

# Massless and massive vector Goldstone bosons in nonlinear quantum electrodynamics

J. L. Chkareuli\*, Z. R. Kepuladze†

*E. Andronikashvili Institute of Physics and I. Chavchavadze State University  
0177 Tbilisi, Georgia*

October 25, 2006

## Abstract

The spontaneous Lorentz invariance violation (SLIV) developing in QED type theories with the nonlinear four-vector field constraint  $A_\mu^2 = M^2$  (where  $M$  is a proposed scale of the Lorentz violation) is considered in the case when the internal  $U(1)$  charge symmetry is also spontaneously broken. We show that such a SLIV pattern induces the genuine vector Goldstone boson which appears massless when the  $U(1)$  symmetry is exact and becomes massive in its broken phase. However, for both of phases an apparent Lorentz violation is completely canceled out in all the observable processes so that the physical Lorentz invariance in theory is ultimately restored.

## 1 Introduction

Lorentz invariance and its spontaneous violation seems to play a special role with respect to the internal local symmetries observed in particle physics[1, 2]. This violation could generally cause the occurrence of the corresponding massless Nambu-Goldstone modes which are believed to be photons or other gauge fields. At the same time the spontaneous Lorentz invariance violation (SLIV) has attracted considerable attention in the last years as an interesting phenomenological possibility appearing in the framework of various quantum field and string theories[3, 4, 5, 6, 7].

The effective theoretical laboratory for the SLIV consideration happens to be some simple class of the QED type models for the starting massive vector field  $A_\mu$  where, in one way or another, the nonlinear dynamical constraint of type

$$A_\mu^2 = M^2 \tag{1}$$

( $M$  is a proposed scale of the SLIV) is appeared. This constraint means in essence the vector field  $A_\mu$  develops the vacuum expectation value (VEV) and Lorentz symmetry  $SO(1, 3)$  formally breaks down to  $SO(3)$  or  $SO(1, 2)$  depending on the sign of the  $M^2$ . Such models, from the SLIV point of view was studied by Nambu[8] independently of the dynamical mechanism which could cause the spontaneous Lorentz violation. For this purpose he applied the technique of nonlinear symmetry realizations which appeared successful in handling the spontaneous breakdown of chiral symmetry, particularly, as it appears in the nonlinear  $\sigma$  model[9]. It was shown, while only

---

\*e-mail: chkareuli@iphac.ge

†e-mail: kepuladze@iphac.ge

in the tree approximation and for the time-like SLIV ( $M^2 > 0$ ), that the non-linear constraint (1) implemented into standard QED Lagrangian containing the charged fermion  $\psi(x)$

$$\mathcal{L}_{QED} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \bar{\psi}(i\gamma\partial + m)\psi - eA_\mu\bar{\psi}\gamma^\mu\psi \quad (2)$$

as some supplementary condition<sup>1</sup> appears in fact as a possible gauge choice which amounts to a temporal gauge for the superlarge (as it is intuitively expected) SLIV scale  $M$ . At the same time, the  $S$ -matrix remains unaltered under such a gauge convention. This particular gauge allows one to interpret QED in terms of the SLIV with the VEV of vector field of the type  $\langle A_\mu \rangle_0 = (M, 0, 0, 0)$ . The SLIV, however, is proved to be superficial as it affects only the gauge of vector potential  $A_\mu$  at least in the tree approximation[8].

Recently[10], this result has been extended to the one-loop approximation and for both the time-like ( $M^2 > 0$ ) and space-like ( $M^2 < 0$ ) Lorentz violation. All the contributions to the photon-photon, photon-fermion and fermion-fermion interactions violating the physical Lorentz invariance happen to be exactly cancelled with each other in the manner observed by Nambu a long ago for the simplest tree-order diagrams. This means that the constraint  $A_\mu^2 = M^2$  having been treated as the nonlinear gauge choice at a tree (classical) level remains as a gauge condition when quantum effects are taken into account as well. So, in accordance with Nambu's original conjecture one can conclude that the physical Lorentz invariance is left intact at least in the one-loop approximation provided we consider the standard QED Lagrangian (2) (with its gauge invariant  $F_{\mu\nu}F^{\mu\nu}$  kinetic term and minimal photon-fermion coupling) taken in the flat Minkowskian space-time<sup>2</sup>.

We consider here the spontaneous Lorentz violation in the framework of QED with the nonlinear four-vector field constraint (1) in the case when the internal  $U(1)$  charge symmetry is also spontaneously broken so that the massless vector Goldstone boson (photon) having been generated through the SLIV becomes then massive in the  $U(1)$  symmetry Higgs phase. For this purpose one needs to extend the starting Lagrangian  $\mathcal{L}_{QED}$  (2) by the scalar field part

$$\mathcal{L}(\varphi) = |D_\mu\varphi|^2 - m_\varphi^2\varphi^*\varphi - \frac{\lambda_\varphi}{2}(\varphi^*\varphi)^2 \quad (3)$$

where  $D_\mu\varphi = (\partial_\mu - ieA_\mu)\varphi$  is a standard covariant derivative for the charged scalar field  $\varphi$  from which the above Goldstonic photon gets its mass. We show again that the apparent Lorentz violation caused by the nonlinear SLIV constraint (1) is completely canceled out in all the physical processes in the same manner as in the massless QED case considered earlier [8, 10].

The paper is organized in the following way. We consider first the massive non-linear QED Lagrangian (Sec.2) appeared once the dynamical constraint (1) is explicitly implemented into Lagrangians (2, 3) and internal  $U(1)$  symmetry is spontaneously broken so that the photon becomes massive. We derive the general Feynman rules for the basic photon-photon and photon-fermion interactions, as well as the rules related with Higgs sector of theory. The model appears in essence two-parametric containing the electric charge  $e$  and inverse SLIV scale  $1/M$  as the perturbation parameters so that the SLIV interactions are always proportional some powers of them. Then in Sec.3 various SLIV processes such the massive photon scattering off the

<sup>1</sup>The shortest way to obtain this supplementary condition  $A_\mu^2 = M^2$  could be an inclusion the "standard" quartic vector field potential  $P(A) = \frac{m_A^2}{2}A_\mu^2 - \frac{\lambda_A}{4}(A_\mu^2)^2$  into the QED Lagrangian (2) as can generally be motivated[3] from the superstring theory. This unavoidably causes the spontaneous breakdown of Lorentz symmetry in a regular way which goes in parallel with a linear  $\sigma$  model for pions[9]. As a result, one has a massive Higgs mode (with mass  $\sqrt{2}m_A$ ) together with massless goldstones associated with photons. Furthermore, just as in the pion model one can go from the linear model for the SLIV to the non-linear one taking a limit  $\lambda_A \rightarrow \infty$ ,  $m_A^2 \rightarrow \infty$  (while keeping the ratio  $m_A^2/\lambda_A$  to be finite) provided that this limit exists. This immediately leads to the constraint (1) for vector potential  $A_\mu$  with  $M^2 = \frac{m_A^2}{\lambda}$ , as it appears from its equation of motion to be satisfied.

<sup>2</sup>For some alternative possibility see the paper[11]

charged fermion, Higgs boson decays and photon-photon scattering are considered in detail. All these effects, both in the tree and one-loop approximation, appear in fact vanishing so that the physical Lorentz invariance is ultimately restored. And, finally, we give our conclusions in Sec.4.

## 2 The Lagrangian and Feynman rules

### 2.1 The Lagrangian: U(1) symmetry phase

We consider simultaneously both of the above-mentioned SLIV cases, time-like or space-like, introducing some properly oriented unit Lorentz vector  $n_\mu$  ( $n_\mu^2 \equiv n^2 = \pm 1$ ) so as to have the following general parametrization for the vector potential  $A_\mu$  in the Lagrangian (2) of the type

$$A_\mu^2 = n^2 M^2, \quad A_\mu = a_\mu + n_\mu (n \cdot A) \quad (4)$$

(hereafter  $M^2$  is defined strictly positive) where the  $a_\mu$  is pure Goldstonic mode

$$n \cdot a = 0 \quad (5)$$

while the Higgs mode (or the  $A_\mu$  component in the vacuum direction) is given by the scalar product  $n \cdot A$ . Substituting this parametrization into the vector field constraint (1) one comes to the equation for  $n \cdot A$  (taking, for simplicity, the positive sign for the square root and expanding it in powers of  $\frac{a_\nu^2}{M^2}$ )

$$n \cdot A = [(M^2 - n^2 a_\nu^2)]^{\frac{1}{2}} = M - \frac{n^2 a_\nu^2}{2M} + O(1/M^2) \quad (6)$$

We proceed further putting that new parametrization (4) into our basic Lagrangians (2) and (3), then expand it in powers of  $\frac{a_\nu^2}{M^2}$  and make the appropriate redefinition of the fermion and scalar fields according to

$$\psi \rightarrow e^{ieM(n \cdot x)} \psi, \quad \varphi \rightarrow e^{ieM(n \cdot x)} \varphi \quad (7)$$

so that the bilinear fermion and scalar terms,  $eM\bar{\psi}(\gamma \cdot n)\psi$  and  $\varphi^*[ieM(\overleftrightarrow{\partial} \cdot n) + e^2 n^2 M^2]\varphi$ , appearing, respectively, from the expansion of the fermion and charged scalar current interactions in the Lagrangians (2, 3) are exactly cancelled by an analogous terms stemming now from their kinetic terms (the abbreviation  $\overleftrightarrow{\partial}$  means, as usual,  $\varphi^* \overleftrightarrow{\partial} \varphi = \varphi^*(\partial\varphi) - (\partial\varphi^*)\varphi$ ). So, we eventually arrive at the nonlinear SLIV Lagrangian for the Goldstonic  $a_\mu$  field (denoting its strength tensor by  $f_{\mu\nu} = \partial_\mu a_\nu - \partial_\nu a_\mu$ )

$$\begin{aligned} \mathcal{L}(a, \psi, \varphi) = & -\frac{1}{4}f_{\mu\nu}f^{\mu\nu} - \frac{1}{2}\lambda(n \cdot a)^2 - \frac{n^2}{4M}f_{\mu\nu}[(n^\mu\partial^\nu - n^\nu\partial^\mu)a_\rho^2] + \\ & + \bar{\psi}(i\gamma\partial + m)\psi - ea_\mu\bar{\psi}\gamma^\mu\psi + \frac{en^2 a_\rho^2}{2M}\bar{\psi}(\gamma \cdot n)\psi + \\ & + |(\partial_\mu - ie a_\mu)\varphi|^2 - \frac{ien^2 a_\rho^2}{2M}[\varphi^*(\overleftrightarrow{\partial} \cdot n)\varphi] - P(\varphi) \end{aligned} \quad (8)$$

explicitly including its orthogonality condition  $n \cdot a = 0$  through the term which can be treated as the gauge fixing term (taking the limit  $\lambda \rightarrow \infty$ ). Note that there are presented only the terms of the first order in  $\frac{a_\nu^2}{M^2}$  in the Lagrangian and also retained the former notations for the fermion  $\psi$  and scalar field  $\varphi$  (with its unchanged potential  $P(\varphi)$  included, as is given in the starting Lagrangian (3)).

The Lagrangian (8) completes the nonlinear  $\sigma$  model type construction for quantum electrodynamics for the charged fermion and scalar fields. The model contains the massless vector Goldstone boson modes (keeping the massive Higgs mode frozen), and in the limit  $M \rightarrow \infty$  is indistinguishable from conventional QED taken in the general axial (temporal or pure axial) gauge. So, for this part of the Lagrangian  $\mathcal{L}(a, \psi, \varphi)$  given by the zero-order terms in  $1/M$  the spontaneous Lorentz violation only means the noncovariant gauge choice in otherwise the gauge invariant (and Lorentz invariant) theory. Remarkably, furthermore, also all the other terms in the  $\mathcal{L}(a, \psi, \varphi)$  (8), though being by themselves the Lorentz and  $C(CPT)$  violating ones, cause no the physical SLIV effects which appear strictly cancelled in all the physical processes involved. As shows the explicit calculations, there is a full equivalence of such a model with a conventional quantum electrodynamics at least in the tree [8] and one-loop[10] approximation taken for the pure fermionic part in the Lagrangian (8). The same conclusion can obviously be expected for its scalar part as well. This seems to confirm that the starting vector field condition  $A_\mu^2 = n^2 M^2$  which results in the nonlinear QED model (8) is the gauge choice rather non-trivial dynamical constraint which might come to the physical Lorentz violation.

## 2.2 The Lagrangian: $U(1)$ symmetry broken phase

Let us now turn to the case of the spontaneous Lorentz violation when the accompanying internal  $U(1)$  symmetry in the SLIV Lagrangian  $\mathcal{L}(a, \varphi)$  (8) is also spontaneously broken. For this purpose one replaces, as usual, the scalar mass squared  $m_\varphi^2 \rightarrow -m_\varphi^2$  in its potential  $P(\varphi)$  so that the scalar  $\varphi$  now develops the VEV

$$\varphi = \frac{1}{\sqrt{2}}(\eta(x) + v)e^{i\xi(x)/v}, \quad v^2 = 2m_\varphi^2/\lambda_\varphi \quad (9)$$

where for the scalar field  $\varphi$  the standard polar parametrization is used with the proper Higgs and Goldstone modes,  $\eta(x)$  and  $\xi(x)$ , involved. Putting the shifted scalar field (9) into the Lagrangian (8) one comes to the final SLIV theory with the broken  $U(1)$  symmetry

$$\begin{aligned} \mathcal{L}(a, \psi, \eta, \xi) = & \mathcal{L}(a, \psi) + \frac{1}{2}(\partial_\mu \eta)^2 + \frac{1}{2}(\eta + v)^2[\partial_\mu(\xi/v) - ea_\mu]^2 + \\ & + \frac{en^2 a_\rho^2}{2M}(\eta + v)^2(\partial \cdot n)(\xi/v) + P(\eta) \end{aligned} \quad (10)$$

where  $\mathcal{L}(a, \psi)$  stands for the vector field and fermion part (both linear and nonlinear) as is given in the Lagrangian  $\mathcal{L}(a, \psi, \varphi)$  (8), while  $P(\eta)$  denotes an ordinary polynomial of the scalar Higgs component  $\eta$  appeared. One can see that the vector Goldstone boson  $a_\mu$  acquires the mass term  $\frac{1}{2}(e^2 v^2)a_\mu^2$ . However, apart from that, there appears the scalar-vector (goldston-goldston) mixing term in the Lagrangian  $\mathcal{L}(a, \psi, \eta, \xi)$ . In an ordinary Higgs mechanism case such a mixing term can easily be removed by choosing a proper unitary gauge. However, it is hardly possible in the SLIV case where, as is seen from the above Lagrangian  $\mathcal{L}(a, \psi, \varphi)$  (8), one has already come to the axial gauge choice for the vector Goldstonic boson  $a_\mu$  once the spontaneous Lorentz violation occurred. So, one may not put now extra (unitary) gauge to get rid of the scalar Goldstone field  $\xi(x)$ . Nonetheless, this field, if it were appeared in the theory, would correspond to the unphysical particle in the sense that it could not appear as incoming or outgoing lines in Feynman graphs, as was recently argued[12] in the context of Standard Model taken in the axial gauge. This can be seen at once by diagonalizing the bilinear  $a - \xi$  mixing term in our Lagrangian  $\mathcal{L}(a, \psi, \eta, \xi)$  (10) so that the  $\xi$  field disappears in it, while leading to the more complicated form for the  $a$  boson gauge fixing term. In this connection, the option of an existence of the starting  $\xi$  field in the Lagrangian (10), while having in momentum space the diagonalized  $a$  and  $\xi$  propagators, happens to be more convenient and transparent and we take this way in what follows.

### 2.3 The Feynman rules

Actually, rewriting the  $a$ - $\xi$  mixing term in the Lagrangian  $\mathcal{L}(a, \psi, \eta, \xi)$  (10) in momentum space and diagonalizing it by the substitution

$$\xi(k) \rightarrow \xi(k) + i\mu \frac{k^\mu a_\mu(k)}{k^2} \quad (11)$$

where  $\mu = ev$  is the vector  $a$  boson mass, one has for this term

$$\frac{1}{2}(e\eta/\mu + 1)^2 \left[ -ik^\mu \xi(k) + \mu \left( \frac{k^\mu k^\nu}{k^2} - g^{\mu\nu} \right) a_\nu(k) \right]^2 \quad (12)$$

with the new pure  $\xi(k)$  and  $a_\mu(k)$  states appeared. Note that just this transversal bilinear form for the  $a$  boson in (12) together with its kinetic terms and the gauge fixing condition in the Lagrangian  $\mathcal{L}(a, \psi, \varphi)$  (8) determines eventually the diagonalized propagator for the massive  $a$  boson of the type (in the limit  $\lambda \rightarrow \infty$ )

$$D_{\mu\nu}^{(a)}(k) = \frac{-i}{k^2 - \mu^2 + i\epsilon} \left( g_{\mu\nu} - \frac{n_\mu k_\nu + k_\mu n_\nu}{n \cdot k} + \frac{n^2 k_\mu k_\nu}{(n \cdot k)^2} \right) \quad (13)$$

whose numerator is happened to be the same as for the axially gauged massless vector boson. Meanwhile the propagator for the massless scalar field  $\xi$  amounts to

$$D^{(\xi)}(k) = \frac{i}{k^2} \quad (14)$$

For the vector boson  $a_\mu$  being orthogonal to  $n^\mu$ , one can choose a basis of two transverse (in momentum space) components  $\epsilon_\mu^{(t)}(k)$  ( $t = 1, 2$ )

$$n^\mu \epsilon_\mu^{(t)}(k) = 0, \quad k^\mu \epsilon_\mu^{(t)}(k) = 0 \quad (15)$$

and the ‘preferred’ component  $\epsilon_\mu^{(n)}(k)$  determined by the particular SLIV direction  $n^\mu$

$$\epsilon_\mu^{(n)}(k) = N(k_\mu - n_\mu \frac{n \cdot k}{n^2}), \quad n^\mu \epsilon_\mu^{(n)}(k) = 0, \quad k^\mu \epsilon_\mu^{(n)}(k) = N(\mu^2 - \frac{(n \cdot k)^2}{n^2}) \quad (16)$$

where the normalization factor  $N$  is proposed to be chosen in such way that the sum of all the polarizations amounts to the numerator of the  $a$  boson propagator (13).

Supplementing this propagator and ordinary Feynman rules by the rules concerning the Lorentz violating interactions (see also[10]) in the Lagrangians  $\mathcal{L}(a, \psi, \eta, \xi)$  (10) and  $\mathcal{L}(a, \psi, \varphi)$  (8), particularly, those for the contact  $a^2$ -fermion vertex

$$i \frac{e g_{\mu\nu} n^2}{M} (\gamma \cdot n) \quad (17)$$

and the  $a^3$  vertex (rewriting it first as the  $-\frac{n^2}{M}(\partial_\mu a_\nu n^\mu a_\rho \partial^\nu a^\rho)$ )

$$-\frac{in^2}{M} [(k_1 \cdot n)k_{1,\alpha}g_{\beta\gamma} + (k_2 \cdot n)k_{2,\beta}g_{\alpha\gamma} + (k_3 \cdot n)k_{3,\gamma}g_{\alpha\beta}] \quad (18)$$

(where the second index in the each photon 4-momentum  $k_1$ ,  $k_2$  and  $k_3$  denotes its Lorentz component) one is ready to calculate some of the low-order (in  $1/M$ ) processes related with the  $a$  boson and fermion. Note that the scalar field  $\xi$  is not coupled to fermions and, therefore, is not considered in the the  $a$ -boson-fermion interactions. However, one should include into

consideration another  $a^3$  vertex which appears from the  $a^2 - \xi$  coupling in the final Lagrangian  $\mathcal{L}(a, \psi, \eta, \xi)$  (10) once the  $a - \xi$  diagonalization (11) in momentum space has been carried out:

$$\frac{in^2\mu^2}{M} \left[ \frac{(k_1 \cdot n)}{k_1^2} k_{1,\alpha} g_{\beta\gamma} + \frac{(k_2 \cdot n)}{k_2^2} k_{2,\beta} g_{\alpha\gamma} + \frac{(k_3 \cdot n)}{k_3^2} k_{3,\gamma} g_{\alpha\beta} \right] \quad (19)$$

One can see that for the  $a$  bosons being on the mass shell,  $k_{1,2,3}^2 = \mu^2$ , the vertices (18) and (19) exactly cancel each other.

The other rules related with interactions of scalar Higgs and Goldstone fields,  $\eta$  and  $\xi$ , will be given in the next section.

### 3 SLIV processes in massive QED

We show now by a direct calculation of the tree level amplitude for Compton scattering of the massive vector Goldstone  $a$  boson off the charged fermion and other processes that the spontaneous Lorentz violation, being superficial in the massless nonlinear QED with an exact  $U(1)$  symmetry involved[8, 10], is still left hidden even though this symmetry is spontaneously broken and the photon is getting mass.

#### 3.1 Vector boson scattering on fermion

The Lorentz violating part of the elastic  $a$ -boson-fermion scattering is, as follows from the Lagrangians  $\mathcal{L}(a, \psi, \eta, \xi)$  (10) and  $\mathcal{L}(a, \psi, \varphi)$  (8), the only SLIV fermionic process which appears in the lowest  $1/M$  order. This process is concerned with two diagrams one of which is given by the direct contact  $a^2$ -fermion vertex (17), while another corresponds to the  $a$  boson exchange induced by the  $a^3$  couplings (18) and (19). Owing to the above-mentioned mutual cancellation of these  $a^3$  couplings for the on-shell  $a$  bosons, only the third terms in them contributes in the case considered so that one comes to the simple matrix element  $i\mathcal{M}$  for the these two diagrams

$$i\mathcal{M} = i\frac{en^2}{M} \bar{u}(p_2) \left[ (\gamma \cdot n) + i\left(1 - \frac{\mu^2}{k^2}\right) (kn) k_\beta \gamma_\alpha D_{\alpha\beta}^{(a)}(k) \right] u(p_1) \cdot [\epsilon(k_1) \cdot \epsilon(k_2)] \quad (20)$$

where the spinors  $u(p_{1,2})$  and polarization vectors  $\epsilon(k_{1,2})$  stand for the ingoing and outgoing fermions and  $a$  bosons, respectively, while  $k$  is the 4-momentum transfer  $k = p_2 - p_1 = k_1 - k_2$ . After further simplifications in the square bracket related with the explicit form of the  $a$  boson propagator  $D_{\alpha\beta}^{(a)}(k)$  (13) and the fermion current conservation  $\bar{u}(p_2)(\hat{p}_2 - \hat{p}_1)u(p_1) = 0$ , one is finally led to the total cancellation of the Lorentz violating contributions to the Compton scattering of the massive vector Goldstone boson  $a$

$$i\mathcal{M}_{SLIV}(a + \psi \rightarrow a + \psi) = 0 \quad (21)$$

One could say that such a result may be in some sense expected since from the SLIV point of view the massive QED which we considered here is hardly differed from the massless one[8, 10]. Actually, the fermion current conservation, which happens crucial for the above cancellation, works in both of cases depending no whether the internal  $U(1)$  symmetry is exact or spontaneously broken. The fermion sector (being no coupled to the charged scalar from the outset (8)) still possesses this symmetry at least in tree level approximation thus leading to the SLIV cancellation.

#### 3.2 Higgs boson decays

Remarkably, the situation is not changed in the Higgs sector where the  $U(1)$  symmetry related with the starting charged scalar field seems to be directly broken and, therefore, the physical

SLIV might appear. Let us examine, for sure, the Lorentz violating Higgs boson decay  $\eta \rightarrow 3a$  which also appears in the lowest  $1/M$  order if the masses of the  $\eta$  and  $a$  bosons are properly arranged,  $m_\eta > 3\mu$  (or  $e < \sqrt{2\lambda_\varphi/9}$  according to Eq.(9)).

As one can see from the Lagrangian  $\mathcal{L}(a, \psi, \eta, \xi)$  (10) with the substitution (11) already made, this decay goes through the contact  $\eta - a^3$  coupling leading to the matrix element

$$i\mathcal{M}_{cont} = i\frac{en^2}{M\mu}\eta(k) \sum_{l,m,n} P^{lmn} [\epsilon(k_m) \cdot \epsilon(k_n)] [k_l \cdot \epsilon(k_l)] (k_l \cdot n) \quad (22)$$

where the external 4-momenta  $k_{l,m,n}$  ( $l, m, n = 1, 2, 3$ ) of all three  $a$ -bosons with the polarization vectors  $\epsilon(k_{l,m,n})$  are supposed to be picked up according the symmetrical projection operator  $P^{lmn}$  ( $l, m, n = 1, 2, 3$ ) introduced which takes the nonzero value 1 for only the non-equal index values ( $l \neq m \neq n$ ), and also the on-shell condition  $(k_{l,m,n})_\alpha^2 = \mu^2$  has been used; furthermore,  $\eta(k)$  stands for the Higgs boson wave function and the total energy-momentum conservation is supposed,  $k = k_l + k_m + k_n$ .

Apart from this contact diagram, the  $\eta \rightarrow 3a$  decay stems via the pole diagrams corresponding to the intermediate  $a$  and  $\xi$  boson exchange. They are diagrams where the  $\eta$  decays first into two  $a$  bosons or into  $a$  and  $\xi$  bosons (with momenta  $k_1$  and  $k_2$ ) due to the normal Lorentz invariant vertexes stemming from Eq.(12)

$$2ie\mu \left( \frac{k_1^\mu k_1^\nu}{k_1^2} - g^{\mu\nu} \right) \left( \frac{k_2^\mu k_2^\rho}{k_2^2} - g^{\mu\rho} \right) \quad (23)$$

$$2ek_1^\mu \left( \frac{k_2^\mu k_2^\rho}{k_2^2} - g^{\mu\rho} \right) \quad (24)$$

followed then by the virtual Lorentz violating transitions  $a \rightarrow 2a$  and  $\xi \rightarrow 2a$  given, respectively, by the  $a^3$  couplings (18,19) and by the  $a^2 - \xi$  vertex in the Lagrangian (10)

$$\frac{n^2\mu}{M}(n \cdot k)g^{\mu\nu} \quad (25)$$

These six pole diagrams (three diagrams for the each type exchange) correspond, respectively, to the cases when one of  $a$  bosons with 4-momentum  $k_l$  ( $l = 1; 2; 3$ ) is produced directly, whereas two other  $a$  bosons with momenta  $k_m$  and  $k_n$  ( $m, n = 2, 3; 1, 3; 1, 2$ ) appear from the virtual  $a$  and  $\xi$  boson.

Using the above projection operator  $P^{lmn}$  one can calculate the decay amplitude according to all these pole diagrams simultaneously. Note that that all the momenta in the above Feynman rules are measured ingoing so that for the outgoing  $\xi$  state the vertexes (24) and (25) should get a minus sign. Again, owing to the already mentioned mutual cancellation of the  $a^3$  vertexes (18) and (19) for the on-shell  $a$  bosons, only one of their terms contributes in the  $a$  boson exchange diagrams. Remarkably, the non-pole contribution in these  $a$  boson exchange terms appears to be completely cancelled (when gauge fixing condition  $n \cdot \epsilon(k_{l,m,n}) = 0$  is used) with the contact diagram contribution  $i\mathcal{M}_{cont}$  (13), while the the pole contribution terms are happened to be exactly cancelled with the terms stemming from the intermediate  $\xi$  boson diagrams. So, one eventually has that the total amplitude for the Lorentz-violating  $\eta \rightarrow 3a$  decay is certainly vanished

$$i\mathcal{M}_{SLIV}(\eta \rightarrow 3a) = 0 \quad (26)$$

### 3.3 Other processes

In the next  $1/M^2$  order the Lorentz violating  $a - a$  scattering is also appeared. Its amplitude is concerned with the  $a$  boson exchange diagram and the contact  $a^4$  interaction diagram following

from the higher terms in  $\frac{a_\mu^2}{M^2}$  in the Lagrangian (8). Again, these two diagrams are exactly cancelled giving no the physical Lorentz violating contributions.

The same conclusion seems to be derived for the higher order processes including both the tree diagrams and the loops concerning the  $a$  bosons and fermions. Actually, as in the massless QED case considered earlier [10], the corresponding one-loop matrix elements, when they do not vanish by themselves, amount to the differences between pairs of the similar integrals whose integration variables are shifted relative to each other by some constants (being in general arbitrary functions of the external four-momenta of the particles involved) that in the framework of the dimensional regularization leads to their total cancellation.

## 4 Conclusions

We have shown that the Lorentz violation pattern developing due to the nonlinear four-vector field constraint  $A_\mu^2 = M^2$  in the QED type theories induces the genuine vector Goldstone boson which appears massless in the Coulomb phase of theory when the internal  $U(1)$  charge symmetry is exact and becomes massive in its Higgs phase once this  $U(1)$  symmetry spontaneously breaks. However, for both of phases an apparent Lorentz violation is completely canceled out in all the observable processes so that the physical Lorentz invariance in theory is ultimately restored. Remarkably, although the scalar Goldstone mode  $\xi$  related with the scalar field  $\varphi$  is not excluded in the massive nonlinear electrodynamics case (since one can not put the proper unitary gauge in addition to the existing axial one (5) determined by the SLIV) it does not appear as the physical particle - the pole at  $k^2 = 0$  that occurs in its propagator (14) is always canceled by the poles in the interaction vertices (18, (19) of the vector Goldstone boson  $a$  emerged.

So, for the QED like theories the spontaneous Lorentz symmetry breaking owing to the nonlinear four-vector field constraint  $A_\mu^2 = M^2$  (or to its more familiar linearized form  $A_\mu = a_\mu + n_\mu M$ ) is in fact superficial both in massless or massive photon case even if the quantum corrections in terms of the one-loop contributions are included into consideration<sup>3</sup>. This happens to correspond only to fixing the non-covariant gauge for the vector field in a special manner admitting an existence unphysical scalar Goldstone mode  $\xi$  in the theory provided that one starts with an ordinary QED type model (2) with its gauge invariant  $F_{\mu\nu}F^{\mu\nu}$  kinetic term and minimal photon-matter couplings taken in the flat Minkowskian space-time.

## Acknowledgments

We would like to thank Colin Froggatt, Rabi Mohaptra and Holger Nielsen for useful discussions and comments. One of us (J.L.C.) is grateful for the warm hospitality shown to him during his visit to Center for Particle and String Theory at University of Maryland where part of this work was carried out.

## References

- [1] W. Heisenberg, Rev. Mod. Phys. **29**, 269 (1957); J.D. Bjorken, Ann. Phys. (N.Y.) **24**, 174 (1963); T. Eguchi, Phys.Rev. D **14**, 2755 (1976).
- [2] J.L. Chkareuli, C.D. Froggatt and H.B. Nielsen, Phys. Rev. Lett. **87**, 091601 (2001); Nucl. Phys. B **609**, 46 (2001); Per Kraus and E.T. Tomboulis, Phys. Rev. D **66**, 045015 (2002); A. Jenkins, Phys. Rev. D **69**, 105007 (2004) .

---

<sup>3</sup>Remarkably, such theories are proved to belong to some general class of models for which the special theorem on the SLIV non-observability appears to work[13].



- [3] V.A. Kostelecky and S. Samuel, Phys. Rev. D **39**, 683 (1989); V.A. Kostelecky and R. Potting, Nucl. Phys. B **359**, 545 (1991); D. Colladay and V. A. Kostelecky, Phys. Rev. D **58**, 116002 (1998); V.A. Kostelecky, Phys. Rev. D **69**, 105009 (2004); R. Bluhm and V.A. Kostelecky, Phys. Rev. D **71**, 065008 (2005). For an excellent overview of various theoretical ideas and phenomenology, see *CPT and Lorentz Symmetry*, ed. A. Kostelecky (World Scientific, Singapore, 1999, 2002, 2005).
- [4] S.M. Carroll, G.B. Field and R. Jackiw, Phys. Rev. D **41**, 1231,1990; R. Jackiw and V.A. Kostelecky, Phys. Rev. Lett. **82**, 3572, (1999).
- [5] S. Coleman and S.L. Glashow, Phys. Rev. D **59**, 116008 (1999).
- [6] J. W. Moffat, Found. Phys. **23**, 411 (1993); Int. J. Mod.Phys. D**2**, 351 (1993); Int. J. Mod. Phys. D**12**, 1279 (2003).
- [7] O. Bertolami and D.F. Mota, Phys. Lett. B **455**, 96 (1999).
- [8] Y. Nambu, Progr. Theor. Phys. Suppl. Extra 190 (1968).
- [9] S. Weinberg, *The Quantum Theory of Fields*, v.2, Cambridge University Press, 2000.
- [10] A.T. Azatov and J.L. Chkareuli, Phys. Rev. D **73**, 065026 (2006).
- [11] J.L. Chkareuli, C.D. Froggatt, R.N. Mohapatra and H.B. Nielsen, hep-th/0412225.
- [12] C. Dams and R. Kleiss, Eur. Phys. Journ. C **34**, 419 (2004).
- [13] J.L. Chkareuli, C.D. Froggatt and H.B. Nielsen, hep-th/0610186, Phys. Rev. Lett. (to appear).