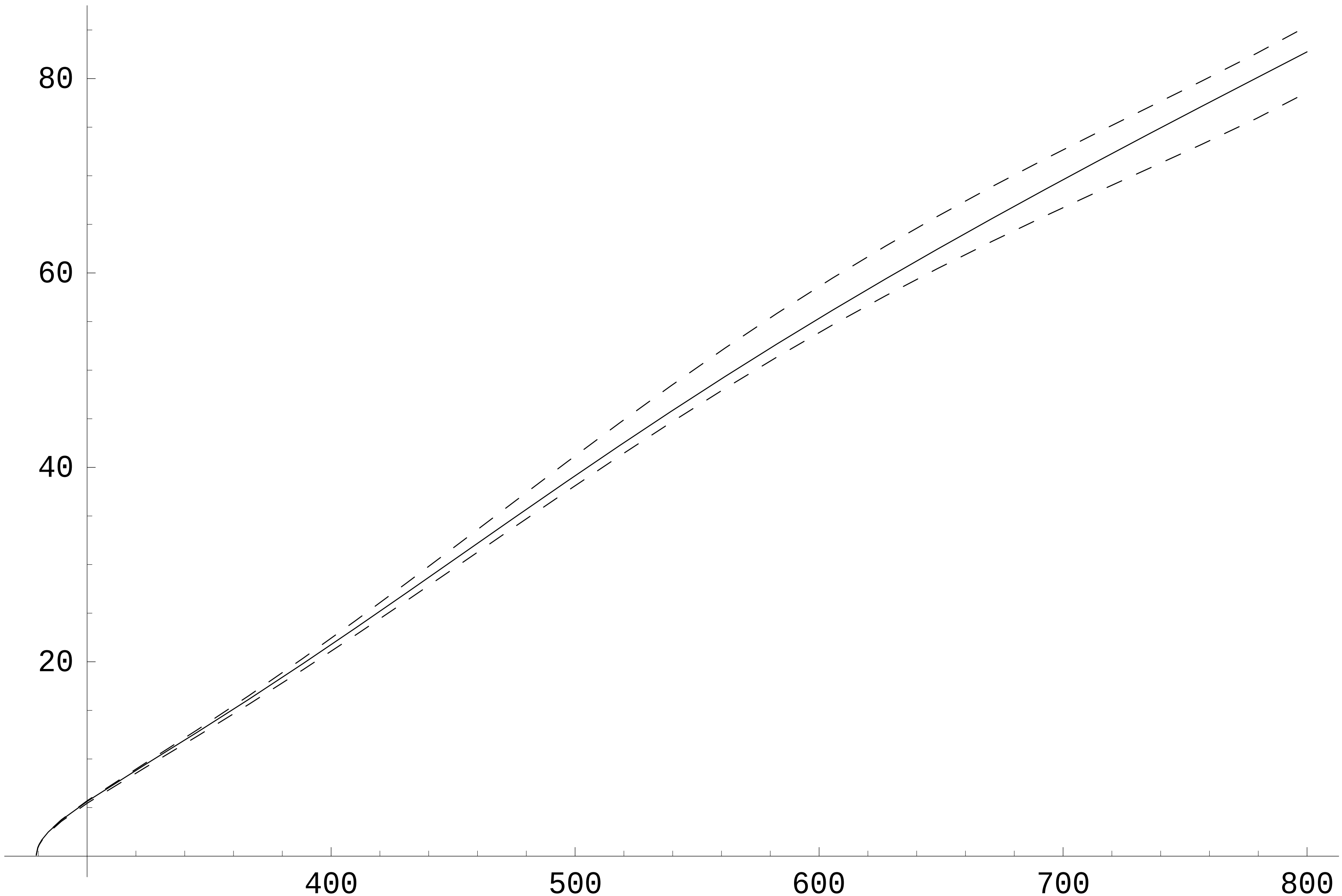
δ_0^0, comparison with BNL data



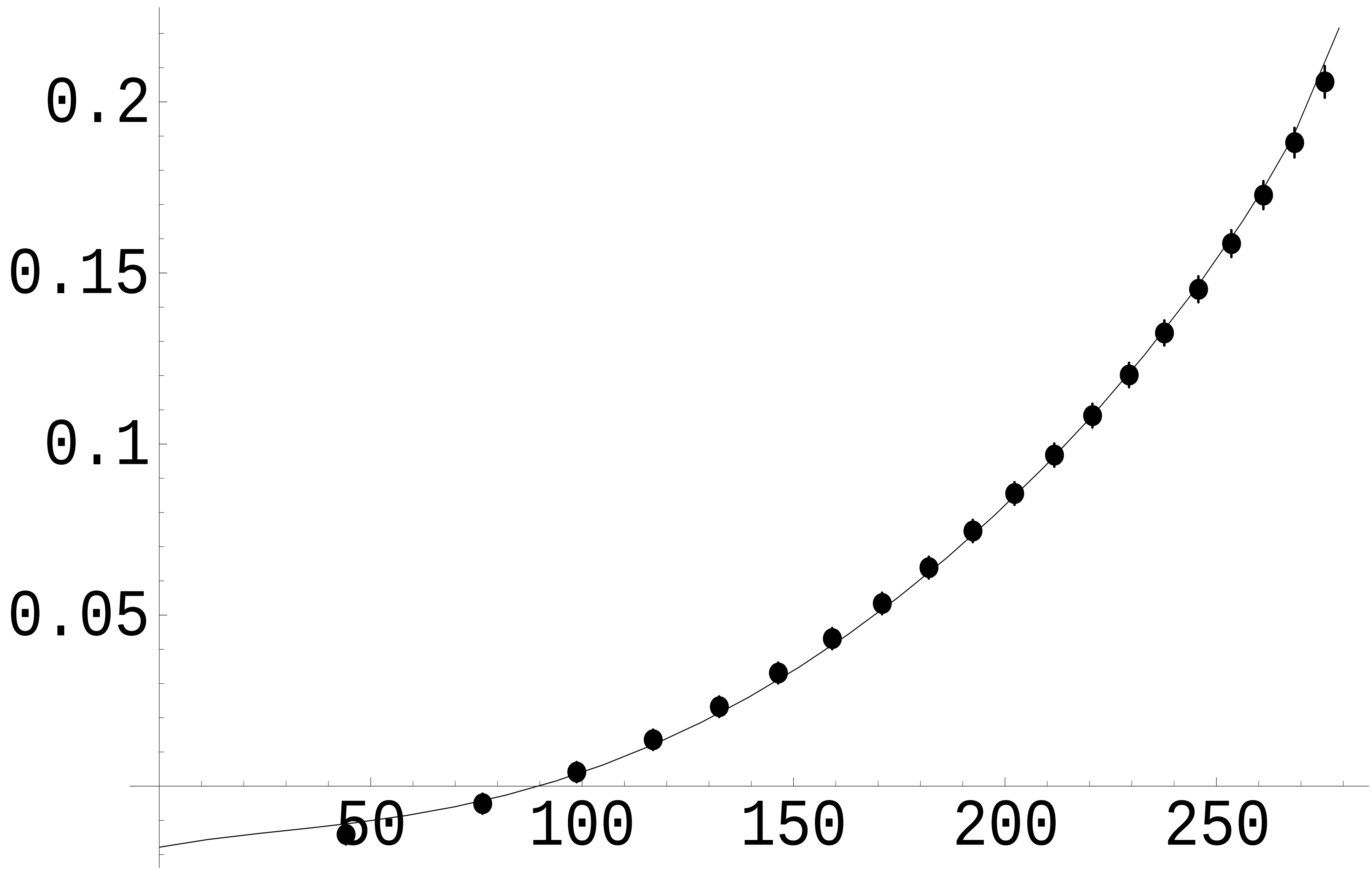
δ_0^0, comparison with NA48 data



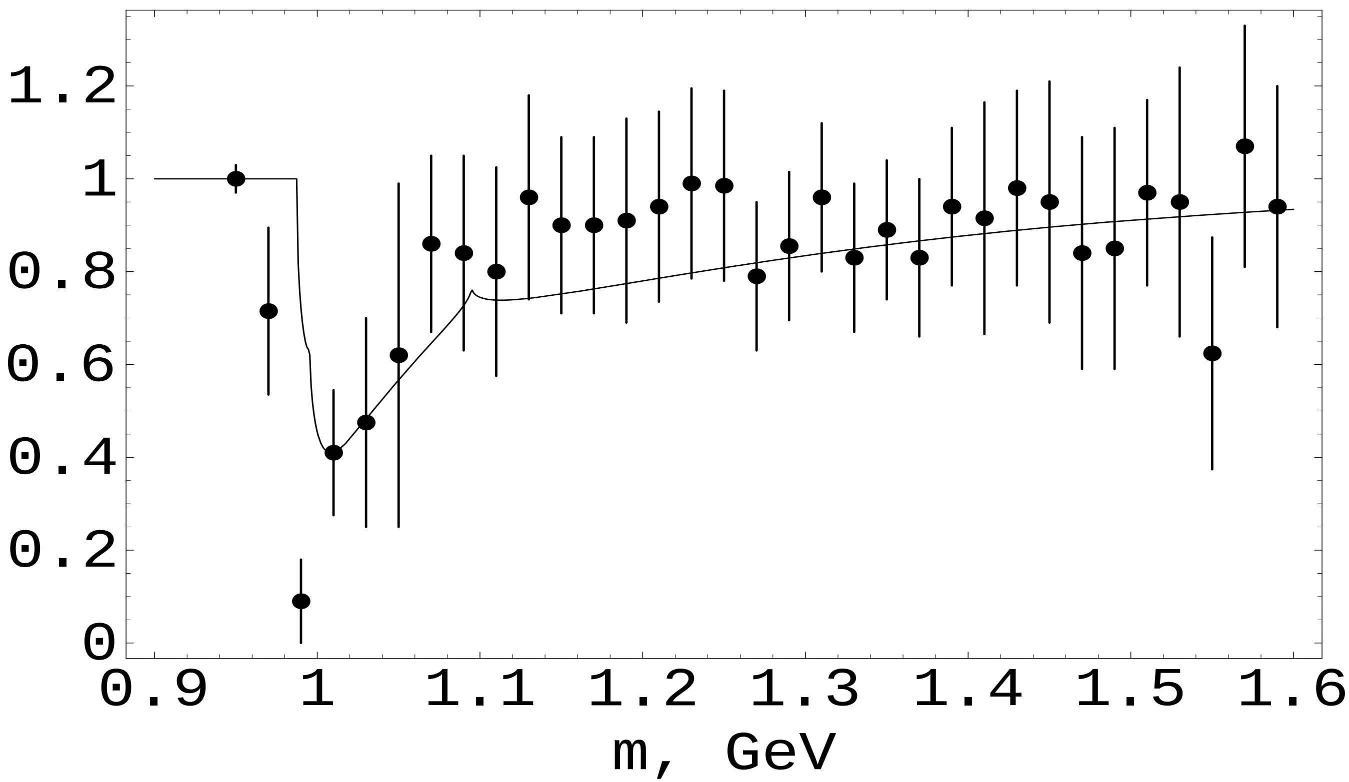
δ_0^0, comparison with CGL band



T_0^0, • is Heiri Leutwyler's calculation



Inelasticity, η_0^0



$$\eta_0^0 =$$

$$\sqrt{1 - 4\rho_{K^+}|T_0^0(\pi\pi \to K^+K^-)|^2 - 4\rho_{K^0}|T_0^0(\pi\pi \to K^0\bar{K}^0)|^2}$$

$$\rho_{K^+} \equiv \rho_{K^+K^-} \equiv \rho_{K^+K^-}(m) = \sqrt{1 - 4m_{K^+}^2/m^2}, \ldots$$

The store for the $\pi\pi$ scattering

$$T_0^0 = \frac{\eta_0^0 e^{2i\delta_0^0} - 1}{2i\rho_{\pi\pi}(m)} = \frac{\eta_0^0 e^{2i(\delta_B^{\pi\pi}+\delta_{res})} - 1}{2i\rho_{\pi\pi}(m)} = T_{bg} + e^{2i\delta_B^{\pi\pi}} T_{res}\,,$$

$$T_{bg} = \frac{e^{2i\delta_B^{\pi\pi}} - 1}{2i\rho_{\pi\pi}(m)},\quad \mathrm{T_{res}} = \frac{\eta_0^0 \mathrm{e}^{2\imath\delta_{\mathrm{res}}} - 1}{2\imath\rho_{\pi\pi}(\mathrm{m})} = \sum_{\mathrm{R,R'}} \frac{\mathrm{g_{R\pi\pi} G^{-1}_{RR'} g_{R'\pi\pi}}}{16\pi}$$

$$g_{R\pi\pi} = \sqrt{3/2}\, g_{R\pi^+\pi^-} = \sqrt{3}\, g_{R\pi^0\pi^0},\qquad R\,,\ R' = f_0\,,\ \sigma$$

$$\mathrm{G_{RR'}} \equiv \mathrm{G_{RR'}(m)} = \begin{pmatrix} D_{f_0}(m) & -\Pi_{f_0\sigma}(m) \\ -\Pi_{f_0\sigma}(m) & D_\sigma(m) \end{pmatrix},$$

$$D_R(m) = m_R^2 - m^2 + [Re\Pi_R(m_R^2) - \Pi_R(m^2)]\,,$$

$$Re\Pi_R(m_R^2) - \Pi_R(m^2) = \textstyle\sum_{ab}[Re\Pi_R^{ab}(m_R^2) - \Pi_R^{ab}(m^2)],$$

$$\Pi_{f_0\sigma}(m) = \textstyle\sum_{ab} \Pi^{ab}_{f_0\sigma}(m) + C_{f_0\sigma},\quad ab = \pi\pi\,,\ K\bar K\,,\ \ldots$$

Four-quark Model

The **nontrivial** nature of the well-established light scalar resonances $f_0(980)$ and $a_0(980)$ is no longer denied practically anybody. In particular, there exist numerous evidences in favour of the $q^2\bar{q}^2$ structure of these states. As for the nonet as a whole, even a **dope's** look at PDG Review gives an idea of the **four-quark** structure of the light scalar meson nonet [a], $\sigma(600)$, $\kappa(700-900)$, $f_0(980)$, and $a_0(980)$, inverted in comparison with the classical P-wave $q\bar{q}$ tensor meson nonet, $f_2(1270)$, $a_2(1320)$, $K_2^*(1420)$, $\phi_2'(1525)$.

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[a] To be on the safe side, notice that the linear σ model does not contradict to non-$q\bar{q}$ nature of the low lying scalars because Quantum Fields can contain different virtual particles in different regions of virtuality.

Four-quark Model

Really, while the scalar nonet cannot be treated as the P-wave $q\bar{q}$ nonet in the naive quark model, it can be easy understood as the $q^2\bar{q}^2$ nonet, where $\sigma(600)$ has no strange quarks, $\kappa(700-900)$ has the s quark, $f_0(980)$ and $a_0(980)$ have the $s\bar{s}$-pair, R.L. Jaffe.

The scalar mesons $a_0(980)$ and $f_0(980)$, discovered more than thirty years ago, became the hard problem for the naive $q\bar{q}$ model from the outset. Really, on the one hand the almost exact degeneration of the masses of the isovector $a_0(980)$ and isoscalar $f_0(980)$ states revealed seemingly the structure similar to the structure of the vector ρ and ω or tensor $a_2(1320)$ and $f_2(1270)$ mesons,

Four-quark Model

but on the other hand the strong coupling of $f_0(980)$ with the $K\bar{K}$ channel as if suggested a considerable part of the strange pair $s\bar{s}$ in the wave function of $f_0(980)$.

In 1977 R.L. Jaffe noted that in the MIT bag model, which incorporates confinement phenomenologically, there are light four-quark scalar states. He suggested also that $a_0(980)$ and $f_0(980)$ might be these states with symbolic structures

$$a_0^0(980) = (us\bar{u}\bar{s} - ds\bar{d}\bar{s})/\sqrt{2} \quad \text{and}$$
$$f_0(980) = (us\bar{u}\bar{s} + ds\bar{d}\bar{s})/\sqrt{2}.$$

From that time $a_0(980)$ and $f_0(980)$ resonances came into beloved children of the light quark spectroscopy.

Radiative Decays of ϕ-Meson

Ten years later we showed that the study of the radiative decays $\phi \to \gamma a_0 \to \gamma\pi\eta$ and $\phi \to \gamma f_0 \to \gamma\pi\pi$ can shed light on the problem of $a_0(980)$ and $f_0(980)$ mesons. Over the next ten years before experiments (1998) the question was considered from different points of view. Now these decays have been studied not only theoretically but also experimentally. The first measurements have been reported by the SND and CMD-2 Collaborations which obtain the following branching ratios

$$BR(\phi \to \gamma\pi^0\eta) = (0.88 \pm 0.14 \pm 0.09) \cdot 10^{-4} \text{ SND},$$

$$BR(\phi \to \gamma\pi^0\pi^0) = (1.221 \pm 0.098 \pm 0.061) \cdot 10^{-4} \text{ SND},$$

$$BR(\phi \to \gamma\pi^0\eta) = (0.9 \pm 0.24 \pm 0.1) \cdot 10^{-4} \text{ CMD-2},$$

$$BR(\phi \to \gamma\pi^0\pi^0) = (0.92 \pm 0.08 \pm 0.06) \cdot 10^{-4} \text{ CMD-2}.$$

Radiative Decays of ϕ-Meson

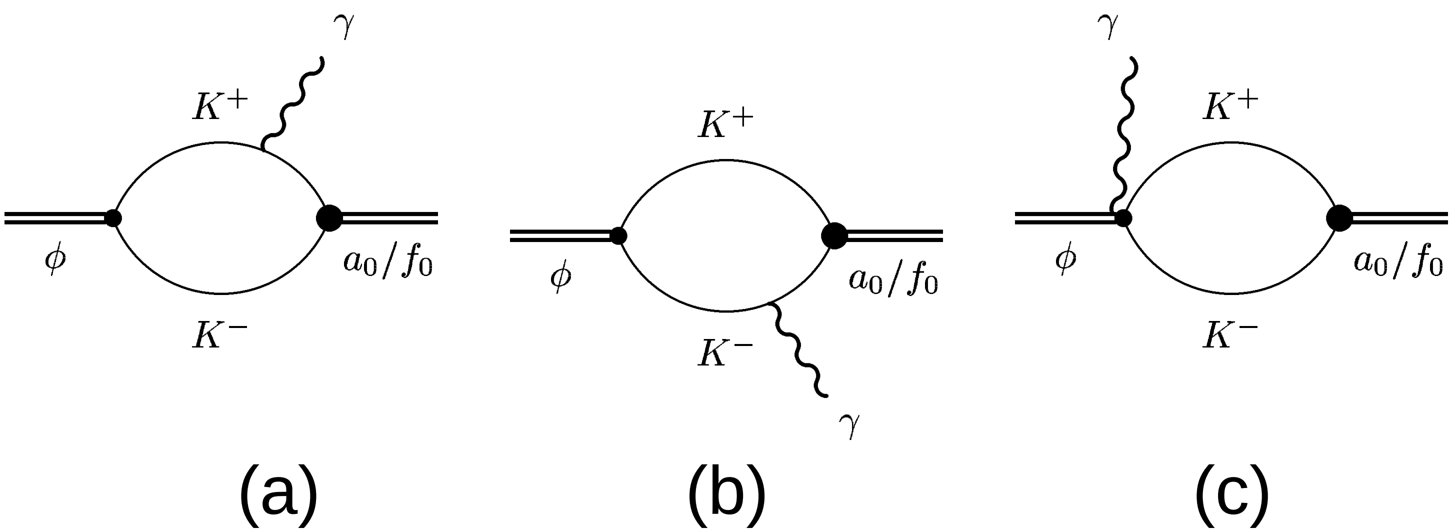
More recently the KLOE Collaboration has measured

$$BR(\phi \to \gamma\pi^0\eta(\to \gamma\gamma)) = (0.851 \pm 0.051 \pm 0.057) \cdot 10^{-4}$$
$$BR(\phi \to \gamma\pi^0\eta(\to \pi^+\pi^-\pi^0)) = (0.796 \pm 0.060 \pm 0.040) \cdot 10^{-4}$$
$$BR(\phi \to \gamma\pi^0\pi^0) = (1.09 \pm 0.03 \pm 0.05) \cdot 10^{-4}$$

in agreement with the Novosibirsk data but with a considerably smaller error.

Note that $a_0(980)$ is produced in the radiative ϕ meson decay as intensively as $\eta'(958)$ containing $\approx 66\%$ of $s\bar{s}$, responsible for $\phi \approx s\bar{s} \to \gamma s\bar{s} \to \gamma\eta'(958)$. It is a clear qualitative argument for the presence of the $s\bar{s}$ pair in the isovector $a_0(980)$ state, i.e., for its four-quark nature.

K^+K^--Loop Model



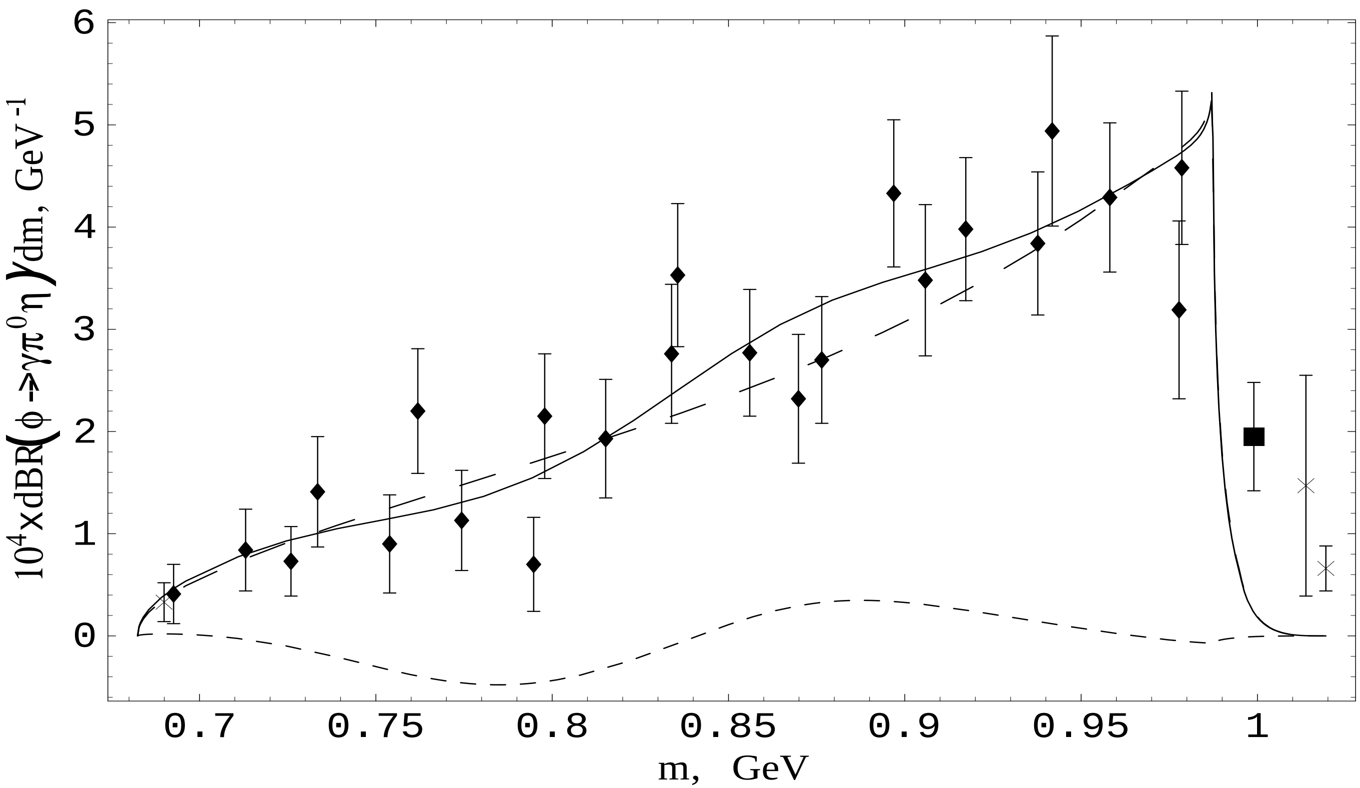
When basing the experimental investigations, we suggested one-loop model $\phi \to K^+K^- \to \gamma a_0(980)$(or $f_0(980)$). This model is used in the data treatment and is ratified by experiment.

K^+K^--Loop Mechanism of Creation

Below we argue on gauge invariance grounds that the present data give the conclusive arguments in favor of the K^+K^--loop transition as the principal mechanism of $a_0(980)$ and $f_0(980)$ meson production in the ϕ radiative decays. This enables to conclude that production of the lightest scalar mesons $a_0(980)$ and $f_0(980)$ in these decays is caused by the four-quark transitions, resulting in strong restrictions on the large N_C expansions of the decay amplitudes. The analysis shows that these constraints give new evidences in favor of the four-quark nature of $a_0(980)$ and $f_0(980)$ mesons.

The data are described in the model $\phi \to (\gamma a_0 + \pi^0\rho) \to \gamma\pi^0\eta$ and $\phi \to [\gamma(f_0 + \sigma) + \pi^0\rho] \to \gamma\pi^0\pi^0$.

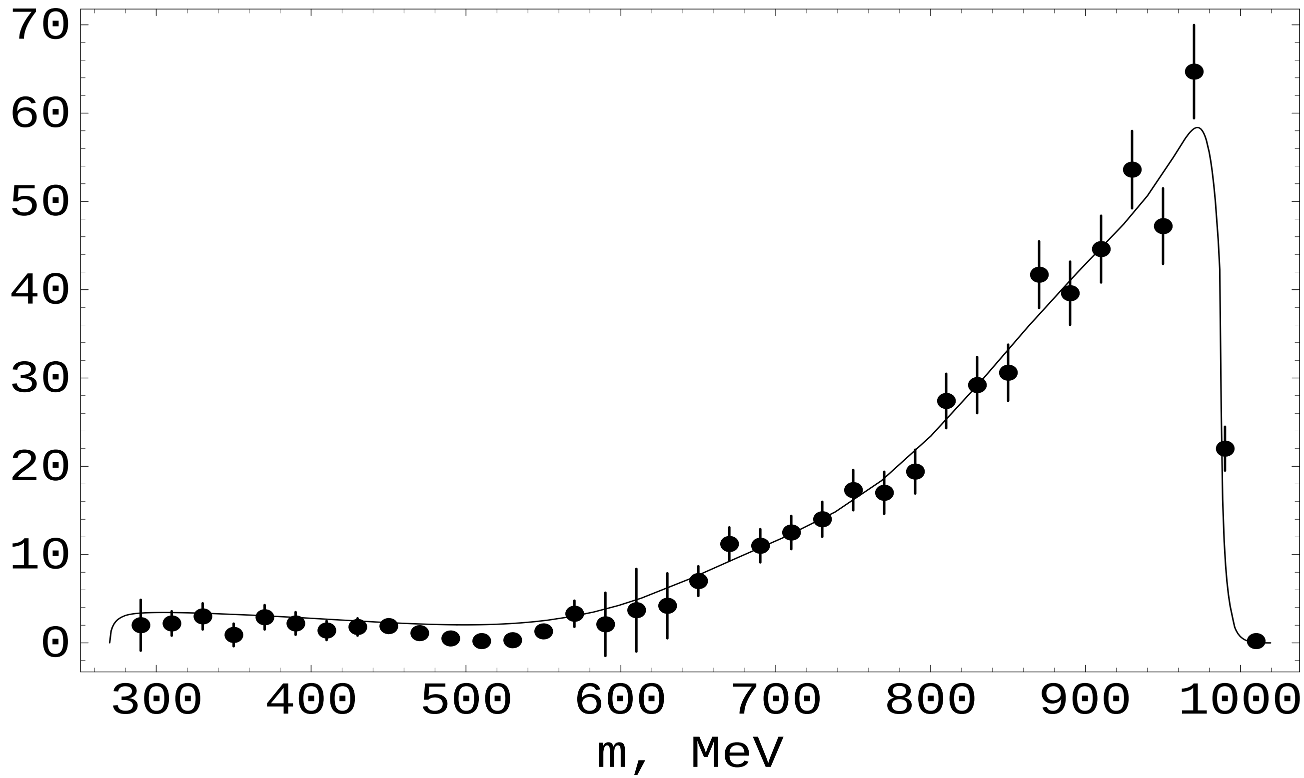
$\phi \to \gamma\pi^0\eta$, KLOE



$$\frac{dBR(\phi \to K^+K^- \to \gamma a_0 \to \gamma\pi^0\eta\,,\, m)}{dm} =$$

$$= \frac{4|g(m)|^2\omega(m)p_{\pi\eta}(m)}{\Gamma_\phi\, 3(4\pi)^3 m_\phi^2}\left|\frac{g_{a_0K^+K^-}g_{a_0\pi\eta}}{D_{a_0}(m)}\right|^2$$

$\phi \to \gamma\pi^0\pi^0$, KLOE



$$\frac{dBR(\phi \to K^+K^- \to \gamma(\sigma + f_0) \to \gamma\pi^0\pi^0\,, m)}{dm} =$$

$$= \frac{16|g(m)|^2\omega(m)p_{\pi\eta}(m)}{\Gamma_\phi\, 3\pi m_\phi^2}\left|T_0^0(K^+K^- \to \pi^0\pi^0)\right|^2$$

Spectra and Gauge Invariance

To describe the experimental spectra

$$\mathrm{S_R(m)} \equiv \frac{\mathrm{dBR}(\phi \to \gamma \mathrm{R} \to \gamma \mathrm{ab}\,,\, \mathrm{m})}{\mathrm{dm}} =$$

$$\frac{2}{\pi} \frac{\mathrm{m}^2 \Gamma(\phi \to \gamma \mathrm{R}\,,\, \mathrm{m}) \Gamma(\mathrm{R} \to \mathrm{ab}\,,\, \mathrm{m})}{\Gamma_\phi |\mathrm{D_R(m)}|^2}$$

$$= \frac{4|\mathrm{g_R(m)}|^2 \omega(\mathrm{m}) \mathrm{p_{ab}(m)}}{\Gamma_\phi\, 3(4\pi)^3 \mathrm{m}_\phi^2} \left| \frac{\mathrm{g_{Rab}}}{\mathrm{D_R(m)}} \right|^2,$$

$$\mathrm{R} = \mathrm{a_0},\ \mathrm{f_0},\ \mathrm{ab} = \pi^0 \eta,\ \pi^0 \pi^0,$$

the function $|g_R(m)|^2$ should be smooth (almost constant) in the range $m \leq 0.99$ GeV. But the problem issues from gauge invariance which requires that

Spectra and Gauge Invariance

$$A\left[\phi(p) \to \gamma(k) R(q)\right]$$
$$= G_R(m) \left[p_\mu e_\nu(\phi) - p_\nu e_\mu(\phi)\right] \left[k_\mu e_\nu(\gamma) - k_\nu e_\mu(\gamma)\right].$$

Consequently, the function

$$g_R(m) = -2(pk) G_R(m) = -2\omega(m) m_\phi G_R(m)$$

is proportional to the photon energy $\omega(m) = (m_\phi^2 - m^2)/2m_\phi$ (at least!) in the soft photon region.

Stopping the function $(\omega(m))^2$ at $\omega(990\ \text{MeV}) = 29$ MeV with the help of the form-factor $1/\left[1 + (R\omega(m))^2\right]$ requires $R \approx 100$ GeV^{-1}. It seems to be incredible to explain such a huge radius in hadron physics. Based on rather great $R \approx 10$ GeV^{-1}, one can

Spectra and Gauge Invariance

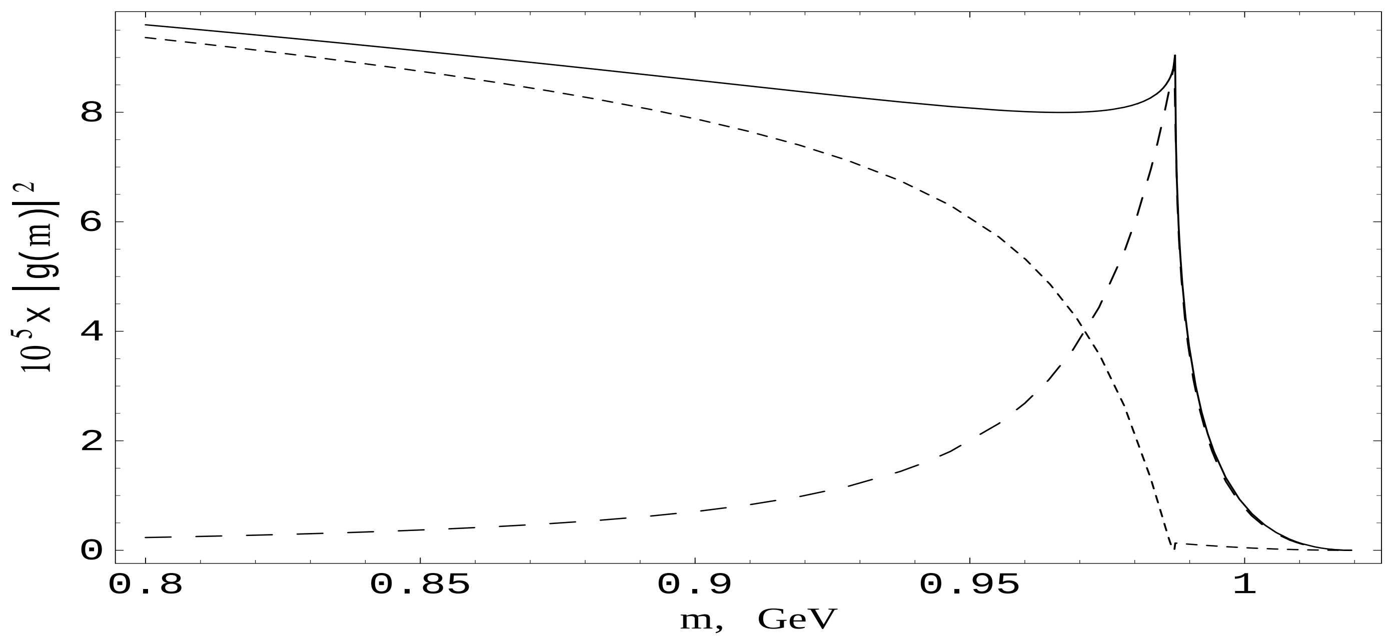
obtain an effective maximum of the mass spectrum only near 900 MeV.

To exemplify this trouble let us consider the contribution of the isolated R resonance: $g_R(m) = -2\omega(m) m_\phi G_R(m_R)$. Let also the mass and the width of the R resonance equal 980 MeV and 60 MeV, then $S_R(920\,\text{MeV}) : S_R(950\,\text{MeV}) : S_R(970\,\text{MeV}) : S_R(980\,\text{MeV}) = 3 : 2.7 : 1.8 : 1$.

So stopping the $g_R(m)$ function is the crucial point in understanding the mechanism of the production of $a_0(980)$ and $f_0(980)$ resonances in the ϕ radiative decays.

The K^+K^--loop model $\phi \to K^+K^- \to \gamma R$ solves this problem in the elegant way: fine threshold phenomenon is discovered.

New Threshold Phenomenon



The universal in K^+K^--loop model function $|g(m)|^2 = |g_R(m)/g_{RK^+K^-}|^2$ is drawn with the solid line. The contribution of the imaginary part is drawn with the dashed line. The contribution of the real part is drawn with the dotted line.

K^+K^--Loop Mechanism is established

In truth this means that $a_0(980)$ and $f_0(980)$ are seen in the radiative decays of ϕ meson owing to K^+K^- intermediate state.

So, the mechanism of production of $a_0(980)$ and $f_0(980)$ mesons in the ϕ radiative decays is established at a physical level of proof.

The real part of the $\phi \to \gamma R$ amplitude contains two different contribution. One is caused by intermediate momenta (a few GeV) in the loops and the other is caused by super high momenta in the loops. At $\omega(m) = 0$ these contribution eliminate each other. With increasing $\omega(m)$ the contribution from intermediate momenta decreases rapidly enough. The contribution from super high momenta is constant and causes the $\phi \to \gamma R$ amplitude in a great part.

Four-quark Transition

Both real and imaginary parts of the $\phi \to \gamma R$ amplitude are caused by the K^+K^- intermediate state. The imaginary part is caused by the real K^+K^- intermediate state while the real part is caused by the virtual compact K^+K^- intermediate state, i.e., we are dealing here with the four-quark transition.[a]

Needless to say, radiative four-quark transitions can happen between two $q\bar{q}$ states as well as between $q\bar{q}$ and $q^2\bar{q}^2$ states but their intensities depend strongly on a type of the transitions.

[a] It will be recalled that the imaginary part of every hadronic amplitude describes a multi-quark transition.

Four-quark Transition and OZI rule

A radiative four-quark transition between two $q\bar{q}$ states requires creation and annihilation of an additional $q\bar{q}$ pair, i.e., such a transition is forbidden according to the Okubo-Zweig-Iizuka (OZI) rule, while a radiative four-quark transition between $q\bar{q}$ and $q^2\bar{q}^2$ states requires only creation of an additional $q\bar{q}$ pair, i.e., such a transition is allowed according to the OZI rule.

The consideration of this problem from the large N_C expansion standpoint, using the G.'t Hooft rules : $g_s^2 N_C \to const$ at and a gluon is equivalent to a quark-antiquark pair ($A_j^i \sim q^i\bar{q}_j$), supports the suppression of a radiative four-quark transition between two $q\bar{q}$ states in comparison with a radiative four-quark transition between $q\bar{q}$ and $q^2\bar{q}^2$ states.

$a_0(980)/f_0(980) \to \gamma\gamma$ & $q^2\bar{q}^2$-Model

Recall that twenty six years ago the suppression of $a_0(980) \to \gamma\gamma$ and $f_0(980) \to \gamma\gamma$ was predicted in our work based on the $q^2\bar{q}^2$ model,

$\Gamma(a_0(980) \to \gamma\gamma) \sim \Gamma(f_0(980) \to \gamma\gamma) \sim 0.27$ keV.

Experiment supported this predicton

$$\Gamma(a_0 \to \gamma\gamma) = (0.19 \pm 0.07^{+0.1}_{-0.07})/B(a_0 \to \pi\eta) \text{ keV, Crystal Ball}$$

$$\Gamma(a_0 \to \gamma\gamma) = (0.28 \pm 0.04 \pm 0.1)/B(a_0 \to \pi\eta) \text{ keV, JADE.}$$

When in the $q\bar{q}$ model it was anticipated

$$\Gamma(a_0 \to \gamma\gamma) = (1.5 - 5.9)\Gamma(a_2 \to \gamma\gamma)$$
$$= (1.5 - 5.9)(1.04 \pm 0.09) \text{ keV.}$$

The wide scatter of the predictions is connected with different reasonable guesses of the potential form.

$f_0(980)/a_0(980) \to \gamma\gamma$

The $a_0 \to K^+K^- \to \gamma\gamma$ model describes adequately data and correspond the four-quark transition $a_0 \to q^2\bar{q}^2 \to \gamma\gamma$, $\langle\Gamma(a_0 \to K^+K^- \to \gamma\gamma\rangle \approx 0.3$ keV.

$$\Gamma(f_0 \to \gamma\gamma) = (0.31 \pm 0.14 \pm 0.09) \text{ keV, Crystal Ball,}$$
$$\Gamma(f_0 \to \gamma\gamma) = (0.24 \pm 0.06 \pm 0.15) \text{ keV, MARK II.}$$

When in the $q\bar{q}$ model it was anticipated

$$\Gamma(f_0 \to \gamma\gamma) = (1.7 - 5.5)\Gamma(f_2 \to \gamma\gamma)$$
$$= (1.7 - 5.5)(2.8 \pm 0.4) \text{ keV.}$$

The wide scatter of the predictions is connected with different reasonable guesses of the potential form.

Dynamics of $\gamma\gamma \to \pi\pi$

$$T_S(\gamma\gamma \to \pi^+\pi^-,\ s) = T_S^{Born}(\gamma\gamma \to \pi^+\pi^-,\ s)$$

$$+8\alpha \mathrm{I}_{\pi^+\pi^-}(\mathrm{s})\, \mathrm{T_S}(\pi^+\pi^- \to \pi^+\pi^-,\ \mathrm{s})$$

$$+8\alpha \mathrm{I}_{\mathrm{K^+K^-}}(\mathrm{s})\, \mathrm{T_S}(\mathrm{K^+K^-} \to \pi^+\pi^-,\ \mathrm{s})$$

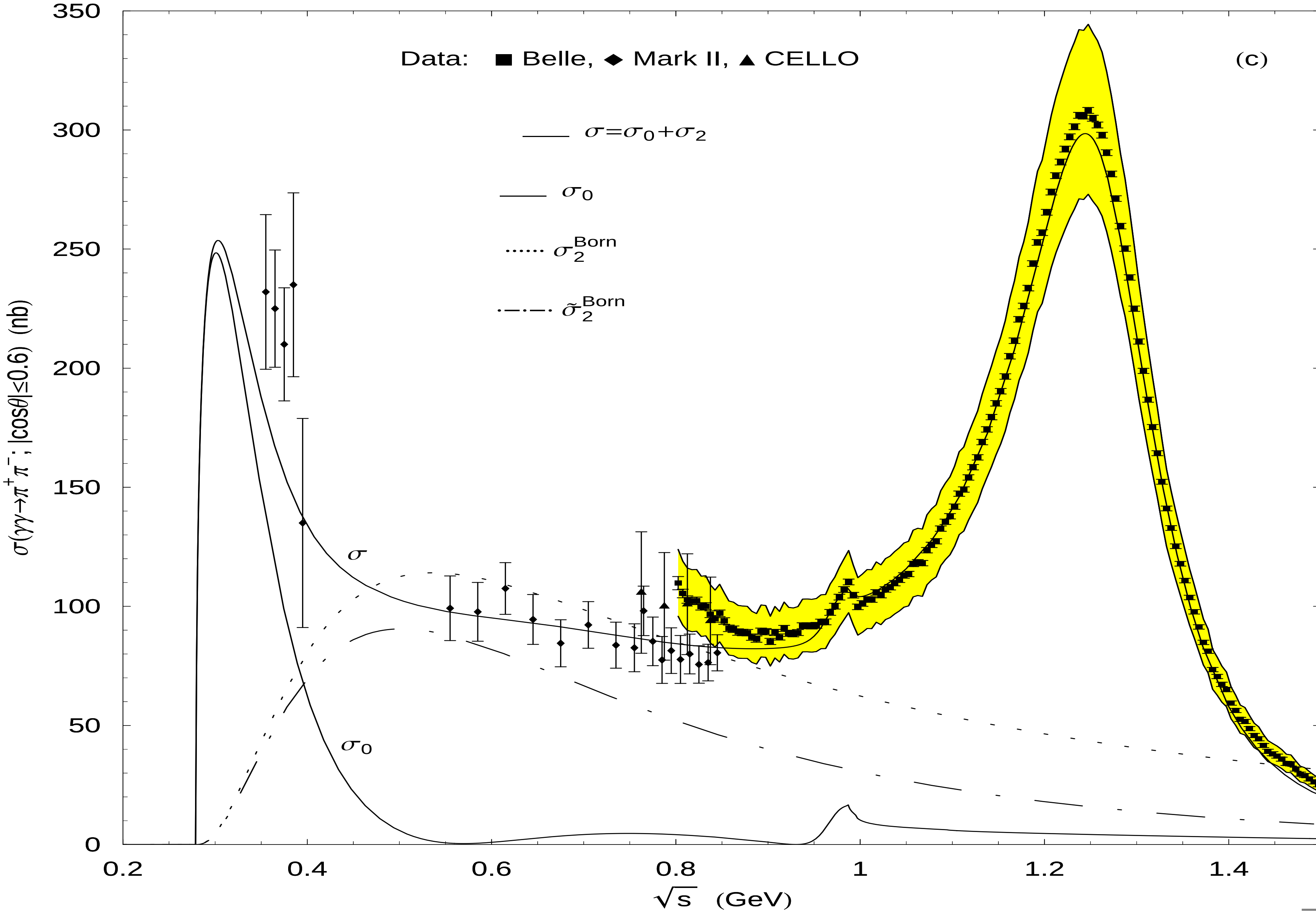
$$+\mathrm{T_S^{direct}}(\gamma\gamma \to \mathrm{res} \to \pi^+\pi^-),$$

$$T_S(\gamma\gamma \to \pi^0\pi^0,\ s) = 8\alpha \mathrm{I}_{\pi^+\pi^-}(\mathrm{s})\, \mathrm{T_S}(\pi^+\pi^- \to \pi^0\pi^0,\ \mathrm{s})$$

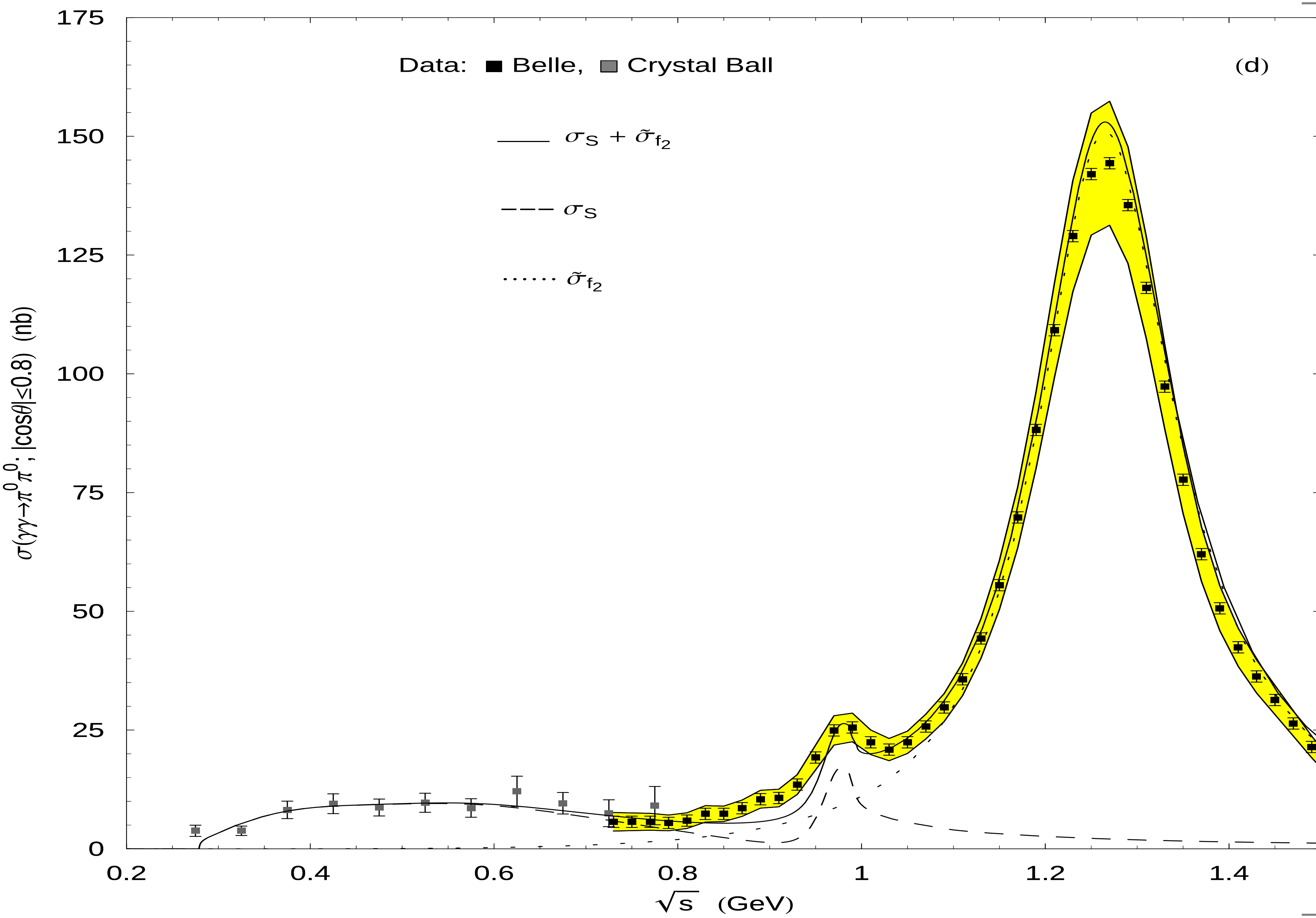
$$+8\alpha \mathrm{I}_{\mathrm{K^+K^-}}(\mathrm{s})\, \mathrm{T_S}(\mathrm{K^+K^-} \to \pi^0\pi^0,\ \mathrm{s})$$

$$+\mathrm{T_S^{direct}}(\gamma\gamma \to \mathrm{res} \to \pi^0\pi^0).$$

The Belle data on $\gamma\gamma \to \pi^+\pi^-$



The Belle data on $\gamma\gamma \to \pi^0\pi^0$



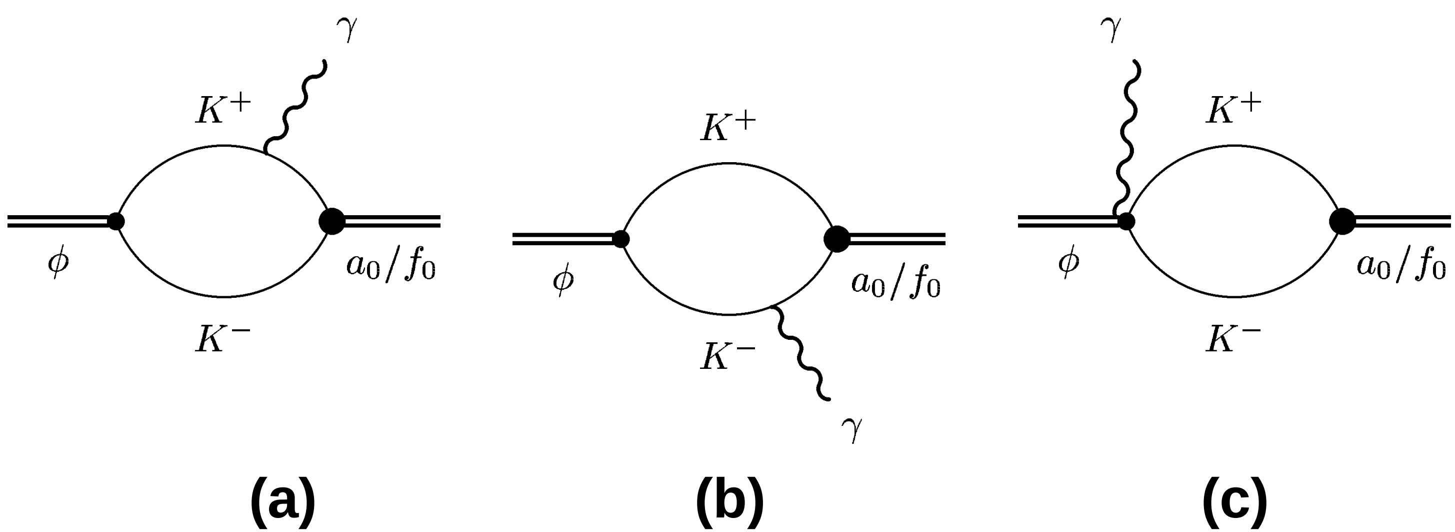
$f_0 \to K^+K^- \to \gamma\gamma$ approximation

$$\langle \Gamma_{f_0 \to K^+K^- \to \gamma\gamma} \rangle \approx 0.2 \text{ keV}$$

The direct $f_0 \to \gamma\gamma$ & $\sigma \to \gamma\gamma$ decays

$$\Gamma^{direct}_{\sigma \to \gamma\gamma} \ll 0.1\text{keV}, \quad \Gamma^{direct}_{f_0 \to \gamma\gamma} \ll 0.1\text{keV}$$

Why a_0 and f_0 are not $K\bar{K}$ molecules



$$T\left[\phi(p) \to \gamma a_0(q)/f_0(q)\right] = (a) + (b) + (c)$$

Every diagram is divergent hence should be regularized in a gauge invariant manner, for example, the Pauli-Wilars one.

$$\overline{T}\left[\phi(p) \to \gamma a_0(q)/f_0(q), M\right] = \overline{(a)} + \overline{(b)} + \overline{(c)}\,,$$

$$\overline{T}\left[\phi(p) \to \gamma a_0(q)/f_0(q), M\right] = \epsilon^{\nu}(\phi)\epsilon^{\mu}(\gamma)\overline{T}_{\nu\mu}(p,q) =$$

$$\epsilon^{\nu}(\phi)\epsilon^{\nu}(\gamma)\left[\overline{a}_{\nu\mu}(p,q) + \overline{b}_{\nu\mu}(p,q) + \overline{d}_{\nu\mu}(p,q)\right]$$

Why a_0 and f_0 are not $K\bar{K}$ molecules

$$\overline{a}_{\nu\mu}(p,q) =$$

$$-\frac{i}{\pi^2}\int\left\{\frac{(p-2r)_\nu(p+q-2r)_\mu}{(m_K^2-r^2)[m_K^2-(p-r)^2][m_K^2-(q-r)^2]} - \frac{(p-2r)_\nu(p+q-2r)_\mu}{(M^2-r^2)[M^2-(p-r)^2][M^2-(q-r)^2]}\right\}dr$$

$$\overline{b}_{\nu\mu}(p,q) =$$

$$-\frac{i}{\pi^2}\int\left\{\frac{(p-2r)_\nu(p-q-2r)_\mu}{(m_K^2-r^2)[m_K^2-(p-r)^2][m_K^2-(q-r)^2]} - \frac{(p-2r)_\nu(p+q-2r)_\mu}{(M^2-r^2)[M^2-(p-r)^2][M^2-(p-q-r)^2]}\right\}dr$$

Why a_0 and f_0 are not $K\bar{K}$ molecules

$$\overline{d}_{\nu\mu}(p,q) = -\frac{i}{\pi^2}\, 2g_{\nu\mu} \int dr \times$$

$$\left\{ \frac{1}{(m_K^2 - r^2][m_K^2 - (q-r)^2]} - \frac{1}{(M^2 - r^2)[M^2 - (q-r)^2]} \right\}$$

where M is the regulator field mass, $M \to \infty$ in the end

$$\overline{T}(\phi \to \gamma a_0/f_0, M \to \infty) \to T^{Phys}(\phi \to \gamma a_0/f_0)$$

We can shift the integration variables in the regularized amplitudes and easy check the gauge invariance condition

$$\epsilon^\nu(\phi) k^\mu \overline{T}_{\nu\mu}(p,q) = \epsilon^\nu(\phi)(p-q)^\mu \overline{T}_{\nu\mu}(p,q) = 0.$$

It is instructive to consider how the gauge invariance condition

$\epsilon^\nu(\phi)\epsilon^\mu(\gamma)\overline{T}_{\nu\mu}(p,p) = 0$ holds true.

Why a_0 and f_0 are not $K\bar{K}$ molecules

$$\epsilon^\nu(\phi)\epsilon^\mu(\gamma)\overline{T}_{\nu\mu}(p,p) =$$
$$\epsilon^\nu(\phi)\epsilon^\mu(\gamma)T^{m_K}_{\nu\mu}(p,p) - \epsilon^\nu(\phi)\epsilon^\mu(\gamma)T^{M}_{\nu\mu}(p,p) =$$
$$(\epsilon(\phi)\epsilon(\gamma))(1-1) = 0$$

The superscript m_K refers to the non-regularized amplitude and the superscript M refers to the the regulator field contribution. So, the contribution of the (a), (b), and (d) diagrams does not depend on a particle mass in the loops (m_K or M) at $p = q$. [a]

But, the physical meaning of these contributions is radically different. The regulator field contribution is caused fully by high momenta ($M \to \infty$) and teaches us how to allow for high K virtualities in

[a] A typical example of such integrals is $2\int_0^\infty \frac{m^2 x}{(x+m^2)^3}dx = 1$.

Why a_0 and f_0 are not $K\bar{K}$ molecules

gauge invariant way. It is clear that

$$\epsilon^{\nu}(\phi)\epsilon^{\mu}(\gamma)\mathrm{T}_{\nu\mu}^{\mathrm{M}\to\infty}(\mathrm{p},\mathrm{q}) \to \epsilon^{\nu}(\phi)\epsilon^{\mu}(\gamma)\mathrm{T}_{\nu\mu}^{\mathrm{M}\to\infty}(\mathrm{p},\mathrm{p}) \equiv$$
$$\epsilon^{\nu}(\phi)\epsilon^{\mu}(\gamma)T_{\nu\mu}^{M}(p,p) \equiv (\epsilon(\phi)\epsilon(\gamma)).$$

So, the regulator field contribution tends to the subtraction constant when $M \to \infty$. The finiteness of this constant hides its high momentum origin and gives rise to an illusion of a nonrelativistic physics in the decays under discussion for some theorists.

$Re(\epsilon^{\nu}(\phi)\epsilon^{\mu}(\gamma)T_{\nu\mu}^{m_K}(p,q))$ decreases rapidly enough with increase the photon energy $(p^2 - q^2)/2\sqrt{p^2}$ and, consequently, $Re(T^{Phys}[\phi(p) \to \gamma a_0(q)/f_0(q)])$ is caused in a great part by the subtraction constant at $\sqrt{q^2} < 980$ MeV, see Fig. on 49 page.

THANK YOU