Relativistic theory of inverse beta-decay of polarized neutron in strong magnetic field

S. A. Shinkevich,* A. I. Studenikin†

Department of Theoretical Physics, Moscow State University
119992, Moscow, Russia

Abstract

The relativistic theory of the inverse beta-decay of polarized neutron, $\nu_e + n \to p + e^-$, in strong magnetic field is developed. We examine the cross section for different energies and directions of propagation of the initial neutrino accounting for neutrons polarization, effects of the proton momentum quantization in the magnetic field and also for the proton recoil motion. It is shown that in the super-strong magnetic field the totally polarized neutron matter is transparent for neutrinos propagating antiparallel to the direction of polarization. The effect of nucleons anomalous magnetic moments in strong magnetic fields is also discussed. The developed relativistic approach can be used for calculations of cross sections of the other URCA processes in strong magnetic fields.

1 Introduction

It is by now widely recognized that strong magnetic fields can be a significant factor relevant to diverse astrophysical and cosmological environments. The presence of strongest magnetic fields in proto-neutron stars and pulsars is well established. The surface magnetic fields for many radio-pulsars, that can be estimated by the observed synchrotron radiation, are of the order of $B \sim 10^{12} - 10^{14} G$. There are also so-called magnetars whose surface magnetic fields are two or three orders of magnitude higher. Very strong magnetic fields are also supposed to exist in the early Universe (see e.g.[1]). Such fields can influence the primordial nucleosynthesis [2, 3, 4] and affect the rate of ^4He production.

Under the influence of strong magnetic fields the direct URCA processes like

$$n \to p + e + \bar{\nu}_e, \quad \nu_e + n \leftrightharpoons e + p, \quad p + \bar{\nu}_e \leftrightharpoons n + e^+ \tag{1}$$

can be modified. These reactions play important role in the neutron star evolution so that the presence of strong magnetic fields significantly change the star cooling rate [8, 7, 5, 6, 9]. It is worth to be noted here also a recent study of neutrino processes (1) in strong magnetic fields of the order $10^{16} G$ and implication for supernova dynamics [10].

The direct URCA processes have gained a lot of attention because of the asymmetry in the neutrino emission, which can arise in the presence of strong magnetic fields. Various authors have argued that asymmetric neutrino emission during the first seconds after the massive star collapse could provide explanations for the observed pulsar velocities. A lot of different mechanisms for the asymmetry in the neutrino emission from a pulsar has been studied previously (see e.g. [11, 12, 13, 16, 17, 15, 18, 19, 14]). For more complete references on the neutrino mechanisms of the pulsar kicks see the review paper [20, 21].

---
*e-mail: shinkevich@gmail.com
†e-mail: studenik@srd.sinp.msu.ru

It is worth to be noted here that the angular dependence of the neutrino emission in URCA processes was first considered for the neutron beta-decay neutrinos in [22, 23]. In these papers the probability of the polarized neutron beta-decay in the presence of a magnetic field was derived, as well as the asymmetry in the neutrino emission was studied for the first time.

The neutron beta-decay have been studied in different electromagnetic field configurations. The first attempts to consider the beta-decay in the field of an electromagnetic wave have been undertaken in [25] and [26]. The relativistic theory of the beta-decay of the neutron (accounting for the proton recoil motion) in the strong magnetic field has been developed in [28]. Many important technical details of the calculations, also useful for the studies performed in the present paper, can be found in [27]. The rates of the two inverse processes in eqs.(1) in the presence of a magnetic field has been derived in [12].

The present paper is devoted to a detailed study of the inverse beta-decay of neutron in a magnetic field

$$\nu_e + n \to p + e^-. \tag{2}$$

The process $\nu n \to pe$ in a magnetic field has been discussed previously by several authors. The contribution of this process to the conditions for beta-equilibrium in the presence of magnetic fields has been considered in [32]. The dependence of the cross section on the magnetic field has been also discussed [15] in the context of the pulsar kick in the case when the asymmetric magnetic field arises just after the star collapse.

A reasonable interest to the inverse beta-decay of neutron in magnetic fields has been stimulated by a believe that it can be relevant for the neutrino opacity in the proto-neutron star stage after supernova collapse. In [18], as well as in [19], the calculations for the cross section have been performed under the assumption that the magnetic field gives contribution to the phase space integrals only, whereas the process matrix element have been considered unaffected by the magnetic field.

The first attempt to calculate modification of the neutrino sphere in pulsar due to the asymmetry in the $\nu n \to pe$ cross section accounting for the magnetic field modifications of the matrix element has been undertaken in [5]. However, in this paper, as well as in [6], the transition to the electron lowest Landau level has been discussed. In the paper [7] the angular asymmetry of the cross section has been calculated only to the first order in the magnetic field.

An important effect of anisotropy in the cross section of the inverse beta-decay has been recently considered in a series of papers [33, 34, 35] where the process $\nu n \to pe$ has been studied in presence of a background magnetic field and the initial neutron polarization has been also accounted for. However, some of the final results of refs.[33, 34] for the cross section do not coincide with corresponding results of ref. [35].

The present paper is devoted to a detailed evaluation of the inverse beta-decay of polarized neutron cross section in a magnetic field. For both of the charged particles (e and p) wave functions we use the exact solutions of the Dirac equation in the presence of a magnetic field so that we also exactly account for the magnetic field influence on the proton. The incoming neutrino is supposed to be relativistic and effects of neutrino non-zero mass are neglected. We do not set any special limit on the neutrino energies, however it is supposed that the four-fermion weak interaction theory is relevant in our case. For astrophysical applications, and for supernovas in particular, it is of interest to consider the neutrino energies in the range of $\varkappa \sim 1 - 30 MeV$.

In our consideration we account for the proton momentum quantization in the magnetic field and for the proton recoil motion so that we develop here the relativistic theory of the inverse beta-decay. We also suppose that the Z and W bosons are not affected by the magnetic field. The contribution of nucleons anomalous magnetic moments in strong magnetic fields is discussed. The former effect can be easily incorporated into our calculations by the corresponding shift of the nucleons masses (see also [8, 7, 1, 36]). We also show that in the case of very strong magnetic fields the process, due to the anomalous magnetic moments, can be forbidden.

2   Cross section of inverse beta-decay

We start with the well known four-fermion Lagrangian,

$$\mathcal{L} = \frac{G}{\sqrt{2}} \left[\overline{\psi}_p \gamma_\mu (1 + \alpha\gamma_5)\psi_n \right] \left[\overline{\psi}_e \gamma^\mu (1 + \gamma_5)\psi_\nu \right], \tag{3}$$

where $G = G_F \cos\theta_c$, θ_c is the Cabibbo angle, and $\alpha = 1.26$ is the ratio of the axial and vector constant. The total cross section of the process can be written as

$$\sigma = \frac{L^3}{T} \sum_{phase\ space} |M|^2, \tag{4}$$

where summation is performed over the pase space of the final particles. The matrix element of the process is given by

$$M = \frac{G}{\sqrt{2}} \int \left[\overline{\psi}_p \gamma_\mu (1 + \alpha\gamma_5)\psi_n \right] \left[\overline{\psi}_e \gamma^\mu (1 + \gamma_5)\psi_\nu \right] dx\,dy\,dz\,dt. \tag{5}$$

We account for the influence of the background magnetic field on the matrix element (5). The corresponding calculations are performed by using the exact solutions of the Dirac equation in the magnetic field for the relativistic electron and proton. Without loss of generality, a constant magnetic field $\vec{B}$ is taken along the z-direction. We use the notations of our previous study [28] of the beta-decay of the polarized neutron in a magnetic field with the proton recoil effects have been accounted for.

The electron wave function $\psi_e(m, n, s, p_0, p_2, p_3)$ can be written in the form

$$\psi_e = \frac{1}{L} \begin{pmatrix} C_1 U_{n-1}(\eta) \\ iC_2 U_n(\eta) \\ C_3 U_{n-1}(\eta) \\ iC_4 U_n(\eta) \end{pmatrix} e^{-i(p_0 t - p_2 y - p_3 z)}, \quad \eta = x\sqrt{\gamma} + \frac{p_2}{\sqrt{\gamma}}, \quad \gamma = eB, \tag{6}$$

where $U_n(\eta)$ are Hermite functions of order n, e is the absolute value of the electron charge, p_0, p_2 and p_3 are the electron energy and momentum components, respectively. The energy spectrum

$$p_0 = \sqrt{m^2 + 2\gamma n + p_3^2}, \tag{7}$$

depends on the discreet number $n = 0, 1, 2, ...$ denoting the Landau levels (m is the electron mass). The spin coefficients C_i are

$$C_{1,3} = \frac{1}{2}\sqrt{1 + s\frac{m}{\tilde{p}_\perp}}\,\sqrt{1 \pm s\frac{\tilde{p}_\perp}{p_0}}, \quad C_{2,4} = \mp\frac{1}{2}s\sqrt{1 - s\frac{m}{\tilde{p}_\perp}}\,\sqrt{1 \mp s\frac{\tilde{p}_\perp}{p_0}}, \tag{8}$$

and $\tilde{p}_\perp = \sqrt{m^2 + 2\gamma n}$. The spin number can have only the values ± 1, $s = +1$ when the electron spin is directed along the magnetic field $\vec{B}$, and $s = -1$ in the opposite case. The electrons on all Landau levels with $n \geq 1$ can have two different spin polarizations. However, in the lowest Landau state ($n = 0$) the electron spin can have the only orientation given by $s = -1$, so that the electrons moving along the direction of the magnetic field are left-handed polarized, whereas the electrons moving in the opposite direction are right-handed polarized.

The proton wave function $\psi_p(m', n', s', p_0', p_2', p_3')$ can be expressed in a similar form

$$\psi_p = \frac{1}{L} \begin{pmatrix} C_1' U_{n'}(\eta') \\ -iC_2' U_{n'-1}(\eta') \\ C_3' U_{n'}(\eta') \\ -iC_4' U_{n'-1}(\eta') \end{pmatrix} e^{-i(p_0' t - p_2' y - p_3' z)}. \tag{9}$$

3

The dashed quantities correspond to the proton mass, number of the Landau state, energy and momentum components. Again the proton spin values are $s' = \pm 1$, however now at the lowest Landau level the spin orientation is along the magnetic field $\vec{B}$.

The initial neutron and neutrino are supposed to be not affected by the magnetic field, and we use the plane waves for their wave functions. The polarized neutron wave function is

$$\psi_n = \frac{1}{2L^{3/2}} \begin{pmatrix} N_1 \\ N_2 \\ N_3 \\ N_4 \end{pmatrix} e^{-i(p_0^n t - \vec{p}_n \vec{r})}, \tag{10}$$

where the neutron spin coefficients are

$$N_{1,3} = s_n \sqrt{1 \pm \frac{m_n}{p_0^n}} \sqrt{1 \pm s_n \cos\theta_n}\, e^{\mp i\varphi_n/2}, \quad N_{2,4} = \sqrt{1 \mp \frac{m_n}{p_0^n}} \sqrt{1 \mp s_n \cos\theta_n}\, e^{\pm i\varphi_n/2}. \tag{11}$$

Here m_n, p_0^n and $\vec{p}_n$ are the neutron mass, energy, momentum, and θ_n, φ_n are the polar and azimuthal neutron momentum angles. We perform our calculations in the rest frame of the neutron, so that we shall take below $p_0^n = m_n$ and $N_3 = N_4 = 0$.

The neutrino wave function can be chosen in the form

$$\psi_\nu = \frac{1}{2L^{3/2}} \begin{pmatrix} f_1 \\ f_2 \\ -f_1 \\ -f_2 \end{pmatrix} e^{-i(\varkappa t - \vec{\varkappa}\vec{r})}, \quad f_1 = -e^{-i\varphi_\nu}\sqrt{1 - \cos\theta_\nu}, \quad f_2 = \sqrt{1 + \cos\theta_\nu}, \tag{12}$$

where $\varkappa$, $\vec{\varkappa}$ are the neutrino energy and momentum, respectively ($\varkappa = |\vec{\varkappa}|$). The neutrino polar and azimuthal angles are denoted as φ_ν and θ_ν.

Putting these wave functions to the matrix element of the process (5), we can perform the integrations over time t and coordinates y, z and obtain squared norm of the matrix element

$$|M|^2 = (2\pi)^3 TL^2 |\tilde{M}|^2 \delta(p_0' + p_0 - m_n - \varkappa)\delta(p_2' + p_2 - \varkappa_2)\delta(p_3' + p_3 - \varkappa_3), \tag{13}$$

where

$$|\tilde{M}|^2 = 2G^2 \Big[(\alpha - 1)^2 f_1^2 N_1^2 (C_1 - C_3)^2 (C_1' + C_3')^2 I_{n',n-1}^2(\rho)$$

$$+ (C_2 - C_4)^2 \Big\{ 4f_1^2 N_2^2 (\alpha C_1' - C_3')^2 + (\alpha + 1)^2 f_2^2 N_1^2 (C_1' - C_3')^2 \Big\} I_{n',n}^2(\rho)$$

$$+ (\alpha - 1)^2 f_2^2 N_2^2 (C_2 - C_4)^2 (C_2' + C_4')^2 I_{n'-1,n}^2(\rho) + (C_1 - C_3)^2$$

$$\times \Big\{ 4f_2^2 N_1^2 (\alpha C_2' - C_4')^2 + (\alpha + 1)^2 f_1^2 N_2^2 (C_2' - C_4')^2 \Big\} I_{n'-1,n-1}^2(\rho)$$

$$+ 4(\alpha + 1)(C_1 - C_3)(C_2 - C_4) \Big\{ f_2^2 N_1^2 (C_1' - C_3')(\alpha C_2' - C_4')$$

$$+ f_1^2 N_2^2 (C_2' - C_4')(\alpha C_1' - C_3') \Big\} I_{n',n}(\rho) I_{n'-1,n-1}(\rho) \Big]. \tag{14}$$

The phase space factor in the general expression (4) for the electron and proton in the presence of a magnetic field is

$$\sum_{phase\ space} = \int \frac{L}{2\pi} dp_2 \frac{L}{2\pi} dp_3 \frac{L}{2\pi} dp_2' \frac{L}{2\pi} dp_3' \sum_{n=0,n'=0} \sum_{s=\pm 1, s'=\pm 1} g_n g_{n'}, \tag{15}$$

where $g_0 = 1$, and $g_k = 2$ for $k \geq 1$ are the degeneracies of the Landau energy levels for the electron and proton. After integrations over the proton momentum component p_2' and the electron momentum component p_3 we get the laws of conservation for the two momentum

4

components, $p_3 = \varkappa_3 - p_3'$, $p_2' = \varkappa_2 - p_2$. The integration over the electron momentum component p_2 is performed by taking in to account the specific for the motion in a magnetic field degeneracy of the electron energy. Finally we obtain the cross section of the inverse beta-decay of the polarized neutron in a magnetic field with the proton recoil motion effect being accounted for,

$$\sigma = \frac{eB}{32\pi} \sum_{s,s'} \sum_{n,n'} \int_{\infty}^{\infty} |\tilde{M}|^2 \delta_0(p_0' + p_0 - m_n - \varkappa)\big|_{p_3 = \varkappa_3 - p_3'} dp_3', \tag{16}$$

where $|\tilde{M}|^2$ is given by (14) with p_3 being substituted by $\varkappa_3 - p_3'$ because we have already perform the integration over the component of the electron momentum p_3 with use of the corresponding δ-function. The remaining integration over the component p_3' of the proton momentum is performed with use of the $\delta_0\big(\varphi(p_3')\big)$-function. The argument $\varphi(p_3')$, being equated with zero, gives the law of energy conservation for the particles in the process,

$$m_n + \varkappa = \sqrt{m^2 + 2\gamma n + (\varkappa_3 - p_3')^2} + \sqrt{m'^2 + 2\gamma n' + p_3'^2}. \tag{17}$$

There are the two roots of the equation $\varphi(p_3'^{(i)}) = 0$,

$$p_3'^{(1,2)} = \frac{1}{2\big[(m_n + \varkappa)^2 - \varkappa_3^2\big]} \left\{ \varkappa_3 \Big[(m_n + \varkappa)^2 + \tilde{p}_\perp'^2 - \tilde{p}_\perp^2 - \varkappa_3^2\Big] \right.$$

$$\left. \pm (m_n + \varkappa)\sqrt{\Big[(m_n + \varkappa)^2 - \tilde{p}_\perp'^2 - \tilde{p}_\perp^2 - \varkappa_3^2\Big]^2 - 4\tilde{p}_\perp'^2\tilde{p}_\perp^2} \right\}. \tag{18}$$

Finally we obtain the cross section of the inverse beta-decay of the polarized neutron in a magnetic field, with the effects of the Landau quantization of the proton momentum and of the proton recoil motion being accounted for exactly,

$$\sigma = \frac{eB}{32\pi} \sum_{s,s'} \sum_{n,n'} \sum_{i=1,2} \frac{|\tilde{M}^{(i)}|^2}{\left| \frac{p_3^{(i)}}{p_0^{(i)}} - \frac{p_3'^{(i)}}{p_0'^{(i)}} \right|}, \tag{19}$$

where one of the sums is performed over the roots $p_3'^{(i)}$ (18), and

$$p_3^{(i)} = \varkappa_3 - p_3'^{(i)}, \quad p_0'^{(i)} = \sqrt{\tilde{p}_\perp'^2 + p_3'^{(i)2}}, \quad p_0^{(i)} = \sqrt{\tilde{p}_\perp^2 + (\varkappa_3 - p_3'^{(i)})^2}. \tag{20}$$

The squared matrix element $|\tilde{M}^{(i)}|^2$ is given by eq. (14) where the substitution $p_3' \to p_3'^{(i)}$ must be done.

3   Critical magnetic field

Now let us consider eq. (17) in detail that gives the energy conservation law by accounting for the presence of a magnetic field. Due to the particular properties of the energy spectra of the electron and proton in a magnetic field, we can introduce two critical values of the magnetic field strength. First let us determine the critical electron magnetic field, B_{cr}, from the condition that in the external field $B \geq B_{cr}$ the electron can occupy only the lowest Landau level with $n = 0$. From (17) we get that for a fixed maximum neutrino energy $\varkappa_{max}$ and for a fixed strength of the magnetic field, the maximum number of the available electron Landau level is

$$n_{max} = \text{int}\left[\frac{(\Delta + \varkappa_{max})^2 - m^2}{2eB}\right], \quad \Delta = m_n - m'. \tag{21}$$
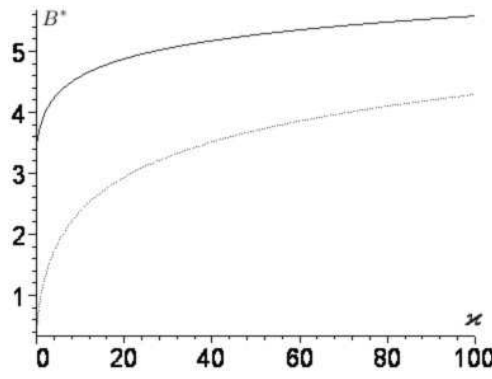
5

Figure 1: Dependence of the electron critical magnetic field B_{cr} (dashed line) and the proton critical magnetic field B'_{cr} (solid line) on the initial neutrino energy $\varkappa$ (MeV). The logarithmic scale is used: $B^* = \log \frac{B}{B_0}$, where $B_0 = \frac{m^2}{e}$.

From the condition $n_{max} < 1$ (it means that the electron can occupy only the lowest Landau level with $n = 0$) we get

$$B_{cr} = \frac{(\Delta + \varkappa_{max})^2 - m^2}{2e}. \tag{22}$$

The critical proton magnetic field, B'_{cr}, was determined from the condition that in the external field $B \geq B'_{cr}$ the proton can occupy only the lowest Landau level with the number $n' = 0$. Again, from (17) we get that for a fixed maximum neutrino energy $\varkappa_{max}$ and for a fixed strength of the magnetic field, the maximum number of the available proton Landau level is

$$n'_{max} = \text{int}\left[\frac{(\varkappa_{max} + m_n - m)^2 - m'^2}{2eB}\right]. \tag{23}$$

The proton can occupy only the Landau level with $n' = 0$ if the magnetic field strength exceeds the proton critical field

$$B'_{cr} = \frac{(\varkappa_{max} + m_n - m)^2 - m'^2}{2e}. \tag{24}$$

Thus, B_{cr} and B'_{cr} depend on the maximum available neutrino energy. For different neutrino energies we have the following values of the electron and proton critical magnetic field:

$$B_{cr} \approx 8.3 \times 10^{16} G, \quad B'_{cr} \approx 5 \times 10^{18} G, \quad \varkappa_{max} = 30 \ MeV \tag{25}$$

$$B_{cr} \approx 1.1 \times 10^{16} G, \quad B'_{cr} \approx 1.7 \times 10^{18} G, \quad \varkappa_{max} = 10 \ MeV \tag{26}$$

$$B_{cr} \approx 1.2 \times 10^{14} G, \quad B'_{cr} \approx 1.3 \times 10^{17} G, \quad \varkappa_{max} \ll m. \tag{27}$$

In Fig.1 we plot the values of the critical fields B_{cr} (dashed line) and B'_{cr} (solid line) as functions of the initial neutrino energy $\varkappa$.

From the above we conclude that there are the three ranges of the magnetic field strength which we call: 1) the weak field ($B \leq B_{cr}$), 2) the strong field ($B_{cr} < B < B'_{cr}$), and 3) the super-strong field ($B'_{cr} \leq B$). For the most of the weak field range ($B \ll B_{cr}$) the electron n and proton n' Landau numbers can have very large values. Inside the strong field range ($B_{cr} < B \ll B'_{cr}$) only the proton number n' can have very large values, whereas the electron number is always zero. In the super-strong fields the both Landau numbers are zero, $n = n' = 0$.

4 Cross section in super-strong, strong and weak magnetic fields

4.1 Cross section in super-strong magnetic field

Let us start with consideration of the super-strong magnetic field $B \geq B'_{cr}$. In this case the both numbers of the Landau levels for the electron and proton are zero. The squared matrix

element $|\tilde{M}|^2$ in eq. (14) is reduced to

$$|\tilde{M}_{n=n'=0}|^2 \;=\; 2G^2(C_2 - C_4)^2\Big\{4f_1^2N_2^2\alpha C_1' - C_3')^2 + (\alpha + 1)^2 f_2^2 N_1^2(C_1' - C_3')^2\Big\}e^{-\rho}. \quad (28)$$

Putting back in (19), we obtain the cross section of the process in the presence of the super-strong magnetic field

$$\sigma_{n=n'=0} \;=\; \frac{eBG^2}{8\pi}e^{-\frac{\varkappa_\perp^2}{2\gamma}} \sum_{i=1,2} \frac{\left(1 + \frac{p_3^{(i)}}{p_0^{(i)}}\right)}{\left|\frac{p_3^{(i)}}{p_0^{(i)}} - \frac{p_3'^{(i)}}{p_0'^{(i)}}\right|}\Big\{a^{(i)} + b^{(i)}\cos\theta + s_n(b^{(i)} + a^{(i)}\cos\theta)\Big\}, \quad (29)$$

where

$$a^{(i)} = 3 + 2\alpha + 3\alpha^2 - 2(1 - \alpha^2)\frac{m'}{p_0'^{(i)}} - (1 + 6\alpha + \alpha^2)\frac{p_3'^{(i)}}{p_0'^{(i)}},$$

$$b^{(i)} = -1 + 2\alpha - \alpha^2 + 2(1 - \alpha^2)\frac{m'}{p_0'^{(i)}} - (1 - \alpha)^2\frac{p_3'^{(i)}}{p_0'^{(i)}}. \quad (30)$$

The effect of the proton motion, which appears in this case exceptional due to the proton recoil in z-direction, is accounted exactly in eqs. (29) and (30). It is worth to be mentioned that the derived expression for the cross section in the super-strong magnetic field $B \geq B_{cr}'$ can be applied for neutrinos with arbitrary (also ultra-high) energies (note that following to eq. (24) the value of B_{cr}' is increasing with the neutrino energy increase).

If we neglect the proton momentum parallel or antiparallel to the magnetic field, we get

$$\sigma_{n=n'=0}\Big|_{p_0'=m'} = \frac{eBG^2}{4\pi}e^{-\frac{\varkappa_\perp^2}{2\gamma}}\{a + b\cos\theta + s_n(b + a\cos\theta)\}\frac{\Delta + \varkappa}{\sqrt{(\Delta + \varkappa)^2 - m^2}}, \quad (31)$$

where

$$a = 1 + 2\alpha + 5\alpha^2, \quad b = 1 + 2\alpha - 3\alpha^2. \quad (32)$$

For $\alpha = 1.26$ one can get $a = 11.5$ and $b = -1.24$. Note that the same coefficients a and b determine the neutrino asymmetry in the probability of the direct neutron beta-decay in the super-strong magnetic field [28, 14]. In the case of moderate neutrino energies $\varkappa^2 \ll eB$ (the last inequality is valid in the super-strong magnetic field $B \geq B_{cr}'$ for the range of the neutrino energies $\varkappa \leq 30\ MeV$) the exponential term in (29) must be substituted for unit.

In the case of non-polarized neutrons we have to overage the cross section over the neutron spin

$$\sigma_{unpol.} = \frac{1}{2}\sum_{s_n=\pm 1}\sigma(s_n). \quad (33)$$

We also can use the obtained expressions for the cross section in analysis of the neutrino interaction with partially polarized neutron matter when the numbers of neutrons (per unit volume) with the two different spin polarizations are N_+ and N_-, respectively. The partially polarized neutron matter can be characterized by the neutrons polarization S determined as

$$S = \frac{N_+ - N_-}{N_+ + N_-}. \quad (34)$$

All the obtained above formulas for the cross section can be used for the case of partially polarized neutron matter if one substitutes s_n for S.

In Fig.2, we have plotted, for neutrino energy $\varkappa = 10\ MeV$, the cross section in the magnetic field $B = \tilde{B}_{cr}'$, normalized to the cross section in the field-free case, as a function of neutrons
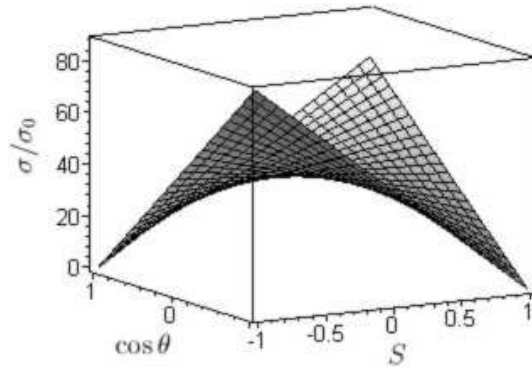
7

Figure 2: The cross section σ in super-strong magnetic field $B = B'_{cr}$, normalized to the cross section σ_0 in the field-free case, for neutrinos with energy of $\varkappa = 10\ MeV$ as functions of the direction of the neutrino momentum $\cos\theta$ and polarization of neutrons S.
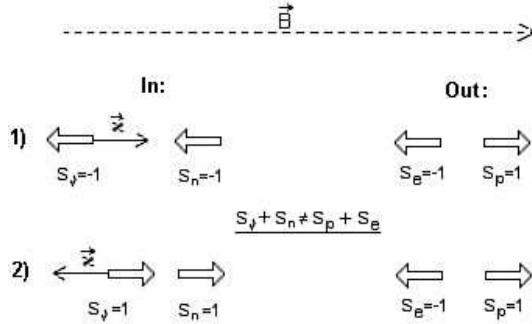

Figure 3: Initial and final particles spin orientations for the two directions of the neutrino propagation ($\cos\theta = \pm 1$) in the super-strong magnetic field $B \geq B'_{cr}$. The broad arrows represent the particles spin orientations, the solid arrows show directions of the neutrino propagation, and the dashed arrow shows the direction of the magnetic field vector. The cross section is zero when the sum of the spin numbers of the initial particles s_ν, s_n is not equal to the sum of the spin numbers of the final particles s_e, s_p.

polarization S and of $\cos\theta$ (θ is the angle the neutrino momentum makes with the magnetic field). The cross section (31) vanishes to zero for the cases when the direction of the neutrons total polarization is antiparallel to the direction of the neutrino momentum, $S\cos\theta = -1$.

Thus, for these two cases the neutron matter is transparent for neutrinos. These phenomenon appears due to the Landau quantization of the momentum and the spin properties of the charged particles in the strong and super-strong magnetic fields. In the field $B \geq B'_{cr}$ the final electron and proton can move only parallel to the fixed line that is given by the magnetic field vector. For the the neutrino also moving along this line and the neutron being at rest, the law of angular momentum conservation reduces to the law of "spin number conservation". In Fig.3 we present an illustration of the law of the "spin number conservation".

4.2   Cross section in strong magnetic field

In the case of strong magnetic fields $B_{cr} \leq B < B'_{cr}$, the electron can only occupy the lowest Landau level with $n = 0$, whereas there could be many Landau levels available for the proton. The maximum number of the proton Landau level is estimated as

$$n'_{max} = \text{int}\left[\frac{(m_n + \varkappa - m)^2 - m'^2}{2eB}\right] \approx \text{int}\left[\frac{m'(\Delta + \varkappa - m)}{eB}\right]. \tag{35}$$

8

For the squared matrix element of the process we get from (14)

$$|\tilde{M}_{n=0}|^2 = 2G^2(C_2 - C_4)^2\Big[(\alpha-1)^2 f_2^2 N_2^2(C_2' + C_4')^2 I_{n'-1,0}^2(\rho)+$$
$$+ \Big\{4f_1^2 N_2^2(\alpha C_1' - C_3')^2 + (\alpha+1)^2 f_2^2 N_1^2(C_1' - C_3')^2\Big\} I_{n',0}^2(\rho)\Big]. \quad (36)$$

Putting the squared matrix element (36) to the general formula for the cross section, eq. (19), we get the expression for the cross section,

$$\sigma_{n=0} = \frac{eBG^2}{8\pi} \sum_{n'=0}^{n'_{max}} \sum_{i=1,2} \frac{\left(1 + \frac{p_3^{(i)}}{p_0^{(i)}}\right)}{\left|\frac{p_3^{(i)}}{p_0^{(i)}} - \frac{p_3'^{(i)}}{p_0'^{(i)}}\right|} \Bigg\{ \Big[(1+\alpha)^2\Big(1 - \frac{p_3'^{(i)}}{p_0'^{(i)}}\Big)(1+S)(1+\cos\theta)$$
$$+ 2\Big[1 + \alpha^2 - (1-\alpha^2)\frac{m'}{p_0'^{(i)}} - 2\alpha\frac{p_3'^{(i)}}{p_0'^{(i)}}\Big](1-S)(1-\cos\theta)\Big] I_{n',0}^2(\rho)$$
$$+ (1-\alpha)^2\Big(1 - \frac{p_3'^{(i)}}{p_0'^{(i)}}\Big)(1-S)(1+\cos\theta)(1-\delta_{n',0})I_{n'-1,0}^2(\rho)\Bigg\}. \quad (37)$$

Together with the expression (18) for $p_3'^{(i)}$, equation (37) gives the cross section for the process in the strong magnetic field exactly accounting for the proton momentum quantization and the proton recoil motion.

A simplification can be achieved if we neglect the proton motion in the plain orthogonal to the magnetic field vector and account only for the proton recoil in z-direction. In this case we get

$$\sigma_{n=0}\Big|_{p_\perp'=0,p_3'\neq0} = \frac{eBG^2}{4\pi} \sum_{i=1,2} \frac{\left(1 + \frac{p_3^{(i)}}{p_0^{(i)}}\right)}{\left|\frac{p_3^{(i)}}{p_0^{(i)}} - \frac{p_3'^{(i)}}{\tilde{p}_3'^{(i)}}\right|} \Bigg\{ \Big(1 + 3\alpha^2 - (1+\alpha)^2\frac{p_3'^{(i)}}{m'}\Big)$$
$$+ \Big(1 - \alpha^2 - (1-\alpha)^2\frac{p_3'^{(i)}}{m'}\Big)\cos\theta + S\Big[2\alpha(1-\alpha) + \Big(2\alpha(1+\alpha) - 4\alpha\frac{p_3'^{(i)}}{m'}\Big)\cos\theta\Big]\Bigg\}. \quad (38)$$

If we also neglect the effect of the proton motion in z-direction, then for the cross section in the strong field $B_{cr} < B \ll B'_{cr}$ we get

$$\sigma_{n=0}\Big|_{p_0'=m'} = \frac{eBG^2}{2\pi} \Bigg\{ 1 + 3\alpha^2 + (1-\alpha^2)\cos\theta$$
$$+ 2\alpha S\Big[1 - \alpha + (1+\alpha)\cos\theta\Big]\Bigg\} \frac{\Delta + \varkappa}{\sqrt{(\Delta+\varkappa)^2 - m^2}}. \quad (39)$$

For the neutron matter totally polarized parallel to the magnetic field vector, $S = 1$, the cross section is vanish.

The cross section in the strong magnetic field $B = B_{cr}$, normalized to the cross section in the field-free case, calculated with use of the exact eq. (37) is shown in Fig.4. Note also that, as it can be seen from the Fig.4, the cross section for $\cos\theta = 1$ and $S = -1$ is also rather small. This is a consequence of smallness of the value $(\alpha - 1)$ because the cross section in this case is proportional to $(1-\alpha)^2 < 0.1$. The neutrino energy is chose to be $\varkappa = 10\ MeV$, that is why the effects of the proton recoil motion cannot be screened.
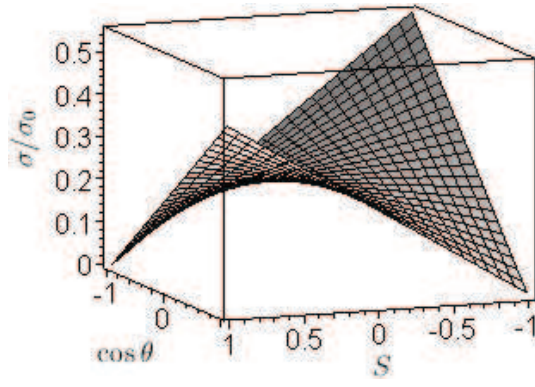
Figure 4: The cross section in the strong magnetic field $B = B_{cr}$, normalized to the cross section in the field-free case, for neutrinos with energy of 10 MeV as functions of the direction of the neutrino momentum $\cos\theta$ and polarization S of neutrons. The cross section in the magnetic field in the case $\cos\theta = -1$, $S = 1$ is exactly zero, whereas the cross section in the case $\cos\theta = 1$, $S = -1$ is not zero, however it is rather small because it is proportional to $(1 - \alpha)^2 < 0.1$.

4.3 Cross section in weak magnetic field

In the case of weak magnetic fields $B < B_{cr}$ many Landau levels become available for the electron so that the electron can have non-zero momentum $p_\perp = \sqrt{2\gamma n}$ in the transverse plane. The maximum allowed value for n is given by (21). In the calculations of the cross section in the presence of a weak magnetic field we perform the summation over the proton Landau number n' up to infinity. The particular contribution to the cross section from the partial process with the electron at the lowest Landau level ($n = 0$) has been already discussed. Therefore, we derive now the fraction $\sigma_{n\geq1}$ of the total cross section that is the sum of the corresponding contributions from the excited electron Landau levels with $n \geq 1$. The final result for the cross section can be expressed as

$$\sigma_{\text{tot}} = \sigma_{n=0} + \sigma_{n\geq1}. \tag{40}$$

Putting the general expression for the squared matrix element (14) to (19), then expanding over $\frac{p_3'}{m'}$ and performing summation over n', we obtain to the first order in the proton recoil motion

$$\sigma_{n\geq1} = \frac{eBG^2}{2\pi} \sum_{n=1}^{n_{max}} \sum_{i=1,2} \frac{1}{\left| \frac{p_3^{(i)}}{p_0^{(i)}} - \frac{p_3'^{(i)}}{p_0'^{(i)}} \right|} \left[1 + 3\alpha^2 + 2\alpha(1-\alpha)\frac{p_3'^{(i)}}{m'}(1 + \cos\theta) + 2S\alpha(1+\alpha)\cos\theta \right.$$

$$\left. + 2(1+\alpha)^2 \frac{\gamma n}{p_0^{(i)} m'}(1 + S\cos\theta) \right]. \tag{41}$$

In the limit of non-moving proton ($p_0' = m'$) the contribution to the cross section for $n \geq 1$ is

$$\sigma_{n\geq1}\Big|_{p_0'=m'} = \frac{eBG^2}{\pi} \left[1 + 3\alpha^2 + 2S\alpha(1+\alpha)\cos\theta \right] \sum_{n=1}^{n_{max}} \frac{\Delta + \varkappa}{\sqrt{(\Delta + \varkappa)^2 - m^2 - 2\gamma n}}. \tag{42}$$

Summing this result with one of eq. (39), we get the result of ref.[35] for the total cross section in the case of weak magnetic field (the proton recoil motion is neglected here)

$$\sigma_{\text{tot}}\Big|_{p_0'=m'} = \frac{eBG^2}{2\pi} \sum_{n=0}^{n_{max}} \left\{ g_n \left[1 + 3\alpha^2 + 2S\alpha(1+\alpha)\cos\theta \right] \right.$$

$$\left. + \delta_{n,0} \left[(1-\alpha^2)\cos\theta + 2S\alpha(1-\alpha) \right] \right\} \frac{\Delta + \varkappa}{\sqrt{(\Delta + \varkappa)^2 - m^2 - 2\gamma n}}. \tag{43}$$
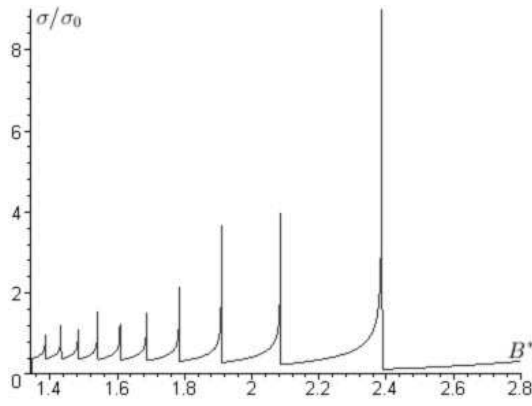
10

Figure 5: The resonance behavior of the cross section in magnetic field, normalized to the field-free case, for neutrino energy $\varkappa = 10 \ MeV$. The logarithmic scale is used: $B^* = \log \frac{B}{B_0}$, where $B_0 = \frac{m^2}{e}$.

As it follows from (42) and (43), the cross section has several resonances (see also [18, 35]). Similar resonances in the probability of the direct beta-decay of the neutron in the magnetic field was first discovered in [22, 23]. In our case the resonances appears, for the given neutrino energy $\varkappa$ and magnetic field strength B, each time when the final electron energy p_0 is exactly equal to one of the allowed $(n \leq n_{max})$ "Landau energies" $\tilde{p}_\perp = \sqrt{m^2 + 2\gamma n}$,

$$p_0 = \varkappa + \Delta = \sqrt{m^2 + 2\gamma n}. \tag{44}$$

In Fig.5 we plot the cross section as a function of B (in the range of not very strong magnetic fields, $B \leq B_{cr}$) for the energy $\varkappa = 10 \ MeV$. Obviously, the similar resonance behavior appears in the cross section as a function of the neutrino energy in a given fixed magnetic field. The number of resonances, which is equal to the number of terms in the sum of eq. (42), increases with the increase of the neutrino energy for a given B. The cross section, calculated without effects of the proton recoil motion, goes to infinity in the resonance points. However, if we plot the cross section with use of eqs. (37) and (41), which accounts for the proton motion, then the infinitely high spikes smooth out.

4.4 Cross section in the absence of magnetic field

The inverse beta-decay in the absence of a magnetic field was considered before by many authors (see, for instance, [38, 39, 40]). The result for the cross section $\nu_e + n \rightarrow e + p$ in the absence of the magnetic field is

$$\sigma_0 = \frac{G^2}{\pi} \left[1 + 3\alpha^2 + 2\alpha S_n(1 + \alpha) \cos\theta \right] (\Delta + \varkappa) \sqrt{(\Delta + \varkappa)^2 - m^2}. \tag{45}$$

When the field is switching off the maximum number of the Landau level n_{max} is increasing to infinity, however the product eBn remains constant,

$$\lim_{\gamma \to 0, n \to \infty} \gamma n = \frac{(\Delta + \varkappa)^2 - m^2}{2}. \tag{46}$$

In the sum over n in (43) the contribution of the lowest Landau level is diminishing in comparison with the contributions of the exited Landau levels $n > 0$. For the estimation of the

11

former we use following the similar procedure described in [22, 23] and have

$$\lim_{\gamma \to 0} \gamma \sum_{n=0}^{n_{max}} \frac{1}{\sqrt{(\Delta + \varkappa)^2 - m^2 - 2\gamma n}} = \lim_{\gamma \to 0} \gamma \left[\int_0^{n_{max}} \frac{dx}{\sqrt{(\Delta + \varkappa)^2 - m^2 - 2\gamma x}} + C \right]$$

$$= \lim_{\gamma \to 0} \gamma \left[\frac{1}{\gamma} \sqrt{(\Delta + \varkappa)^2 - m^2} + C \right] = \sqrt{(\Delta + \varkappa)^2 - m^2}, \quad (47)$$

where C is a function proportional to $\gamma^{-1/2}$. Thus, in the limit $B \to 0$ from (43) we get the cross section of the process in the absence of a magnetic field (45).

4.5 Effects of Anomalous Magnetic Moments of Nucleons

When considering the influence of very strong magnetic fields on the inverse beta-decay of a neutron one should be careful about the effect of magnetic field on anomalous magnetic moments of a neutron and proton. In particular, it is known (see, for instance, [29, 1, 36]) that the interplay between anomalous magnetic moments of the neutron and proton shifts the masses of these particles. These effects are important only for the super-strong magnetic fields, when the corresponding shift of the electron energy due to the electron anomalous magnetic moment is vanishing [41, 42] (see also in [1]).

The energy of the moving proton and the neutron at rest in a magnetic field, with the contributions from the anomalous magnetic moments interaction being accounted for, are given respectively by expressions

$$p_0' = \sqrt{\left(\sqrt{m'^2 + 2eBn'} - s'k_p B \right)^2 + p_3'^2}, \quad p_0^n = m_n - s_n k_n B, \quad (48)$$

where the values of the proton and neutron anomalous magnetic moments

$$k_p = \frac{e}{2m'}\left(\frac{g_p}{2} - 1\right), \quad k_n = \frac{e}{2m_n}\frac{g_n}{2}, \quad (49)$$

are determined by the Lande's $g-$factors: $g_p = 5.58$, $g_n = -3.82$.

Taking into account modified expressions for the proton and neutron energies (48), we can repeat all the described above calculations applying the substitutions

$$m' \to m'^* = m' - k_p B, \quad m_n \to m_n^* = m_n - s_n k_n B. \quad (50)$$

Note that in the super-strong magnetic filed $B \geq B_{cr}'$ there is the only one spin state for the proton with $s' = +1$.

The law of energy conservation (17) shows that in the super-strong magnetic field there is a range of the neutron matter polarization S for which the matter becomes transparent for neutrinos. From (17) we get that the process $\nu_e + n \to e + p$ is forbidden if $(Sk_n - k_p) > 0$ and the magnetic field exceeds the value of B_{forb}:

$$B_{forb} = \frac{\Delta + \varkappa - m}{Sk_n - k_p}. \quad (51)$$

Note that this forbidding effect appears for nearly maximum neutron matter polarizations against the magnetic field, $-1 \leq S < k_p/k_n \approx -0.94$. The values of B_{forb} for different neutrino energies in case of maximum neutron spin polarization $S = -1$ are

$$B_{forb} \approx 8.5 \times 10^{19} G, \quad \varkappa_{max} = 30 \; MeV, \quad (52)$$

$$B_{forb} \approx 3.0 \times 10^{19} G, \quad \varkappa_{max} = 10 \; MeV, \quad (53)$$

$$B_{forb} \approx 2.2 \times 10^{18} G, \quad \varkappa_{max} \ll m. \quad (54)$$

5 Conclusions

We have developed the relativistic theory of the inverse beta-decay of the polarized neutron in a magnetic field. Effects of the proton momentum quantization in the magnetic field have been included. The obtained closed expression for the cross section in the magnetic field exactly accounts for the proton longitudinal and transversal motion. For the three ranges of the magnetic field (which we call the super-strong magnetic field $B \geq B'_{cr}$, the strong field $B_{cr} \leq B < B'_{cr}$, and the weak field $B < B_{cr}$) we have calculated the cross section and discussed its dependence on the neutrino energy and angle θ, as well as on the neutron polarization S.

In description of the proton we have used the the exact solution of the Dirac equation in a magnetic filed. This enables us to get the exact cross section in the case of the super-strong magnetic field $B \geq B'_{cr}$ when the proton can occupy only the lowest Landau level $n' = 0$. We have shown that it is not correct to use the cross section, derived under the assumption that the proton wave function is not modified by the magnetic field, in the case when only one or not many Landau levels are opened for the proton even if the proton motion is neglected. Thus we conclude that the Landau quantization of the proton momentum have to be accounted for not only the super-strong magnetic field, but even for lower magnetic fields when not too many Landau levels are opened for the proton.

We have shown that in the case of the total neutrons polarization ($S = \pm 1$) the cross section is exactly zero in the super-strong magnetic filed if $S \cos \theta = -1$, i.e. in the two cases: 1) $S = 1$, $\cos \theta = -1$, and 2) $S = -1$, $\cos \theta = 1$. Thus, in the super-strong magnetic field the totally polarized neutron matter is transparent for the neutrino propagating in the direction opposite to the direction of the neutrons polarization. In the case of the strong magnetic filed the cross section is exactly zero if $S = 1$ and $\cos \theta = -1$, that confirms the result of ref.[35]. These asymmetries in the cross section appear as a consequence of the angular momentum conservation and of the spin polarization properties of the electron and proton being at the lowest Landau levels in the magnetic field.

It should be noted that the developed relativistic treatment of the cross section can be applied to the other URCA processes with two particles in the initial and final states.

6 Acknowledgements

References

[1] D.Grasso, H.Rubinstein, Magnetic fields of the early Universe, Phys.Rep. 348 (2001) 163.

[2] G.Greenstein, Primordial helium production in "magnetic" cosmologies, Nature 223 (1969) 938.

[3] J.Matese, R.O'Connell, Production of helium in the big-bang expansion of a magnetic universe, ApJ 160 (1970) 451.

[4] D.Grasso, H.Rubinstein, Revisting nucleosynthesis constraints on primordial magnetic fields, Phys.Lett.B 379 (1996) 73; Limits on possible magnetic fields at necleosynthesis time, Astropart.Phys. 3 (1995) 95.

[5] A.Goyl, Effect of magnetic field on electron neutrino sphere in pulsar, Phys.Rev. D59, 101301 (1999), hep-ph/9812473.

[6] A.Gvozdev, I.Ognev, On the possible enhancement of the magnetic field by neutrino re-emission processes in the mantel of a supernova, JETP Lett. 69 (1999) 365, astro-ph/9909154.

[7] P.Arras, D.Lai, Neutrino-nucleon interactions in magnetized neutron-star matter: the effect of parity violation, Phys.Rev. D60 043001 (1999), astro-ph/9811371.

[8] L.B.Leinson, A.Perez, Direct URCA process in neutron stars with strong magnetic fields, JHEP 9809, 020 (1998), astro-ph/9711216.

[9] D.Lai, S.Shapiro, Cold equation of state in a strong magnetic field: effects of inverse β-decay, Astrophysical Journal, 383 (1991) p.745-751.

[10] H.Duan, Y-Zh.Qian, Neutrino processes in strong magnetic fields and implications for supernova dynamics, Phys. Rev. D69 (2004) 123004, astro-ph/0401634.

[11] N.Chugai, Chirality of neutrino and space velocities of pulsars,Sov.Astron.Lett. 10 (1984) 87 (Pis'ma Astron. Zh. 10 (1984) 210).

[12] O.Dorofeev, V.Rodionov, I.Ternov, "Anisotropic neutrino emission in beta processes induced by an intense magnetic field", JETP Lett. 40 (1984) 917 (Pisma Zh.Eksp.Teor.Fiz. 40 (1984) 159); "Anisotropic neutrino emission from beta decays in a strong magnetic field", Sov.Astron.Lett. 11 (1985) 123 (Pis'ma Astron. Zh. 11 (1985) 302).

[13] A.Kusenko, G.Segre, "Velocities of pulsars and neutrino oscillations", Phys.Rev.Lett.77 (196) 4872.

[14] A.Studenikin, The object containing neutrinos motion due to the neutrino distribution asymmetry in β-decay of neutrons in the magnetic field, Sov.J.Astrophys. 28 (No.3) (1988) 639.

[15] G.Bisnovatyi-Kogan, Asymmetric neutrino emission and formation of rapidly moving pulsars, Astron.Astrophys.Trans. 3 (1993) 287, astro-ph/9707120.

[16] E.Akhmedov, A.Lanza, D.Sciama, Resonant spin-flip precession of neutrinos and pusar velocities, Phys.Rev. D56 (1997) 6117, hep-ph/9702436.

[17] H.Nunakawa, V.Semiloz, A.Smirnov, J.W.F.Valle, Neutrino conversions in a polarized medium, Nucl.Phys. B501 (1997) 17, hep-ph/9701420.

[18] E.Roulet, Electron neutrino opacity in magnetised media, JHEP 9801, 013 (1998), hep-ph/9711206.

[19] D.Lai, Y.Z.Qian, Neutrino transport in strongly magnetized proto-neutron stars and the origin of pulsar kicks: II. The effecct of asymmetric magnetic field topology, ApJ 505 (1998) 844, astro-ph/9802345.

[20] K.Bhattacharya, P.Pal, "Neutrino and magnetic fields: a short review", hep-ph/0212118.

[21] A.Kusenko, Pulsar kicks from neutrino oscillations, astro-ph/0409521.

[22] L.Korovina, β-decay of polarized neutron in magnetic field, Izvestija Vuzov. Fizika, astron., 6 (1964) 86.

[23] I.Ternov, B.Lysov, L.Korovina, On the theory of neutron β-decay in external magnetic field, Mosc.Univ.Phys.Bull. (Vest.MSU, Fiz.Astr.) No. 5 (1965) 58.

[24] D.Lai, S.Shapiro, Cold equation of state in a strong magnetic filed: effects of inverse β-decay, ApJ 383 (1991) 754.

[25] A.Gazazian, Proton decay induced by electromagnetic rediation, Izvestija Acedemy Nauk Armenii 18 (1965) 126.

[26] I.Baranov, Beta decay of polarized neutron in intense electromagnetic field, Izvestija Vuzov. Fizika, astron. 4 (1974) 115.

[27] I.Ternov, V.Rodionov, V.Zhulego, A.Studenikin, Polarization effects in beta decay in intence electromagnetic field, Sov.J.Nucl.Phys. 28 (1978) 747 (Yadernaja Fizika 28 (1978) 1454; Beta decay of polarized neutrons in external electromagnetic fields, Ann.d.Phys. 37 (1980) 406.

[28] A.Studenikin, Proton recoil effects in β-decay of polarized neutrons in strong magnetic fields, Sov.J.Nucl.Phys. 49 (1989) 1031 (Yadernaja Fizika 49 (1989) 1665).

[29] M.Bander, H.Rubinstein, Proton decay in large magnetic field, Phys.Lett. B289 (1992) 385; Preprint Uppsala Univ. PT 11, 1991.

[30] G.Zharkov, Radiation of π mesons and beta decay of proton in electromagnetic field, Sov.Phys.JETP 89 (1985) 1489.

[31] A.Studenikin, Weak processes in external electromagnetic fields, PhD Thesis, Moscow State University, 1983, 127 p.

[32] B.Cheng, D.Schramm, J.Truran, Interaction rates at high magnetic fields strengths and high degeneracy, Phys.Lett.B 316 (1993) 521.

[33] K.Bhattacharya, P.Pal, Inverse beta-decay in magnetic fields, hep-ph/9911498.

[34] K.Bhattacharya, P.Pal, Neutrino scattering in strong magnetic fields, hep-ph/0001077.

[35] K.Bhattacharya, P.Pal, Inverse beta-decay of arbitrary polarized neutrons in a magnetic field,Pramana 62 (2004) 1041-1058, hep-ph/0209053.

[36] I.Mamsourov, H.Goudarzi, Inverse β^+-decay of proton in the presence of the strong magnetic field, hep-ph/0404086.

[37] A.A.Sokolov, I.M.Ternov, Synchrotron Radiation, Pergamon Press, New York, 1968.

[38] D.Tubbs, D.Schramm, Neutrino opacities at high temperatures and densities, ApJ 201 (1975) 467.

[39] S.Bruenn, Stellar core collapse - numerical model and infall epoch, ApJ Suppl. 58 (1985) 771.

[40] B.Kerimov, On longitudinal polarization of particles in four-fermion weak interaction, Izv.Ak.Nauk USSR, vol.15, 1 (1961).

[41] I.Ternov, V.Bagrov, V.Bordovitsyn, O.Dorofeev, JETP, 28 (1969) 1206.

[42] J.Schwinger, Particles, sources and fields, Addison-Wesly, Vol.3 (1988).