$\boldsymbol{B \to (\rho, \omega)\gamma}$ Decays and CKM phenomenology

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Abstract

We review and update the branching ratios for the $B \to (\rho, \omega)\gamma$ decays, calculated in the QCD factorization approach in the next-to-leading order (NLO) in the strong coupling α_s and to leading power in $\Lambda_{\rm QCD}/m_b$. The corrections take into account the vertex, hard-spectator and annihilation contributions and are found to be large. Theoretical expectations for the branching ratios, CP-asymmetry, isospin- and $SU(3)_{\rm F}$-violating ratios in the $B \to \rho\gamma$ and $B \to \omega\gamma$ decays are presented and compared with the available data.

1 Introduction

There is considerable theoretical interest in radiative $B \to V\gamma$ decays, where V is a vector meson ($V = K^*$, ρ, ω, ϕ), as these processes are currently under intensive investigations in experiments at the two B-factories, BABAR and BELLE. The present measurements of the branching ratios for $B \to K^*\gamma$ decays from the CLEO [1], BABAR [2], and BELLE [3] collaborations as well as their world averages [4] are presented in Table 1. In getting the isospin-averaged $B \to K^*\gamma$ branching fraction the following life-time weighted definition is adopted:

$$\bar{\mathcal{B}}(B \to K^*\gamma) \equiv \frac{1}{2}\left[\mathcal{B}(B^+ \to K^{*+}\gamma) + \frac{\tau_{B^+}}{\tau_{B^0}}\,\mathcal{B}(B^0 \to K^{*0}\gamma)\right], \tag{1}$$

and the current world average [4] for the B-meson lifetime ratio:

$$\tau_{B^+}/\tau_{B^0} = 1.076 \pm 0.008, \tag{2}$$

has been used in arriving at the numerical results.

The decays $B \to \rho\gamma$ and $B \to \omega\gamma$ have been experimentally searched since a long time, as they are a measure of the underlying quark transition $b \to d\gamma$. Hence, in the standard model (SM), they provide information on the Cabibbo-Kobayashi-Maskawa (CKM) matrix element $|V_{td}|$. In particular, the ratio of the branching ratios $\mathcal{B}[B \to (\rho, \omega)\gamma]/\mathcal{B}(B \to K^*\gamma)$ provides an independent measurement of the CKM matrix element ratio $|V_{td}/V_{ts}|$, to be compared with the corresponding ratio obtained through the ratio of the mixing-induced mass differences $\Delta M_{B_d}/\Delta M_{B_s}$, yielding [5] $|V_{td}/V_{ts}| = 0.2060^{+0.0081}_{-0.0060}(\text{theory}) \pm 0.0007(\text{exp})$.

The first observation of the $B \to (\rho, \omega)\gamma$ decays was announced by the BELLE collaboration last summer [6], and the results are presented in Table 1. Of these, the signal from the $B^0 \to \rho^0\gamma$ decay was established with a significance of 5.2σ while no evidence from the other two decay modes $B^+ \to \rho^+\gamma$ and $B^0 \to \omega\gamma$ was found (their significances are 1.6σ and 2.3σ, respectively). The branching fraction of the charged mode $B^+ \to \rho^+\gamma$ is currently a factor two

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Table 1: Status of B-meson radiative branching fractions (in units of 10^{-6}) after the ICHEP-2006 Conference (Moscow).

Mode	BABAR	BELLE	CLEO	HFAG
$B^+ \to K^{*+}\gamma$	$38.7 \pm 2.8 \pm 2.6$	$42.5 \pm 3.1 \pm 2.4$	$37.6^{+8.9}_{-8.3} \pm 2.8$	40.3 ± 2.6
$B^0 \to K^{*0}\gamma$	$39.2 \pm 2.0 \pm 2.4$	$40.1 \pm 2.1 \pm 1.7$	$45.5^{+7.2}_{-6.8} \pm 3.4$	40.1 ± 2.0
$B^+ \to \rho^+\gamma$	$1.06^{+0.35}_{-0.31} \pm 0.09$	$0.55^{+0.42+0.09}_{-0.36-0.08}$	< 13	$0.87^{+0.27}_{-0.25}$
$B^0 \to \rho^0\gamma$	$0.77^{+0.21}_{-0.19} \pm 0.07$	$1.25^{+0.37+0.07}_{-0.33-0.06}$	< 17	$0.91^{+0.19}_{-0.18}$
$B^0 \to \omega\gamma$	$0.39^{+0.24}_{-0.20} \pm 0.03$	$0.56^{+0.34+0.05}_{-0.27-0.10}$	< 9.2	$0.45^{+0.20}_{-0.17}$
$b \to s\gamma$	$327 \pm 18^{+55}_{-41}$	$355 \pm 32^{+30+11}_{-31-7}$	$321 \pm 43^{+32}_{-29}$	$355 \pm 24^{+9}_{-10} \pm 3$
$B \to K^*\gamma$	40.4 ± 2.5	42.8 ± 2.4	43.3 ± 6.2	41.8 ± 1.7
$B \to (\rho, \omega)\,\gamma$	$1.01 \pm 0.21 \pm 0.08$	$1.32^{+0.34+0.10}_{-0.31-0.09}$	< 14	$1.11^{+0.19}_{-0.18}$

smaller than that of the neutral decay mode $B^0 \to \rho^0\gamma$ – in obvious contradiction with the SM predictions [7,8]. However, one should not try to read too much from the existing data which are statistically limited.

At the ICHEP-2006 Conference in Moscow this summer [9], the BABAR collaboration have also presented the measurements of the $B \to \rho\gamma$ and $B \to \omega\gamma$ branching fractions, which are shown in Table 1. Based on approximately the same statistics as the BELLE collaboration, in the BABAR data both the charged and neutral $B \to \rho\gamma$ decays were observed with the significances 4.1σ and 5.2σ, respectively. There is no evidence for the $B^0 \to \omega\gamma$ decay mode yet (the signal has a significance of 2.3σ). Thus, both the collaborations have observed the $B^0 \to \rho^0\gamma$ mode in good agreement with each other within the experimental errors, while the other two decay modes require more statistics to be established. With limited statistics, one may resort to the following weighted branching fraction for the CKM phenomenology:

$$\mathcal{B}[B \to (\rho, \omega)\,\gamma] \equiv \frac{1}{2}\left\{\mathcal{B}(B^+ \to \rho^+\gamma) + \frac{\tau_{B^+}}{\tau_{B^0}}\left[\mathcal{B}(B^0 \to \rho^0\gamma) + \mathcal{B}(B^0 \to \omega\gamma)\right]\right\}. \tag{3}$$

Both the BABAR and BELLE collaborations have measured this fraction with 5.1σ significance (see Table 1) and within errors their measurements agree.

The other potentially interesting radiative mode is the decay $B^0 \to \phi\gamma$. Dominated by the annihilation-type diagrams, its branching fraction has been estimated at the level of 10^{-11} [10, 11], too small to be measured at present B-meson factories, but, possibly this mode can be targeted by the LHC-b experiment or at a future high-luminosity Super-B factory. The current upper limit on this decay (at 90% C.L.) is reported by the BABAR collaboration [12]:

$$\mathcal{B}_{\rm exp}(B^0 \to \phi\gamma) < 0.85 \times 10^{-6}. \tag{4}$$

No information on this decay from the BELLE collaboration is as yet available.

What concerns the CKM phenomenology, the ratios of the branching fractions are more reliably calculable, as the various uncertainties related to the theoretical input are considerably reduced in these ratios thereby enhancing the precision on the ratio $|V_{td}/V_{ts}|$. One such ratio is defined below together with its current experimental measurements:

$$R_{\rm exp}[(\rho, \omega)\gamma/K^*\gamma] \equiv \frac{\mathcal{B}_{\rm exp}[B \to (\rho, \omega)\gamma]}{\mathcal{B}_{\rm exp}(B \to K^*\gamma)} = \left\{ \begin{array}{ll} 0.024 \pm 0.005, & \text{[BABAR]} \\ 0.032 \pm 0.008 \pm 0.002. & \text{[BELLE]} \end{array} \right. \tag{5}$$

The results presented are consistent with each other within errors.

Ratios of neutral B-meson branching fractions are more favorable for the CKM analysis as they are less sensitive to the annihilation contribution, which is theoretically less tractable but expected to be small for the neutral modes. The BABAR collaboration have presented the measurement of such a ratio [9]:

$$R_{\rm exp}(\rho^0\gamma/K^{*0}\gamma) \equiv \frac{2\,\mathcal{B}_{\rm exp}(B^0\to\rho^0\gamma)}{\mathcal{B}_{\rm exp}(B^0\to K^{*0}\gamma)} = 0.038^{+0.011}_{-0.010}. \qquad (6)$$

In comparison with Eq. (5), the central value in Eq. (6) is substantially larger but due to the large errors the two measurements are compatible with each other. As emphasized by several authors in the past, measurements of these ratios provide a robust determination of the ratio $|V_{td}/V_{ts}|$ of the CKM matrix elements. However, to make an impact on the CKM phenomenology, in particular in the post-ΔM_s observation era, the measurements in radiative B-meson decays have to become an order of magnitude more precise than is currently the case. In view of this, we will constrain the CKM parameters from the SM fits of the unitarity triangle [13, 14], including the measurement of ΔM_s [5], and predict the various branching ratios, their ratios, and asymmetries to be confronted with data in radiative B-decays. This will serve as a stringent test of the SM in this sector.

Several competing theoretical frameworks have been used to study exclusive B-meson decays. The QCD-Factorization approach [15] provides a satisfactory theoretical basis for calculations of two-body radiative B-meson decays [16] and has been applied to the $B\to K^*\gamma$ [7, 8, 17–21], $B\to\rho\gamma$ [7, 8, 19–22] and $B\to\phi\gamma$ [23] modes. There are several other theoretical approaches which have also been used to study two-body radiative B-meson decays. These include the Soft-Collinear Effective Theory (SCET) [11,24,25] and the perturbative QCD (pQCD) approach [10, 26, 27]. In addition, information on various input hadronic quantities is required which is usually taken from the Light-Cone Sum Rules (LCSRs) [28, 29]. All these approaches are in fair agreement with the measured branching ratios of the $B\to K^*\gamma$ decays, and predict the branching ratios of the $B\to\rho\gamma$ and $B\to\omega\gamma$ decays typically of $O(10^{-6})$.

In this paper, we discuss and review the predictions for the branching ratio of the $B\to\rho\gamma$ and $B\to\omega\gamma$ decays obtained in the QCD-Factorization framework. We shall concentrate mainly on the ratio of the branching fractions defined below:

$$R_{\rm th}(\rho\gamma/K^*\gamma) \equiv \frac{\mathcal{B}_{\rm th}(B\to\rho\gamma)}{S_\rho\,\mathcal{B}_{\rm th}(B\to K^*\gamma)}, \qquad R_{\rm th}(\omega\gamma/K^*\gamma) \equiv \frac{2\,\mathcal{B}_{\rm th}(B\to\omega\gamma)}{\mathcal{B}_{\rm th}(B\to K^*\gamma)}, \qquad (7)$$

where $S_\rho = 1$ for the $B^\pm$-meson decay modes and $S_\rho = 1/2$ for the B^0-meson decays. Measurements of the $B\to K^*\gamma$ branching ratios in combination with the theoretical estimates of the ratios in (7) allow us to make predictions for the $B\to\rho\gamma$ and $B\to\omega\gamma$ branching fractions with reduced uncertainties.

In addition to the branching ratios, there are several asymmetries involving isospin-, $SU(3)_{\rm F}$- and CP-violation in the $B\to(\rho,\omega)\gamma$ decays. For example, first measurement of the isospin-violating ratio, defined below, has been presented by the BABAR collaboration this summer [9]:

$$\Delta \equiv \frac{1}{2}\,\frac{\Gamma(B^+\to\rho^+\gamma)}{\Gamma(B^0\to\rho^0\gamma)} - 1 = \frac{\tau_{B^0}}{2\,\tau_{B^+}}\,\frac{\mathcal{B}(B^+\to\rho^+\gamma)}{\mathcal{B}(B^0\to\rho^0\gamma)} - 1 = -0.36\pm 0.27, \qquad (8)$$

which is consistent with zero at 1.3σ. As the isospin-violating ratio Δ depends on the unitarity-triangle angle α due to the interference between the penguin- and annihilation-type contributions (see, Eq. (37) below), its experimental measurement, in principle, will yield an independent determination of this angle. In the SM, constraining the angle α from the unitarity fits, $\alpha = (97.3^{+4.5}_{-5.0})^\circ$ [13], we estimate $\Delta = (2.9\pm 2.1)\%$ in the QCD factorization approach. Estimates of Δ in the pQCD approach [10] are similar though they allow somewhat larger isospin-violation Thus, isospin-violation in the $B\to\rho\gamma$ decays is parametrically small in the SM, being a consequence of the experimentally measured value $|\cos\alpha|_{\rm SM} < 0.2$ and the ratio of the annihilation-to-penguin amplitudes, typically estimated as $|A/P| \le 0.3$. The $SU(3)_{\rm F}$-violating ratio $\Delta^{(\rho/\omega)}$, defined in Eq. (48), is estimated to be likewise small in the SM. With a realistic estimate of the $SU(3)_{\rm F}$-breaking in the form factors, $\zeta_{\omega/\rho} \equiv \xi_\perp^{(\omega)}(0)/\xi_\perp^{(\rho)}(0) = 0.9 \pm 0.1$, we estimate $\Delta^{(\rho/\omega)} = (11 \pm 11)\%$, which is consistent with the current data within large experimental errors. These predictions can be tested in high statistics measurements in the $B \to (\rho, \omega)\gamma$ decays. Finally, we also present the CP-asymmetries (both direct and mixing-induced) in the $B \to \rho\gamma$ and $B \to \omega\gamma$ decays, updating our results presented in Refs. [7,8]. These asymmetries test the underlying dynamical model (the QCD factorization), as shown by comparison with the corresponding existing calculations in the pQCD approach [27].

2 $B \to V\gamma$ Branching Fractions in NLO

The effective Hamiltonian for the $B \to \rho\gamma$ (equivalently $b \to d\gamma$) decays at the scale $\mu = O(m_b)$, where m_b is the b-quark mass, is as follows:

$$\begin{aligned} \mathcal{H}_{\rm eff} &= \frac{G_F}{\sqrt{2}} \Big\{ V_{ub} V_{ud}^* \left[C_1(\mu)\, \mathcal{O}_1^{(u)}(\mu) + C_2(\mu)\, \mathcal{O}_2^{(u)}(\mu) \right] \\ &+ V_{cb} V_{cd}^* \left[C_1(\mu)\, \mathcal{O}_1^{(c)}(\mu) + C_2(\mu)\, \mathcal{O}_2^{(c)}(\mu) \right] \\ &- V_{tb} V_{td}^* \left[C_7^{\rm eff}(\mu)\, \mathcal{O}_{7\gamma}(\mu) + C_8^{\rm eff}(\mu)\, \mathcal{O}_{8g}(\mu) \right] + \ldots \Big\}, \end{aligned} \tag{9}$$

where the set of operators is $(q = u, c)$:

$$\mathcal{O}_1^{(q)} = (\bar{d}_\alpha \gamma_\mu (1 - \gamma_5) q_\beta)\, (\bar{q}_\beta \gamma^\mu (1 - \gamma_5) b_\alpha), \tag{10}$$

$$\mathcal{O}_2^{(q)} = (\bar{d}_\alpha \gamma_\mu (1 - \gamma_5) q_\alpha)\, (\bar{q}_\beta \gamma^\mu (1 - \gamma_5) b_\beta), \tag{11}$$

$$\mathcal{O}_{7\gamma}(\mu) = \frac{e\, m_b(\mu)}{8\pi^2}\, (\bar{d}_\alpha \sigma^{\mu\nu} (1 + \gamma_5) b_\alpha)\, F_{\mu\nu}, \tag{12}$$

$$\mathcal{O}_{8g}(\mu) = \frac{g_s(\mu)\, m_b(\mu)}{8\pi^2}\, (\bar{d}_\alpha \sigma^{\mu\nu} (1 + \gamma_5) T^A_{\alpha\beta} b_\beta)\, G^A_{\mu\nu}. \tag{13}$$

The strong and electroweak four-quark penguin operators are present in the effective Hamiltonian (denoted by ellipses) but are not taken into account due to their small Wilson coefficients.

The effective Hamiltonian sandwiched between the B-meson and the vector meson V states can be expressed in terms of the matrix elements of bilinear quark currents defining a heavy-to-light transition. The general decomposition of the matrix elements on all possible Lorentz structures admits seven scalar functions (form factors): $V^{(V)}(q^2)$, $A_i^{(V)}(q^2)$ $(i = 0, 1, 2)$, and $T_i^{(V)}(q^2)$ $(i = 1, 2, 3)$ of the momentum squared $q^2 = (p_B - p)^2$ transferred from the B-meson to the light vector meson. To be definite, we study the $B \to \rho\gamma$ decay in which the transition matrix elements are defined as follows:

$$\left\langle \rho(p, \varepsilon^*) | \bar{d}\, \gamma^\mu\, b | \bar{B}(p_B) \right\rangle = \frac{2i\, V^{(\rho)}(q^2)}{m_B + m_\rho}\, \varepsilon^{\mu\nu\alpha\beta}\, \varepsilon^*_\nu p_\alpha p_{B\beta}, \tag{14}$$

$$\begin{aligned} \left\langle \rho(p, \varepsilon^*) | \bar{d}\, \gamma^\mu \gamma_5\, b | \bar{B}(p_B) \right\rangle &= A_1^{(\rho)}(q^2)\, (m_B + m_\rho) \left[\varepsilon^{*\mu} - \frac{(\varepsilon^* q)}{q^2}\, q^\mu \right] \\ &- A_2^{(\rho)}(q^2)\, \frac{(\varepsilon^* q)}{m_B + m_\rho} \left[(p_B + p)^\mu - \frac{(m_B^2 - m_\rho^2)}{q^2} q^\mu \right] + 2 m_\rho\, A_0^{(\rho)}(q^2)\, \frac{(\varepsilon^* q)}{q^2}\, q^\mu, \end{aligned} \tag{15}$$

$$\left\langle \rho(p, \varepsilon^*) | \bar{d}\, \sigma^{\mu\nu} q_\nu\, b | \bar{B}(p_B) \right\rangle = 2\, T_1^{(\rho)}(q^2)\, \varepsilon^{\mu\nu\alpha\beta} \varepsilon^*_\nu p_\alpha p_{B\beta}, \tag{16}$$

$$\begin{aligned} \left\langle \rho(p, \varepsilon^*) | \bar{d}\, \sigma^{\mu\nu} \gamma_5 q_\nu\, b | \bar{B}(p_B) \right\rangle &= -i\, T_2^{(\rho)}(q^2)\, [(m_B^2 - m_\rho^2)\, \varepsilon^{*\mu} - (\varepsilon^* q)\, (p_B + p)^\mu] \\ &- i\, T_3^{(\rho)}(q^2)\, (\varepsilon^* q) \left[q^\mu - \frac{q^2}{m_B^2 - m_\rho^2}\, (p_B + p)^\mu \right]. \end{aligned} \tag{17}$$

The heavy quark symmetry in the large energy limit of the vector meson allows to reduce the number of independent form factors to two only: $\xi_\perp^{(\rho)}(q^2)$ and $\xi_\parallel^{(\rho)}(q^2)$. Both of them enter in the analysis of the $B \to \rho \ell^+ \ell^-$ decay. However, for the radiative $B \to \rho\gamma$ decay amplitude, we need only one of them, $\xi_\perp^{(\rho)}(q^2 = 0)$, which is related to the form factors introduced above in the full QCD as follows (terms of order m_ρ^2/m_B^2 are neglected):

$$\frac{m_B}{m_B + m_\rho} V^{(\rho)}(0) = \frac{m_B + m_\rho}{m_B} A_1^{(\rho)}(0) = T_1^{(\rho)}(0) = T_2^{(\rho)}(0) = \xi_\perp^{(\rho)}(0). \tag{18}$$

These relations among the form factors in the symmetry limit are broken by perturbative QCD radiative corrections arising from the vertex renormalization and hard-spectator interaction. To incorporate both types of QCD corrections, a factorization formula for the heavy-to-light transition form factors at large recoil and at leading order in the inverse heavy meson mass was established in Ref. [16]:

$$F_k^{(\rho)}(q^2 = 0) = C_{\perp k}\, \xi_\perp^{(\rho)}(q^2 = 0) + \phi_B \otimes T_k(q^2 = 0) \otimes \phi_\rho, \tag{19}$$

where $F_k^{(\rho)}(q^2 = 0)$ is any of the four form factors in the $B \to \rho$ transitions related by Eq. (18), $C_{\perp k} = C_{\perp k}^{(0)}\,[1 + O(\alpha_s)]$ is the renormalization coefficient, T_k is a hard-scattering kernel calculated in $O(\alpha_s)$, ϕ_B and ϕ_ρ are the light-cone distribution amplitudes (LCDAs) of the B- and ρ-mesons convoluted with the kernel T_k.

In the leading order, the electromagnetic penguin operator $\mathcal{O}_{7\gamma}$ contributes in the $B \to \rho\gamma$ decay amplitude at the tree level. Taking into account the definitions of the $B \to \rho$ transition form factors in the tensor (16) and the axial-tensor (17) currents and the symmetry relation $T_1^{(\rho)}(0) = T_2^{(\rho)}(0)$, the amplitude for the $B \to \rho\gamma$ decay takes the form:

$$\begin{aligned} M^{(0)} &= -\frac{G_F}{\sqrt{2}}\, V_{tb} V_{td}^* \,\frac{e \bar{m}_b(\mu)}{4\pi^2}\, C_7^{(0)\mathrm{eff}}(\mu)\, T_1^{(\rho)}(0) \\ &\times \left[(Pq)(e^* \varepsilon^*) - (e^* P)(\varepsilon^* q) + i\, \mathrm{eps}(e^*, \varepsilon^*, P, q)\right], \end{aligned} \tag{20}$$

where $q = p_B - p$ and e^* are the photon four-momentum and polarization vector, respectively, $P = p_B + p$, and $\mathrm{eps}(e^*, \varepsilon^*, P, q) = \varepsilon^{\mu\nu\alpha\beta}\, e_\mu^* \varepsilon_\nu^* P_\alpha q_\beta$. The corresponding branching ratio can be easily obtained and reads as follows:

$$\mathcal{B}_{\mathrm{th}}^{\mathrm{LO}}(B \to \rho\gamma) = \tau_B\, S_\rho\, \frac{G_F^2 \alpha |V_{tb} V_{td}^*|^2 m_B^3}{32\pi^4} \left[1 - \frac{m_\rho^2}{m_B^2}\right]^3 \bar{m}_b^2(\mu)\, |C_7^{(0)\mathrm{eff}}(\mu)|^2\, |T_1^{(\rho)}(0,\mu)|^2, \tag{21}$$

where $S_\rho = 1$ for the $B^\pm$-meson decay and $S_\rho = 1/2$ for the B^0 decay. The scale (μ)-dependence of the form factor, $T_1^{(\rho)}(0,\mu)$, the b-quark mass, $\bar{m}_b(\mu)$, and the Wilson coefficient, $C_7^{(0)\mathrm{eff}}(\mu)$, in the above expression for the branching ratio are made explicit.

The branching fraction for the $B \to K^*\gamma$ decays can be easily obtained from Eq. (21) by replacing $V_{td} \to V_{ts}$, $m_\rho \to m_{K^*}$, and $T_1^{(\rho)}(0,\mu) \to T_1^{(K^*)}(0,\mu)$, which yields the following expression for the ratio of the branching ratios defined in Eq. (7):

$$R_{\mathrm{th}}^{(0)}(\rho\gamma/K^*\gamma) = S_\rho \left|\frac{V_{td}}{V_{ts}}\right|^2 \left[\frac{m_B^2 - m_\rho^2}{m_B^2 - m_{K^*}^2}\right]^3 \left[\frac{T_1^{(\rho)}(0,\mu)}{T_1^{(K^*)}(0,\mu)}\right]^2. \tag{22}$$

A similar ratio involving the $B^0 \to \omega\gamma$ and $B^0 \to K^{*0}\gamma$ decay widths can be written as follows:

$$R_{\mathrm{th}}^{(0)}(\omega\gamma/K^*\gamma) = \frac{1}{2} \left|\frac{V_{td}}{V_{ts}}\right|^2 \left[\frac{m_B^2 - m_\omega^2}{m_B^2 - m_{K^*}^2}\right]^3 \left[\frac{T_1^{(\omega)}(0,\mu)}{T_1^{(K^*)}(0,\mu)}\right]^2. \tag{23}$$

Apart from the electromagnetic penguins, one also has contributions from the annihilation diagrams to the $B \to \rho\gamma$ and $B \to \omega\gamma$ decay widths which modify the ratios (22) and (23):

$$R_{\rm th}(\rho\gamma/K^*\gamma) = R_{\rm th}^{(0)}(\rho\gamma/K^*\gamma)\,[1+\Delta R(\rho/K^*)]\,, \tag{24}$$

$$R_{\rm th}(\omega\gamma/K^*\gamma) = R_{\rm th}^{(0)}(\omega\gamma/K^*\gamma)\,[1+\Delta R(\omega/K^*)]\,. \tag{25}$$

In the annihilation amplitude, photon radiation from the quarks in the vector meson is compensated by the diagram in which the photon is emitted from the vertex [30–32]. Hence, only the annihilation diagram with the photon emitted from the spectator quark in the B-meson is numerically important. The quantities $\Delta R(\rho/K^*)$ and $\Delta R(\omega/K^*)$ can be parameterized (apart from the CKM factors) by dimensionless factors $\epsilon_A^{(\pm)}$, $\epsilon_A^{(0)}$ and ϵ_A^{ω}:

$$\Delta R(\rho^\pm/K^{*\pm}) = \lambda_u\,\varepsilon_A^{(\pm)}, \qquad \Delta R(\rho^0/K^{*0}) = \lambda_u\,\varepsilon_A^{(0)}, \qquad \Delta R(\omega/K^{*0}) = \lambda_u\,\varepsilon_A^{(\omega)}, \tag{26}$$

$$\lambda_u = \frac{V_{ub}V_{ud}^*}{V_{tb}V_{td}^*} = -\left|\frac{V_{ub}V_{ud}^*}{V_{tb}V_{td}^*}\right| {\rm e}^{i\alpha} = F_1 + iF_2, \tag{27}$$

where $F_1 = -|\lambda_u|\cos\alpha$, $F_2 = -|\lambda_u|\sin\alpha$, and α is one of the inner angles of the unitarity triangle. In the neutral B-meson decays, the parameter ε_A is numerically small due to the color suppression and the unfavorable electric charge of the d-quark, resulting in the estimate $\varepsilon_A^{(0)} = -\varepsilon_A^{(\omega)} = 0.03 \pm 0.01$ [31], obtained with the help of the Light-Cone Sum Rules. For the charged B-meson decays, LCSRs yield a larger value $\varepsilon_A^{(\pm)} = 0.30 \pm 0.07$ [31], which is used in the current analysis.

Both the penguin and annihilation contributions receive QCD corrections. The next-to-leading order (NLO) corrections to the $B \to \rho\gamma$ and $B \to \omega\gamma$ decay widths consist of the following contributions [7]:

1. The NLO correction to the $\overline{\rm MS}$ b-quark mass $\bar{m}_b(\mu)$. We have related $\bar{m}_b(\mu)$ with the pole mass, $m_{b,\rm pole}$, at the renormalization scale μ.

2. The NLO correction to the Wilson coefficient $C_7^{\rm eff}(\mu)$.

3. The factorizable NLO corrections to the $T_1^{(\rho)}(0,\mu)$ and $T_1^{(\omega)}(0,\mu)$ form factors which can be further divided into the vertex and hard-spectator corrections. These two types of corrections are estimated at different scales: the vertex and hard-spectator corrections should be calculated at the hard $\mu_b \sim m_b$ and intermediate $\mu_i \sim \sqrt{\Lambda_{\rm H} m_b}$ ($\Lambda_{\rm H} \simeq 0.5$ GeV) scales, respectively.

4. The nonfactorizable NLO corrections which are also of two types: the vertex and the hard-spectator corrections. The nonfactorizable vertex corrections can be taken from inclusive $B \to X_d\gamma$ decay [33]. The nonfactorizable hard-spectator corrections were calculated by several groups [7, 17, 19].

In addition, the NLO corrections to the annihilation diagrams have also to be taken into account. We have mentioned them in the context of the $B \to \phi\gamma$ decay. For the $B^\pm \to \rho^\pm\gamma$ decay, they can be modeled on the $B^\pm \to \ell^\pm\nu_\ell\gamma$ decay, as based on the large-N_c argument, the non-factorizing contribution is expected to be small [34]. We shall adopt here the annihilation contribution estimates obtained using the QCD LCSRs [31, 32].

The NLO corrections discussed above modify the $B \to \rho\gamma$ and $B \to \omega\gamma$ branching ratios. The result for the charged-conjugate averaged $B^\pm \to \rho^\pm\gamma$ branching fraction can be written in the form:

$$\bar{\mathcal{B}}_{\rm th}(B^\pm \to \rho^\pm\gamma) = \tau_{B^+}\,\frac{G_F^2\alpha|V_{tb}V_{td}^*|^2}{32\pi^4}\,m_{b,\rm pole}^2\,m_B^3\left[1-\frac{m_\rho^2}{m_B^2}\right]^3 \left[\xi_\perp^{(\rho)}(0)\right]^2 C_7^{(0)\rm eff} \tag{28}$$

$$\times\left\{C_7^{(0)\rm eff} + 2A_R^{(1)t} + \epsilon_A^{(\pm)}\,(F_1^2+F_2^2)\,[\epsilon_A^{(\pm)}\,C_7^{(0)\rm eff} + 2A_R^u] + 2F_1\,[A_R^u + \epsilon_A^{(\pm)}\,(C_7^{(0)\rm eff} + A_R^{(1)t})]\right\},$$

where the subscript R denotes the real part of the corresponding quantity. The NLO amplitude $A^{(1)t}(\mu)$ of the decay presented here can be decomposed in three contributing parts [7]:

$$A^{(1)t}(\mu) = A^{(1)}_{C_7}(\mu) + A^{(1)}_{\rm ver}(\mu) + A^{(1)\rho}_{\rm sp}(\mu_{\rm sp}), \tag{29}$$

where the correction due to the b-quark mass, $\bar{m}_b(\mu)$, is included in $A^{(1)}_{\rm ver}(\mu)$. The amplitude $A^{(1)K^*}(\mu)$ for the $B \to K^*\gamma$ decay can be written in a similar form and differs from $A^{(1)t}(\mu)$ by the hard-spectator part $A^{(1)K^*}_{\rm sp}(\mu)$ only [7]. Note that the u-quark contribution $A^u(\mu)$ from the penguin diagrams, which also involves the contribution of hard-spectator corrections, can not be ignored in the $B \to \rho\gamma$ and $B \to \omega\gamma$ decays.

Using the formula (28) for the branching ratio, the dynamical function $\Delta R(\rho/K^*)$, defined by Eq. (24), can be written as follows [7]:

$$\begin{aligned}\Delta R(\rho/K^*) = {} & \left[2\epsilon_A\, F_1 + \epsilon_A^2(F_1^2 + F_2^2)\right]\left(1 - \frac{2A^{(1)K^*}}{C_7^{(0)\rm eff}}\right) - \frac{2A^{(1)K^*}}{C_7^{(0)\rm eff}} \\ & + \frac{2}{C_7^{(0)\rm eff}}\,{\rm Re}\left[A^{(1)\rho}_{\rm sp} - A^{(1)K^*}_{\rm sp} + F_1(A^u + \epsilon_A A^{(1)t}) + \epsilon_A(F_1^2 + F_2^2)A^u\right],\end{aligned} \tag{30}$$

where the NLO corrections in the penguin amplitude and QCD LCSRs for the annihilation amplitude are taken into account. A similar expression with the exchange $\epsilon_A \to \epsilon_A^{(\omega)}$ holds for $\Delta R(\omega/K^*)$ defined in (25).

3 Phenomenology of $B \to \rho\gamma$ and $B \to \omega\gamma$ Decays

Branching Ratios for $B \to \rho\gamma$ and $B \to \omega\gamma$ Decays: For the numerical predictions for the $B \to \rho\gamma$ and $B \to \omega\gamma$ branching ratios, we employ the ratios defined in Eq. (7) and use the experimentally measured values of the $B \to K^*\gamma$ branching fractions from Table 1. A number of input hadronic quantities has been changed compared to our earlier analysis [8] and the changes are desribed below.

Let us start with the discussion of the tensor $B \to V$ transition form factors. The $SU(3)_{\rm F}$-breaking effects in the QCD transition form factors $T_1^{(K^*)}(0)$, $T_1^{(\rho)}(0)$, and $T_1^{(\omega)}(0)$ have been evaluated in a number of different theoretical frameworks. We take the $SU(3)_{\rm F}$-breaking to hold also for the ratio of the soft form factors in the effective theory. Defining $\zeta \equiv \xi_\perp^{(\rho)}(0)/\xi_\perp^{(K^*)}(0)$, and restricting ourselves to the QCD LCSRs, we note that the earlier result in this approach [36], yielding $\zeta = 0.76 \pm 0.06$, has been updated recently yielding $\zeta = 0.86 \pm 0.07$ [28], which we use here for the numerical analysis. In our paper [8], we had assumed the equality of the tensor form factors in the decays $B \to \rho\gamma$ and $B \to \omega\gamma$, which holds in the $SU(3)_{\rm F}$ symmetry limit. Recent estimates within the QCD LSCRs result in modest $SU(3)_{\rm F}$-breaking effect in the form factors, illustrated by the values [35]: $T_1^{(\rho)}(0) = 0.267 \pm 0.021$ and $T_1^{(\omega)}(0) = 0.242 \pm 0.022$. This gives for the ratio $\zeta_{\omega/\rho} \equiv \xi_\perp^{(\omega)}(0)/\xi_\perp^{(\rho)}(0) = 0.9 \pm 0.1$, which in turn yields $\xi_\perp^{(\omega)}(0)/\xi_\perp^{(K^*)}(0) = \zeta\,\zeta_{\rho/\omega} = 0.78 \pm 0.10$. This is used in the analysis of the $B \to \omega\gamma$ decay.

We now discuss the changes connected with the B-, ρ- and K^*-meson distribution amplitudes. In our earlier paper [8], the two-parameter model for the leading-twist B-meson LCDA by Braun, Ivanov and Korchemsky (BIK) [37] was used with the following ranges of the parameters: $\lambda_B^{-1}(1~{\rm GeV}) = (2.15 \pm 0.50)~{\rm GeV}^{-1}$ and $\sigma_B(1~{\rm GeV}) = 1.4 \pm 0.4$, obtained from the sum-rules analysis. Recently, Lee and Neubert [38] have derived model-independent properties of the B-meson LCDA, obtaining explicit expressions for the first two moments as a function of the renormalization scale μ. Based on this analysis, these authors suggest a modified leading-twist B-meson LCDA which is consistent with the moment relations. It was also shown that the BIK model obeys the same moment constraints with the modified values of the two input parameters

$\lambda_B^{-1}(1\text{ GeV}) = (1.79 \pm 0.06)\text{ GeV}^{-1}$ and $\sigma_B(1\text{ GeV}) = 1.57 \pm 0.27$ [38]. Though the functional forms of the two B-meson LCDAs are different, with the indicated values of λ_B^{-1} and σ_B, both the BIK and the Lee-Neubert functions are nearly indistinguishable. Following this work, we use the BIK model with the improved parameters in our analysis. For the B-meson decay constant, the value $f_B = (205 \pm 25)$ MeV [28] is taken. The ρ-meson leading-twist LCDA was taken from Ref. [28] with $f_\perp^{(\rho)}(1\text{ GeV}) = (165 \pm 9)$ MeV and $a_{\perp 2}^{(\rho)}(1\text{ GeV}) = 0.15 \pm 0.07$. The models for the leading-twist LCDAs of the K- and K^*-meson have been updated during the last several years. In the present analysis we use the set of parameters for the K^*-meson LCDA from Ref. [28]: $f_\perp^{(K^*)}(1\text{ GeV}) = (185 \pm 10)$ MeV, $a_{\perp 1}^{(K^*)}(1\text{ GeV}) = 0.04 \pm 0.03$, and $a_{\perp 2}^{(K^*)}(1\text{ GeV}) = 0.11 \pm 0.09$. While $f_\perp^{(K^*)}$ and $a_{\perp 2}^{(K^*)}$ remain approximately the same, the first Gegenbauer moment $a_{\perp 1}^{(K^*)}$ has changed significantly from its previously used value, $a_{\perp 1}^{(K^*)}(1\text{ GeV}) = -0.34 \pm 0.18$. Note that the soft part, $\xi_\perp^{(K^*)}$, of the QCD form factor $T_1^{K^*}(0)$, is practically insensitive to the changes in the K^*-meson LCDA, and the updated value now is $\bar{\xi}_\perp^{(K^*)}(0) = 0.26 \pm 0.02$.

The other sizable changes compared to our previous analysis [8] are in the values of the CKM parameters, which are now input. Taking into account the recent measurement of the ratio $|V_{td}/V_{ts}|$ from the ratio $\Delta M_d/\Delta M_s$ of the B_d^0- and B_s^0-meson mass differences by the CDF collaboration [5], yields $|V_{td}/V_{ts}| = 0.2060^{+0.0081}_{-0.0060}(\text{theory}) \pm 0.0007(\text{exp})$, which we take as $|V_{td}/V_{ts}| = 0.206 \pm 0.008$. In addition, the numerical value of the unitarity-triangle angle $\alpha = (97.3^{+4.5}_{-5.0})^\circ$ is taken from the global CKM fits [13]. We also modify the top quark mass, reflecting the smaller value of the t-quark mass reported recently by the Fermilab collider experiments $m_t = (171.4 \pm 2.1)$ GeV [39].

The main uncertainties in the dynamical functions $\Delta R(\rho/K^*)$ and $\Delta R(\omega/K^*)$ come from the CKM angle α and the soft form factors $\xi_\perp^{(K^*)}(0)$, $\xi_\perp^{(\rho)}(0)$, and $\xi_\perp^{(\omega)}(0)$. Taking into account various parametric uncertainties, it is found that the dynamical functions are constrained in the ranges:

$$\Delta R(\rho^\pm/K^{*\pm}) = 0.057^{+0.057}_{-0.055}, \quad \Delta R(\rho^0/K^{*0}) = 0.006^{+0.046}_{-0.043}, \quad \Delta R(\omega/K^{*0}) = -0.002^{+0.046}_{-0.043}. \tag{31}$$

Thus, these corrections turn out to be below 5% in the radiative decays of the neutral B-meson, and may reach as high as 11% for the charged mode. This explicitly quantifies the statement that the ratios $R_{\rm th}(\rho\gamma/K^*\gamma)$ and $R_{\rm th}(\omega\gamma/K^*\gamma)$ (7) are stable against $O(\alpha_s)$ and $1/m_b$-corrections, in particular for the neutral B-meson decays. Comparison with the corresponding estimates obtained by us in Ref. [8] shows that the central values are now smaller and the errors have decreased due to the various improvements since then. Note that the reduced central values reflect mainly the substantial change in the value of the input parameter $a_{\perp 1}^{(K^*)}$.

With the modified input values specified above, the branching ratios for the radiative B-decays are estimated as follows:

$$\begin{aligned} \bar{\mathcal{B}}_{\rm th}(B^\pm \to \rho^\pm\gamma) &= (1.37 \pm 0.26[\text{th}] \pm 0.09[\text{exp}]) \times 10^{-6}, \\ \bar{\mathcal{B}}_{\rm th}(B^0 \to \rho^0\gamma) &= (0.65 \pm 0.12[\text{th}] \pm 0.03[\text{exp}]) \times 10^{-6}, \\ \bar{\mathcal{B}}_{\rm th}(B^0 \to \omega\gamma) &= (0.53 \pm 0.12[\text{th}] \pm 0.02[\text{exp}]) \times 10^{-6}\ . \end{aligned} \tag{32}$$

In the above estimates, the first error is due to the uncertainties of the theory and the second error is from the experimental data on the $B \to K^*\gamma$ branching fractions. The recent data from the BABAR and BELLE experiments are in the right ball-park compared to the above SM-based predictions. However, the comparison of theory and experiment is not yet completely quantitative due to the paucity of data.

Combining all the above branching fractions (32) together into the isospin- and $SU(3)_{\rm F}$-averaged branching fraction (3), one has the following prediction:

$$\bar{\mathcal{B}}_{\rm th}[B \to (\rho/\omega)\gamma] = (1.32 \pm 0.26) \times 10^{-6}, \tag{33}$$

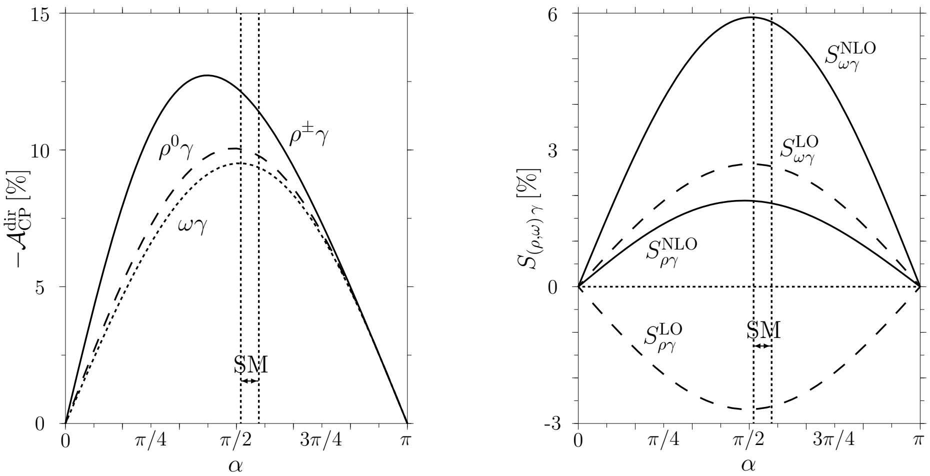

Figure 1: Left figure: Direct CP-asymmetry in the decays $B^\pm \to \rho^\pm\gamma$ (solid curve), $B^0 \to \rho^0\gamma$ (dashed curve) and $B^0 \to \omega\gamma$ (dotted curve) as a function of the unitarity-triangle angle α. Right figure: Mixing-induced CP-asymmetry in the decays $B^0 \to \rho^0\gamma$ (solid curves) and $B^0 \to \omega\gamma$ (dotted curves) in the leading (LO) and next-to-leading (NLO) orders as a function of the unitarity-triangle angle α. The $\pm 1\sigma$ allowed band of α from the SM unitarity fits [13] is also indicated on both plots.

in agreement with the current world average (see Table 1).

The results (32) can be compared with the predictions obtained within the pQCD approach [27]:

$$
\begin{aligned}
\bar{\mathcal{B}}_{\rm th}(B^\pm \to \rho^\pm\gamma) &= (2.5 \pm 1.5) \times 10^{-6}, \\
\bar{\mathcal{B}}_{\rm th}(B^0 \to \rho^0\gamma) &= (1.2 \pm 0.7) \times 10^{-6}, \\
\bar{\mathcal{B}}_{\rm th}(B^0 \to \omega\gamma) &= (1.1 \pm 0.6) \times 10^{-6}.
\end{aligned}
\tag{34}
$$

The central values and the errors in the pQCD approach are typically a factor of two larger than the improved QCDF-based predictions given earlier. An updated analysis of these branching ratios in the pQCD approach will shed light on the current numerical differences.

Direct CP-Asymmetry: The direct CP-asymmetry in the $B^\pm \to \rho^\pm\gamma$ decays is defined as follows:

$$
\mathcal{A}_{\rm CP}(\rho^\pm\gamma) = \frac{\mathcal{B}(B^- \to \rho^-\gamma) - \mathcal{B}(B^+ \to \rho^+\gamma)}{\mathcal{B}(B^- \to \rho^-\gamma) + \mathcal{B}(B^+ \to \rho^+\gamma)}. \tag{35}
$$

In NLO, the direct CP-asymmetry can be written in the form [7, 40]:

$$
\mathcal{A}_{\rm CP}(\rho^\pm\gamma) = \frac{2|\lambda_u| \sin\alpha}{C_7^{(0){\rm eff}}\,(1 + \Delta_{\rm LO})}\, {\rm Im}\left[A^u - \epsilon_A\; A^{(1)t}\right], \tag{36}
$$

where λ_u has been defined in Eq. (27) and $\Delta_{\rm LO}$ is the isospin-violating ratio in the leading order [7, 40]:

$$
\Delta_{\rm LO} = -2\epsilon_A\,|\lambda_u|\cos\alpha + \epsilon_A^2\,|\lambda_u|^2. \tag{37}
$$

Similar definitions and expressions can also be used for the two neutral decay modes $B^0 \to \rho^0\gamma$ and $B^0 \to \omega\gamma$. The dependence of the CP-asymmetry on the angle α for the three decay modes is presented in the left plot in Fig. 1. In the QCDF approach, the SM yields the direct CP-asymmetry to be negative, and the results in the interval $0.21 \le \sqrt{z} = m_c/m_b \le 0.33$ are as follows:

$$
\mathcal{A}_{\rm CP}(\rho^\pm\gamma) = \left(-11.8^{+2.8}_{-2.9}\right)\%, \quad \mathcal{A}_{\rm CP}(\rho^0\gamma) = \left(-9.9^{+3.8}_{-3.4}\right)\%, \quad \mathcal{A}_{\rm CP}(\omega\gamma) = \left(-9.5^{+4.0}_{-3.6}\right)\%. \tag{38}
$$

Being at the level of 10%, the direct CP asymmetry in these decays can be measured at the current B-factories in several years.

The results (38) can be compared with the predictions obtained within the pQCD approach [27]:

$$\mathcal{A}_{\rm CP}(\rho^\pm\gamma) = (17.7 \pm 15.0)\,\%, \quad \mathcal{A}_{\rm CP}(\rho^0\gamma) = (17.6 \pm 15.0)\,\%, \quad \mathcal{A}_{\rm CP}(\omega\gamma) = (17.9 \pm 15.2)\,\%. \tag{39}$$

They are at variance with the results (38) based on the QCD factorization discussed here. In particular, the direct CP-asymmetry is predicted to be positive in the pQCD approach in all decay modes and the central values are typically a factor of two larger while errors are rather large. Measurements of these asymmetries will allow to distinguish the detailed dynamical models illustrated here by the differing predictions of the QCDF and pQCD approaches.

Mixing-Induced CP-Asymmetry: For the time-dependent CP-asymmetries in the neutral B-meson decay modes, the interference of the $B^0 - \bar{B}^0$-mixing and decay amplitudes has to be taken into account, yielding the following characteristic time-dependence of such asymmetries:

$$a_{\rm CP}^{\rho\gamma}(t) = -C_{\rho\gamma}\cos(\Delta M_d\, t) + S_{\rho\gamma}\sin(\Delta M_d\, t), \tag{40}$$

$$a_{\rm CP}^{\omega\gamma}(t) = -C_{\omega\gamma}\cos(\Delta M_d\, t) + S_{\omega\gamma}\sin(\Delta M_d\, t), \tag{41}$$

where ΔM_d is the $B_d^0 - \bar{B}_d^0$ mass difference. The coefficients $C_{\rho\gamma}$ and $C_{\omega\gamma}$ accompanying $\cos(\Delta M_d\, t)$ in Eqs. (40) and (41), up to a sign, coincide with the direct CP-asymmetry discussed above. The second coefficients $S_{\rho\gamma}$ and $S_{\omega\gamma}$, called the mixing-induced CP-asymmetries, are defined as follows:

$$S_{\rho\gamma} = \frac{2\,{\rm Im}(\lambda_{\rho\gamma})}{1 + |\lambda_{\rho\gamma}|^2}, \quad \lambda_{\rho\gamma} \equiv \frac{q}{p}\,\frac{A(\bar{B}^0 \to \rho^0\gamma)}{A(B^0 \to \rho^0\gamma)}, \tag{42}$$

$$S_{\omega\gamma} = \frac{2\,{\rm Im}(\lambda_{\omega\gamma})}{1 + |\lambda_{\omega\gamma}|^2}, \quad \lambda_{\omega\gamma} \equiv \frac{q}{p}\,\frac{A(\bar{B}^0 \to \omega\gamma)}{A(B^0 \to \omega\gamma)}, \tag{43}$$

where the ratio $q/p = {\rm e}^{-2i\beta}$ is a pure phase factor to a good accuracy (experimentally, $|q/p| = 1.0013 \pm 0.0034$ [4]).

The mixing-induced CP-violating asymmetry $S_{\rho\gamma}$ in NLO can be presented in the form [8]:

$$S_{\rho\gamma}^{\rm LO} = -\frac{2|\lambda_u|\,\varepsilon_A^{(0)}\sin\alpha\,(1 - |\lambda_u|\,\varepsilon_A^{(0)}\cos\alpha)}{1 - 2|\lambda_u|\,\varepsilon_A^{(0)}\cos\alpha + |\lambda_u|^2(\varepsilon_A^{(0)})^2}, \tag{44}$$

$$S_{\rho\gamma}^{\rm NLO} = S_{\rho\gamma}^{\rm LO} - \frac{2|\lambda_u|\sin\alpha\,[1 - 2|\lambda_u|\,\varepsilon_A^{(0)}\cos\alpha + |\lambda_u|^2(\varepsilon_A^{(0)})^2\cos(2\alpha)]}{[1 - 2|\lambda_u|\,\varepsilon_A^{(0)}\cos\alpha + |\lambda_u|^2(\varepsilon_A^{(0)})^2]^2}\,\frac{A_R^u - \varepsilon_A^{(0)}A_R^{(1)t}}{C_7^{(0)\rm eff}}, \tag{45}$$

where $A_R^{(1)t}$ and A_R^u are the real parts of the NLO contributions to the decay amplitudes. This expression can be easily rewritten for $S_{\omega\gamma}$. It is seen that, neglecting the weak-annihilation contribution ($\varepsilon_A^{(0)} = 0$), the mixing-induced CP-asymmetry vanishes in the leading order. However, including the $O(\alpha_s)$ contribution, this CP-asymmetry is non-zero even in the absence of the annihilation contribution. The dependence of the mixing-induced CP-asymmetry on the angle α is presented in the right plot in Fig. 1.

The QCDF-based estimates of the mixing-induced CP-asymmetry in the leading and next-to-leading order in α_s in the SM are:

$$S_{\rho\gamma}^{\rm LO} = (-2.7 \pm 0.9)\%, \qquad S_{\rho\gamma}^{\rm NLO} = (1.9^{+3.8}_{-3.2})\%,$$
$$S_{\omega\gamma}^{\rm LO} = (+2.7 \pm 0.9)\%, \qquad S_{\omega\gamma}^{\rm NLO} = (5.9^{+4.1}_{-3.5})\%, \tag{46}$$

showing the tendency of the NLO corrections to compensate the leading order contribution in $S_{\rho\gamma}$ and enhancing it in $S_{\omega\gamma}$. Theoretical uncertainties are rather large and both the values are consistent with being small.

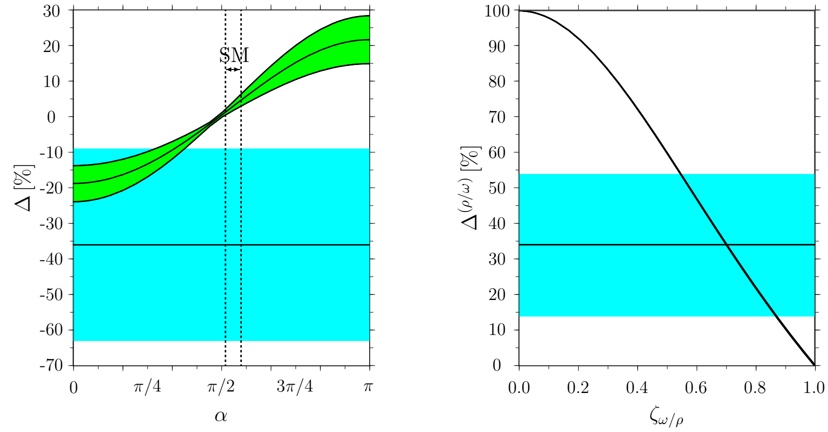

Figure 2: Left figure: The charged-conjugate averaged ratio Δ for $B \to \rho\gamma$ decays (green/dark-shaded region) as a function of the unitarity-triangle angle α. The blue/shaded area is the experimentally measured region by the BABAR collaboration [9]: $\Delta_{\rm exp} = -0.36 \pm 0.27$. Right figure: The $SU(3)_{\rm F}$-violating ratio $\Delta^{(\rho/\omega)}$ as a function of the ratio $\zeta_{\omega/\rho} = \xi_\perp^{(\omega)}(0)/\xi_\perp^{(\rho)}(0)$. The blue/shaded area is the experimental region: $\Delta_{\rm exp}^{(\rho/\omega)} = 0.34 \pm 0.20$, determined from the HFAG averages [4].

Isospin-Violating Ratio: The charge-conjugate averaged quantity Δ for the $B \to \rho\gamma$ decays defined as:

$$\Delta = \frac{1}{4}\left[\frac{\Gamma(B^- \to \rho^-\gamma)}{\Gamma(\bar{B}^0 \to \rho^0\gamma)} + \frac{\Gamma(B^+ \to \rho^+\gamma)}{\Gamma(B^0 \to \rho^0\gamma)}\right] - 1, \qquad (47)$$

is found to be stable against the NLO and $1/m_b$-corrections [7]. In the leading-order, this ratio has been defined in Eq. (37). The NLO corrections do not change the LO result significantly and preserve the main feature – the small value in the vicinity of $\alpha = 90^\circ$, the region favored by the CKM fits [13, 14]. The dependence of the isospin-violating ratio on the angle α is presented in the left plot in Fig. 2. In the expected ranges of the CKM parameters [13], this ratio is estimated as $\Delta = (2.9 \pm 2.1)\%$. A comparison with the result obtained within the pQCD approach [27]: $\Delta = (-5.4 \pm 5.4)\%$, shows that, apart from being somewhat larger in magnitude, Δ has the opposite sign. Thus, while ϵ_A is model-dependent, explicit calculations show that the SM predicts a small isospin-violation in the $B \to \rho\gamma$ decays, as its measure, Δ, is parametrically suppressed (being proportional to $\cos\alpha$, with α close to 90°). A comparison of Δ with the recent BABAR measurement (8) of the same is shown in Fig. 2. As the current experimental errors are rather large, one will have to wait for higher statistics data from the B-factories to draw any quantitative conclusion.

***SU*(3)$_{\mathbf{F}}$-Violating Ratio:** The ratio based on the branching fractions of the neutral $B^0 \to \rho^0\gamma$ and $B^0 \to \omega\gamma$ decay modes may be defined as follows:

$$\begin{aligned} \Delta^{(\rho/\omega)} &\equiv \frac{1}{2}\left[\Delta_B^{(\rho/\omega)} + \Delta_{\bar{B}}^{(\rho/\omega)}\right], \qquad (48) \\ \Delta_B^{(\rho/\omega)} &\equiv \frac{(m_B^2 - m_\omega^2)^3\,\mathcal{B}(B^0 \to \rho^0\gamma) - (m_B^2 - m_\rho^2)^3\,\mathcal{B}(B^0 \to \omega\gamma)}{(m_B^2 - m_\omega^2)^3\,\mathcal{B}(B^0 \to \rho^0\gamma) + (m_B^2 - m_\rho^2)^3\,\mathcal{B}(B^0 \to \omega\gamma)}. \end{aligned}$$

The NLO expression obtained in the $SU(3)_{\rm F}$ symmetry limit, $\zeta_\perp^{(\rho)}(0) = \zeta_\perp^{(\omega)}(0)$, can be written in a simple form [8]:

$$\Delta_{SU(3)}^{(\rho/\omega)} = -\frac{|\lambda_u|\,(\varepsilon_A^{(0)} - \varepsilon_A^{(\omega)})}{C_7^{(0)\rm eff}}\left[(C_7^{(0)\rm eff} - A_R^{(1)t})\cos\alpha + |\lambda_u|\,A_R^u\cos(2\alpha)\right]. \qquad (49)$$

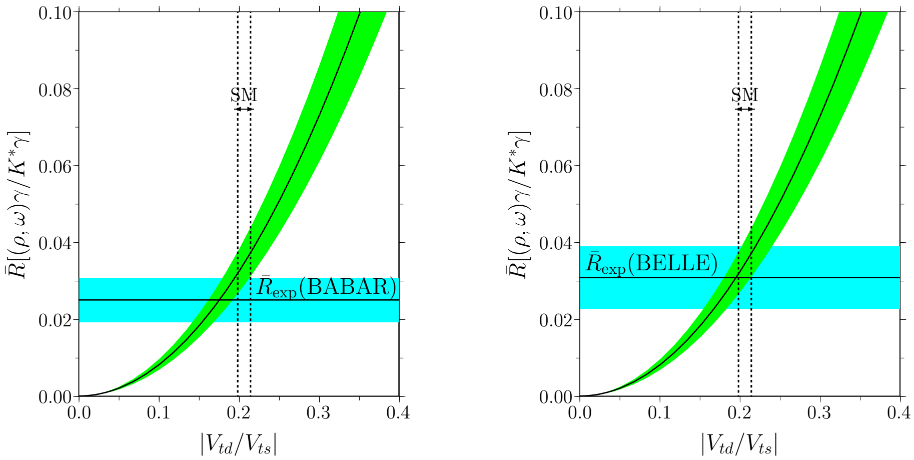

Figure 3: The ratio $\bar{R}[(\rho,\omega)\gamma/K^*\gamma]$ as a function of the ratio $|V_{td}/V_{ts}|$ of the CKM matrix elements. The plots based on the BABAR (left figure) and BELLE (right figure) measurements of the isospin-averaged $B \to (\rho,\omega)\gamma$ branching fractions show their good agreement both with the theoretical estimations of this ratio (green region) and with the recent CDF measurement [5] (the vertical band labeled as "SM") within the 1σ intervals.

The theoretical expression (49) for the ratio $\Delta^{(\rho/\omega)}$ can be improved by including the $SU(3)_{\rm F}$-breaking in the ratio $\zeta_{\omega/\rho}$:

$$\Delta^{(\rho/\omega)} = \frac{1-\zeta_{\omega/\rho}^2}{1+\zeta_{\omega/\rho}^2} + \frac{4\zeta_{\omega/\rho}^2}{(1+\zeta_{\omega/\rho}^2)^2}\,\Delta_{SU(3)}^{(\rho/\omega)} + \mathcal{O}(\alpha_s^2, \varepsilon_A^{(0)}\varepsilon_A^{(\omega)}). \tag{50}$$

The dependence of $\Delta^{(\rho/\omega)}$ on the parameter $\zeta_{\omega/\rho}$ is presented in the right plot in Fig. 2. Based on the recent averages from Table 1, one obtains the following experimental estimate: $\Delta_{\rm exp}^{(\rho/\omega)} = (34 \pm 20)\%$, which is also shown in Fig. 2. Within the range $\zeta_{\omega/\rho} = 0.9 \pm 0.1$, derived from the results of Ref. [35], we estimate: $\Delta^{(\rho/\omega)} = (11 \pm 11)\%$, which is consistent with the experimental value within large errors. We remark that $\Delta^{(\rho/\omega)}$ is dominated by the first term in Eq. (50), as its $SU(3)_{\rm F}$- symmetric value $\Delta_{SU(3)}^{(\rho/\omega)}$ is estimated to be small, $\Delta_{SU(3)}^{(\rho/\omega)} = (2.0 \pm 1.9) \times 10^{-3}$.

4 Summary

Physics of the radiative $B \to \rho\gamma$ and $B \to \omega\gamma$ decays will impact on the CKM phenomenology. A good measure of this is the value of the CKM ratio $|V_{td}/V_{ts}|$, which can be extracted from these decays in conjunction with the $B \to K^*\gamma$ decays. First results along these lines have been obtained by the BABAR and BELLE collaborations, which will become quantitative in due course of time. In addition to the branching fractions resulting in the estimates of the ratio $|V_{td}/V_{ts}|$ (see Fig. 3), the analysis of different asymmetries in these modes will give additional information on the CKM parameters, in particular on the unitarity-triangle angle α, apart from shedding light on the underlying QCD dynamics. In this review, we have taken the attitude that the CKM parameters are well known by now and we use this input to make definite predictions for the branching ratios and various related asymmetries in the $B \to (\rho,\omega)\gamma$ decays. The SM-based predictions are in fair agreement with data and this comparison will become more precise in the coming years.

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