Rare meson decays in R-parity-violating SUSY theories

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Abstract

I discuss rare meson decays $K^+ \to \pi^- \ell^+ \ell'^+$ and $D^+ \to K^- \ell^+ \ell'^+$ $(\ell, \ell' = e, \mu)$ in a supersymmetric extension of the Standard Model with explicit breaking of R-parity. My emphasis is put on trilinear R-parity breaking terms, because in this work, I assume that the bilinear terms are absent at tree level. They will be generated by quantum corrections, but it is expected that the phenomenology will be still dominated by the tree-level trilinear terms. Estimates of the branching ratios for these decays are presented.

In the Standard Model (SM), the lepton L and baryon B number conservation is protected to all orders of perturbation theory due to the accidental $U(1)_L \times U(1)_B$ symmetry existing at the level of renormalizable operators. But for many extensions of the SM this symmetry is absent and the L and B violating processes are not forbidden. The well known mechanism of lepton number (LN) violation is based on the mixing of massive Majorana neutrinos predicted by various Grand unified theories (GUTs) [1]. The Majorana mass term violates LN by $\Delta L = \pm 2$ [2] and can lead to a large number of LN violating processes. Among them, the most sensitive to the LN violation neutrinoless double beta decay $(A, Z) \to (A, Z+2) + e^- + e^-$ (for a recent review, see [3]), rare meson decays (see, e.g., [4, 5])

$$M^+ \to M'^- \ell^+ \ell'^+ \tag{1}$$

and like-sign dilepton production in high-energy hadron-hadron, lepton-hadron collisions (see, e.g., the papers and references therein: $pp \to \ell^\pm \ell'^\pm X$ [4, 6], $e^+ p \to \bar{\nu}_e \ell^+ \ell'^+ X$ [7]) have been extensively studied.

There exists now convincing evidence for oscillations of solar, atmospheric, reactor, and accelerator neutrinos [8]. The oscillations, i.e., periodic neutrino flavor changes, imply that neutrinos have nonzero masses and there is mixing: neutrinos ν_ℓ of specific flavors $\ell = e, \mu, \tau$ are the coherent superposition of the neutrino mass eigen-states ν_N of masses m_N,

$$\nu_\ell = \sum\nolimits_N U_{\ell N} \nu_N. \tag{2}$$

Here the coefficients $U_{\ell N}$ are elements of the unitary leptonic mixing matrix.

Neutrino flavor changes imply lepton family number L_ℓ nonconservation admissible for neutrinos of both types, Dirac and Majorana, but for the Dirac neutrinos, in contrast to the Majorana ones, the total lepton number $L = \sum_\ell L_\ell$ is conserved. The nature of the neutrino mass is one of the main unsolved problems in particle physics. However, oscillation experiments can not distinguish between the two types of neutrinos.

In Refs. [4, 5] the rare decays (1) of the pseudoscalar mesons $M = K, D, D_s, B$ mediated by light ($m_N \ll m_\ell,\ m_{\ell'}$) and heavy ($m_N \gg m_M$) Majorana neutrinos were investigated. It was shown that the present direct experimental bounds on the branching ratios (BRs) are too weak to set reasonable limits on the effective Majorana masses. Taking into account the limits on lepton mixing and neutrino masses obtained from the precision electroweak measurements,

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neutrino oscillations, cosmological data and searches of the neutrinoless double beta decay, I have derived the indirect upper bounds on the BRs that are greatly more stringent than the direct ones.

In this report, I investigate another mechanism of the $\Delta L = 2$ rare decays (1) based on supersymmetric (SUSY) theories with explicit R-parity breaking (for a review, see [9]). I recall that R-parity is defined as $R = (-1)^{3(B-L)+2S}$, where S, L, and B are the spin, the lepton, and baryon numbers, respectively. The SM fields, including additional Higgs boson fields appearing in the extended gauge models, have $R = +1$ while their superpartners have $R = -1$. In the minimal supersymmetric extension of the SM (MSSM), R-parity conservation has been imposed to prevent the L and B violation; it also leads to the production of superpartners in pairs and ensures the stability of the lightest superparticle. However, neither gauge invariance nor supersymmetry require R-parity conservation. There are many generalizations of the MSSM with explicitly or spontaneously broken R-symmetry [9].

Sypersymmetric models with R-parity violation have been extensively discussed in the literature not only because of their great theoretical interest, but also because they have interesting phenomenological and cosmological implications. I consider a SUSY theory with the minimal particle content of the MSSM and explicit R-parity violation ($\not{R}$MSSM).

The most general form for the R-parity and lepton number violating part of the superpotential is given by [9, 10]

$$W_{\not{R}} = \varepsilon_{\alpha\beta}\left(\frac{1}{2}\lambda_{ijk}L_i^\alpha L_j^\beta \bar{E}_k + \lambda'_{ijk}L_i^\alpha Q_j^\beta \bar{D}_k + \epsilon_i L_i^\alpha H_u^\beta\right). \tag{3}$$

Here $i, j, k = 1, 2, 3$ are generation indices, L and Q are $SU(2)$ doublets of left-handed lepton and quark superfields ($\alpha, \beta = 1, 2$ are isospinor indices), $\bar{E}$ and $\bar{D}$ are singlets of right-handed superfields of leptons and down quarks, respectively; H_u is a doublet Higgs superfield (with hypercharge $Y = 1$); $\lambda_{ijk} = -\lambda_{jik}$, λ'_{ijk} and ϵ_i are constants.

This superpotential has only lepton number violating terms and is therefore in agreement with proton stability. In the superpotential (3) the trilinear ($\propto \lambda,\ \lambda'$) and bilinear ($\propto \epsilon$) terms are present. Previously the main attention was paid to the phenomenology of the trilinear $\not{R}$ Yukawa couplings. It was widely believed that the bilinear $\not{R}$ terms can be rotated away by a proper field redefinition. However, it is not the case in the presence of the soft SUSY breaking interactions [9, 10]. The bilinear R-parity violation generically leads to the nonzero vacuum expectation values (VEV) of the sneutrino fields and to the lepton-gaugino-higgsino and slepton-Higgs mixing. In this work, we assume that the bilinear terms are absent at tree level ($\epsilon = 0$). They will be generated by quantum corrections [9], but it is expected that the phenomenology will be still dominated by the tree-level trilinear terms.

The effective Lagrangian describing meson decays of the type (1) is

$$\mathcal{L} = \mathcal{L}_\lambda + \mathcal{L}_{\lambda'} + \mathcal{L}_{\tilde{g}} + \mathcal{L}_{\tilde{\chi}}. \tag{4}$$

The trilinear terms or the R-parity breaking part of the superpotential (3) lead to the following $\Delta L = 1$ lepton-quark operators (the first two terms of the effective Lagrangian):

$$\begin{aligned}
\mathcal{L}_\lambda &= \tfrac{1}{2}\lambda_{ijk}\left[\tilde{\nu}_{Li}\bar{\ell}_{Rk}\ell_{Lj} + \tilde{\ell}_{Lj}\bar{\ell}_{Rk}\nu_{Li} + (\tilde{\ell}_{Rk})^*(\bar{\nu}_{Li})^c\ell_{Lj} - (i \leftrightarrow j)\right] + \text{H.c.},\\
\mathcal{L}_{\lambda'} &= \lambda'_{ijk}\Big[\tilde{\nu}_{Li}\bar{d}_{Rk}d_{Lj} + \tilde{d}_{Lj}\bar{d}_{Rk}\nu_{Li} + (\tilde{d}_{Rk})^*(\bar{\nu}_{Li})^c d_{Lj} - \tilde{\ell}_{Li}\bar{d}_{Rk}u_{Lj}\\
&\qquad - \tilde{u}_{Lj}\bar{d}_{Rk}\ell_{Li} - (\tilde{d}_{Rk})^*(\bar{\ell}_{Li})^c u_{Lj}\Big] + \text{H.c.}
\end{aligned} \tag{5}$$

The third and fourth terms in the effective Lagrangian (4) are corresponding to gluino $\tilde{g}$ and neutralino $\tilde{\chi}^0$ interactions with fermions ψ (quarks q and leptons $\ell = e, \mu, \tau$) and their superpartners ($\tilde{q}$, $\tilde{\ell}$) [11]:

$$\mathcal{L}_{\tilde{g}} = -\sqrt{2}g_3\frac{(\lambda_r)^a{}_b}{2}(\bar{q}_{aL}\tilde{g}^{(r)}\tilde{q}^b_L - \bar{q}_{aR}\tilde{g}^{(r)}\tilde{q}^b_R) + \text{H.c.}, \tag{6}$$

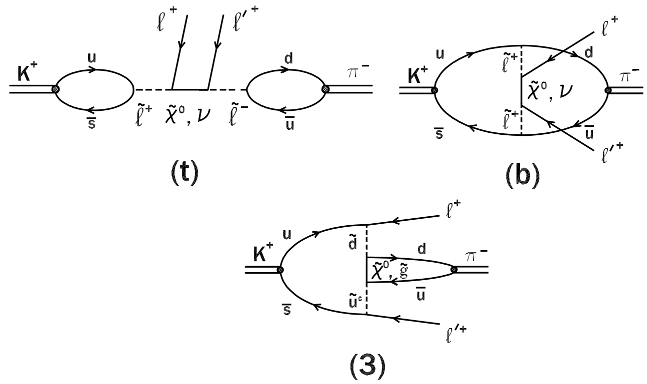

Figure 1: Feynman diagrams for the decay $K^+ \to \pi^- + \ell^+ + \ell'^+$ mediated by Majorana neutrinos ν, neutralinos $\tilde{\chi}^0$, gluinos $\tilde{g}$ with $\tilde{f}$ being the scalar superpartners of the corresponding fermions $f = \ell, u, d$ (leptons and quarks). Bold vertices correspond to Bethe–Salpeter amplitudes for mesons as bound states of a quark and an antiquark. There are also crossed diagrams with interchanged lepton lines.

where λ_r are Gell-Mann matrices ($r = 1, ..., 8$), $a, b = 1, 2, 3$ are color indices of the group $SU(3)_c$;

$$\mathcal{L}_{\tilde{\chi}} = \sqrt{2} g_2 \sum_{\delta=1}^{4} (\epsilon_{L\delta}(\psi) \bar{\psi}_L \tilde{\chi}^0_\delta \tilde{\psi}_L + \epsilon_{R\delta}(\psi) \bar{\psi}_R \tilde{\chi}^0_\delta \tilde{\psi}_R) + \text{H.c.}, \tag{7}$$

where

$$\epsilon_{L\delta}(\psi) = -T_3(\psi) N_{\delta 2} + \text{tg}\theta_W (T_3(\psi) - Q(\psi)) N_{\delta 1},$$
$$\epsilon_{R\delta}(\psi) = Q(\psi) \text{tg}\theta_W N_{\delta 1},$$

$Q(\psi)$ and $T_3(\psi)$ are the electric charge and the third component of the weak isospin of the field ψ, respectively, the coefficients $N_{\delta\sigma}$ are elements of the orthogonal 4×4 neutralino mixing matrix which diagonalizes the neutralino mass matrix.

At first we consider the rare decay $K^+(P) \to \pi^-(P') + \ell^+(p) + \ell'^+(p')$ in the $\not{R}$MSSM (a rough estimate of the width of the decay $\mathrm{B}(K^+ \to \pi^- \mu^+ \mu^+)$ in the same theory was obtained in [12]). The leading order amplitude of the process is described by three types of diagrams shown in Fig. 1.

The total width of the decay is given by [13]

$$\Gamma_{\ell\ell'} = \left(1 - \frac{1}{2}\delta_{\ell\ell'}\right) \int (2\pi)^4 \delta^{(4)} \left(P' + p + p' - P\right) \frac{|A_t + A_b + A_3|^2}{2m_K \cdot 2^3 (2\pi)^9} \frac{d^3P' d^3p\, d^3p'}{P'^0 p^0 p'^0}. \tag{8}$$

Here A_n ($n = t, b, 3$) are contributions of diagrams (n) to the amplitude of the decay. They are expressed as convolutions of leptonic $L^{(n)}$ and hadronic $H^{(n)}$ tensors:

$$A_n = \frac{1}{(2\pi)^8} \int L^{(n)} H^{(n)}\, d^4q d^4q', \tag{9}$$

where q (q') is the quark-antiquark relative 4-momentum in the initial (final) meson.

The hadronic parts of the decay amplitude are calculated with the use of a model for the Bethe–Salpeter amplitudes for mesons [14],

$$\chi_P(q) = \gamma^5 (1 - \delta_M \not{P}) \varphi_P(q), \tag{10}$$

where $\delta_M = (m_1 + m_2)/m_M^2$, m_M is the mass of the meson composed of a quark q_1 and an antiquark $\bar{q}_2$ with the current masses m_1 and m_2, $P = p_1 + p_2$ is the total 4-momentum of the meson, $q = (p_1 - p_2)/2$ is the quark-antiquark relative 4-momentum; $\varphi_P(q)$ is the model-dependent scalar function.

For all mesons in question, $m_M \ll m_{SUSY}$, where $m_{SUSY} \gtrsim 100$ GeV is the common mass scale of superpartners, and for heavy Majorana neutrinos, $m_N \gg m_M$ (the contribution of light neutrinos is strongly suppressed by phenomenology [4, 5]), we can neglect momentum dependence in the propagators (see Fig. 1) and use the effective low-energy current-current interaction. In this approximation the decay amplitude does not depend on the specific form of the functions $\varphi_P(q)$ (see Eq. (10)) and is expressed through the known decay constants of the mesons, f_M, as

$$f_M = 4\sqrt{N_c}\;\delta_M\,(2\pi)^{-4}\int d^4q\,\varphi_P\,(q)\,,$$

where $N_c = 3$ is the number of colors.

For the total width of the decay we obtain

$$\Gamma(K^+\to\pi^-\ell^+\ell'^+) = \left(1-\frac{1}{2}\delta_{\ell\ell'}\right)\frac{f_K^2 f_\pi^2 m_K^3}{2^{12}\pi^3\delta_K^2\delta_\pi^2}\Phi_{\ell\ell'}$$
$$\times\left|\sum_{i,j,k,k',N}(\lambda^*_{ik\ell}\,\lambda^*_{jk'\ell'}+\lambda^*_{ik\ell'}\lambda^*_{jk'\ell})\frac{\lambda'_{k12}\lambda'_{k'11}U_{iN}U_{jN}}{m^2_{\tilde\ell_{Lk}}\,m^2_{\tilde\ell_{Lk'}}\,m_N}\left(1-\frac{1}{2N_c}\right)\right.$$
$$+(\lambda'_{\ell 11}\lambda'_{\ell' 12}+\lambda'_{\ell' 11}\lambda'_{\ell 12})\left[g_2^2\sum_{\delta=1}^{4}\frac{1}{m_{\tilde\chi_\delta}}\left(2\frac{\epsilon^*_{L\delta}(\ell)\epsilon^*_{L\delta}(\ell')}{m^2_{\tilde\ell L}m^2_{\tilde\ell' L}}\left(1-\frac{1}{2N_c}\right)\right.\right.$$
$$\left.\left.\left.+\frac{1}{N_c}\frac{\epsilon_{R\delta}(d)\epsilon^*_{L\delta}(u)}{m^2_{\tilde d_R}\,m^2_{\tilde u_L}}\right)-\frac{4g_3^2}{N_c^2}\frac{1}{m^2_{\tilde d_R}\,m^2_{\tilde u_L}m_{\tilde g}}\right]\right|^2. \qquad (11)$$

Here $\Phi_{\ell\ell'}$ is the reduced phase space integral ($z = (P-P')^2/m_K^2$):

$$\Phi_{\ell\ell'} = \int_{l_+}^{h_-} dz\left(1-\frac{l_+ + l_-}{2z}\right)\left[(h_+ - z)(h_- - z)(l_+ - z)(l_- - z)\right]^{1/2},$$

and the various parameters are defined as follows:

$$h_\pm = (1\pm m_\pi/m_K)^2\,,\quad l_\pm = \left[(m_\ell \pm m_{\ell'})/m_K\right]^2;$$
$$\epsilon_{L\delta}(\psi) = -T_3(\psi)N_{\delta 2} + \tan\theta_W(T_3(\psi)-Q(\psi))N_{\delta 1},$$
$$\epsilon_{R\delta}(\psi) = Q(\psi)\tan\theta_W N_{\delta 1},$$

where $Q(\psi)$, $T_3(\psi)$, and $N_{\delta\sigma}$ are defined after Eq. (7). For the numerical estimates of the branching ratios, $\mathrm{B}_{\ell\ell'} = \Gamma\left(M^+\to M'^-\ell^+\ell'^+\right)/\Gamma_{\text{total}}$, I have used the known values for the couplings, decay constants, meson, lepton and current quark masses [8, 5], and a typical set of the matrix elements $N_{\delta 1}, N_{\delta 2}$ from Ref. [15]. In addition, I have taken all the masses of superpartners to be equal with a common value m_{SUSY}. Taking into account the present bounds on the effective inverse Majorana masses [5], I find that the main contribution to the decay width comes from the exchange by neutralinos and gluinos (see Fig. 1). The results of the calculations with the use of Eq. (11) and an analogous formula for the decays $D^+\to K^-\ell^+\ell'^+$ are shown in the fourth column of Table 1 (here $m_{200} = m_{SUSY}/(200\text{GeV})$). In the second and third columns the present direct experimental upper bounds on the BRs [8] and the indirect bounds for the Majorana mechanism of the rare decays [5] are shown, respectively. My result for the $\mathrm{B}(K^+\to\pi^-\mu^+\mu^+)$ is in agreement with a rough estimate of Ref. [12].

To calculate the upper bounds on the BRs in the $\not{R}$MSSM, I take $m_{200} = 1$ and $|\lambda'_{ijk}\lambda'_{i'j'k'}| \lesssim 10^{-3}$ [16]. It yields

$$\mathrm{B}\left(K^+\to\pi^-\ell^+\ell'^+\right)\lesssim 10^{-23},\ \mathrm{B}\left(D^+\to K^-\ell^+\ell'^+\right)\lesssim 10^{-24}.$$

These estimates are much smaller than the corresponding direct experimental bounds but are close (except for the ee decay mode) to the indirect bounds based on the Majorana mechanism of the decays (see Table 1).

Table 1: The branching ratios $\mathrm{B}_{\ell\ell'}$ for the rare meson decays $M^+ \to M'^- \ell^+ \ell'^+$

Rare decay	Exp. upper bound on $\mathrm{B}_{\ell\ell'}$	Ind. bound on $\mathrm{B}_{\ell\ell'}(\nu_M\mathrm{SM})$	$\mathrm{B}_{\ell\ell'} \times m_{200}^{10}$ ($\not{R}$MSSM)
$K^+ \to \pi^- e^+ e^+$	6.4×10^{-10}	5.9×10^{-32}	$1.3 \times 10^{-17} \lvert\lambda'_{111}\lambda'_{112}\rvert^2$
$K^+ \to \pi^- \mu^+ \mu^+$	3.0×10^{-9}	1.1×10^{-24}	$4.7 \times 10^{-18} \lvert\lambda'_{211}\lambda'_{212}\rvert^2$
$K^+ \to \pi^- e^+ \mu^+$	5.0×10^{-10}	5.1×10^{-24}	$4.3 \times 10^{-18} \lvert\lambda'_{111}\lambda'_{212} + \lambda'_{211}\lambda'_{112}\rvert^2$
$D^+ \to K^- e^+ e^+$	4.5×10^{-6}	1.5×10^{-31}	$1.4 \times 10^{-18} \lvert\lambda'_{122}\lambda'_{111} - 0.39\lambda'_{121}\lambda'_{112}\rvert^2$
$D^+ \to K^- \mu^+ \mu^+$	1.3×10^{-5}	8.9×10^{-24}	$1.3 \times 10^{-18} \lvert\lambda'_{222}\lambda'_{211} - 0.39\lambda'_{221}\lambda'_{212}\rvert^2$
$D^+ \to K^- e^+ \mu^+$	1.3×10^{-4}	2.1×10^{-23}	$6.5 \times 10^{-19} \lvert(\lambda'_{122}\lambda'_{211} + \lambda'_{222}\lambda'_{111})$ $-0.39(\lambda'_{121}\lambda'_{212} + \lambda'_{221}\lambda'_{112})\rvert^2$

References

[1] P. Langacker, *Nucl. Phys.* **B** *(Proc. Suppl.)* **100**, 383 (2001).

[2] B. Kayser, F. Gibrat-Debu, F. Perrier, *"The Physics of the Massive Neutrinos"* (World Scientific, Singapore, 1989).

[3] A.S. Barabash, *JINST* **1**, P07002 (2006); hep-ex/0602037.

[4] A. Ali, A.V. Borisov, N.B. Zamorin, *Eur. Phys. J.* **C21**, 123 (2001).

[5] A. Ali, A.V. Borisov, M.V. Sidorova, *Phys. At. Nucl.* **69**, 475 (2006).

[6] A. Ali, A.V. Borisov, N.B. Zamorin, in *"Frontiers of Particle Physics"* (Proceedings of the 10th Lomonosov Conference on Elementary Particle Physics), ed. by A. Studenikin (World Scientific, Singapore, 2003), p. 74.

[7] A. Ali, A.V. Borisov, D.V. Zhuridov, *Phys. At. Nucl.* **68**, 2061 (2005).

[8] Particle Data Group: W.-M. Yao et al., *J. Phys.* **G33**, 1 (2006).

[9] R. Barbier et al., *Phys. Rep.* **420**, 1 (2005).

[10] D. Aristizabal Sierra, M. Hirsch, W. Porod, *JHEP* **0509**, 033 (2005).

[11] H.E. Haber, G.L. Kane, *Phys. Rep.* **117**, 75 (1985).

[12] L.S. Littenberg, R. Shrock, *Phys. Lett.* **B491**, 285 (2000).

[13] L.B. Okun, *"Leptons and Quarks"* (Nauka, Moscow, 1990) [in Russian].

[14] J.G. Esteve, A. Morales, R. Núñes-Lagos, *J. Phys.* **G9**, 357 (1983).

[15] B.C. Allanach, A. Dedes, H.K. Dreiner, *Phys. Rev.* **D69**, 115002 (2004).

[16] F. Tahir, M. Anwar Mughal, K. Ahmed, *Europhys. Lett.* **54**, 580 (2001).