Realistic renormalizable SO(10)

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Abstract

In this talk I will shortly describe some relevant topics studied in connection with the idea of grand unification. Among them a special role is played by the constraints on the Yukawa structure of the theory. I will review the state of the art of some of the most promising examples of models based on the SO(10) gauge group.

1 Introduction

If one wants to see what are the consequences of grandunification to the flavour structure, one needs to completely ignore possible flavour symmetries as well as all other internal symmetries, gauge, global or discrete. SU(5) is a bad theory of neutrino mass, so we will consider only SO(10) grand unified theories. If one considers all the operators allowed by SO(10) to the Yukawa couplings, there are too many model parameters, and so no prediction is really possible. One option is to assume that the minimal number of parameters must be employed. It has been shown that 4 (3 of them nonrenormalizable) operators are enough in models with 10 and 45 Higgs representations only [1]. Although this is an important piece of information and it has been the starting point of a lot of model building, it is difficult to see a reason for some operators (of different dimensions) to be present in the superpotential and other not, without using some sort of flavour symmetry, so these type of models are considered in the next subsection. On the other hand, a self consistent way of truncating the large number of SO(10) allowed operators without relying on extra symmetries is to consider only the renormalizable ones. This is exactly what we will assume.

In SO(10) a generation of fermions live in a 16 dimensional spinorial representation. This gives the right quantum numbers to the 15 fermions of one generation of the standard model plus a SM singlet righthanded neutrino ν^c. SO(10) thus predicts the existence of three righthanded neutrinos (one for each generation), in contrast with SU(5), in which there is no ad-hoc need for SU(5) singlets. The seesaw mechanism [2] is thus naturally incorporated in SO(10). What remains to be done is however give a large mass to the three ν^c and then integrate them out.

Since we assumed there are no nonrenormalizable operators at tree order, there are just two ways of giving mass to ν^c: by a nonzero vev of the Higgs $\overline{126}$, or generate an effective nonrenormalizable operator radiatively [3]. While the first option is at least in principle always possible, the second one cannot apply to models with low-energy supersymmetry, due to the nonrenormalization of the superpotential. We will consider in turn both of them, but let us first see how can a general renormalizable Yukawa look like in SO(10). From group theory there are just three possible types of Yukawas, schematically written as

$$\mathcal{L}_Y = 16_F^T \left(10_H Y_{10} + 120_H Y_{120} + \overline{126}_H Y_{126}\right) 16_F \ , \tag{1}$$

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where the matter fields 16_F come in three generations, Y_{10} and Y_{126} are 3×3 symmetric complex matrices and Y_{120} is a 3×3 antisymmetric complex matrix, and 10_H, 120_H and $\overline{126}_H$ are the fundamental, three indices completely antisymmetric and five indices completely antisymmetric and anti-self-dual representations of SO(10).

2 Elementary $\overline{\mathbf{126}}_{\mathbf{H}}$

One needs to add another GUT Higgs representation, since a single Yukawa matrix can not give any mixing. The best studied option is adding a 10 dimensional Higgs representation, which contains a $(2,2,1)$ representation under the Pati-Salam decomposition. This would give an equality among the down quark mass matrix M_D and the charged lepton mass matrix M_E. This gets corrected by the $(2,2,15)$ in $\overline{126}$ [4, 5]. At the same time the large vev (SM singlet) $(1,3,10)$ and a tiny vev $(3,1,\overline{10})$ induced by the MSSM Higgs vevs [4, 6] give rise to the so called seesaw of type I (or canonical) and seesaw of type II (or noncanonical) respectively.

The mass matrices at M_{GUT} are thus

$$\begin{aligned} M_D &= v_{10}^d Y_{10} + v_{126}^d Y_{126} \ , && (2)\\ M_U &= v_{10}^u Y_{10} + v_{126}^u Y_{126} \ , && (3)\\ M_E &= v_{10}^d Y_{10} - 3v_{126}^d Y_{126} \ , && (4)\\ M_N &= -M_{\nu_D} M_{\nu_R}^{-1} M_{\nu_D} + M_{\nu_L} \ , && (5)\\ & && (6) \end{aligned}$$

where

$$\begin{aligned} M_{\nu_D} &= v_{10}^u Y_{10} - 3v_{126}^u Y_{126} \ , && (7)\\ M_{\nu_R} &= v_R Y_{126} \ , && (8)\\ M_{\nu_L} &= v_L Y_{126} \ . && (9) \end{aligned}$$

These relations are valid at M_{GUT}, so it is there that their validity must be tested. The analysis done so far used the results of renormalization group running from M_Z to M_{GUT} from [7].

The first attempts in fitting the mass matrices assumed the domination of the type I seesaw, and the nonsupersymmetric [5] or low-energy supersymmetric [8].

A new impetus to the whole program was given by the observation that in case type II seesaw dominates (a way to enforce it is to use a 54 dimensional Higgs representation [9]) the neutrino mass, an interesting relation in these type of models between $b-\tau$ unification and large atmpospheric mixing angle can be found [10]. The argument is very simple and it can be traced to the relation [11]

$$M_N \propto M_D - M_E \ , \tag{10}$$

which follows directly from (2), (4) and (9), if only the second term (type II) in (5) is considered. Considering only the heaviest two generations as an example and taking the usually good approximation of small second generation masses and small mixing angles, one finds all the elements of the righthandside small except the 22 element, which is proportional to the difference of two big numbers, $m_b - m_\tau$. Thus, a large neutrino atmospheric mixing angle is linked to the smallness of this 22 matrix element, and thus to $b-\tau$ unification. To notice that in these types of models such $b-\tau$ unification is no more automatic due to the presence of the $\overline{126}$, which breaks SU(4)$_C$. It is however a quite good prediction of the RGE running in the case of low-energy supersymmetry.

The numerical fitting were able to reproduce also a large solar mixing angle both in case of type II [12] or mixed seesaw [13], predicting also a quite large $|U_{e3}| \approx 0.17-0.18$ mixing element, close to the experimental upper bound. The difficulty in fitting the CKM CP violating phase in the first quadrant was overcome by new solutions found in [14], mainaining the prediction of large $|U_{e3}| \leq 0.1$ matrix element.

All these fittings were done assuming no constraints coming from the Higgs sector. Regading it, it was found that the minimal supersymmetric model [15] has only 26 model parameters [16], on top of the usual supersymmetry breaking soft terms, as in the MSSM. When one considers this minimal model, the vevs in the mass formulae (2)-(9) are not completely arbitrary, but are connected by the restrictions of the Higgs sector. This has been first noticed in [17] showing a possible clash with the positive results of the unconstrained Yukawa sector studied in [14]. The issue has been pursued in [18], showing that in the region of parameter space where the fermion mass fitting is successfull, there are necessarily intermediate scale thresholds which spoil perturbativity of the RGE evolution of the gauge couplings.

To definitely settle the issue two further checks should be done: a) the χ^2 analysis used in the fitting procedure should be implemented at M_Z, where the errors are better known, and not at M_{GUT}. Preliminary results of such an analysis seem not to be better [19]. b) Another issue is to consider also the effect of the possible increased gauge couplings on the Yukawas [20]. Only after these two checks will be done, this minimal model could be ruled out.

Some topics have to be still mentioned in connection with the above: the important calculation of the mass spectrum and Clebsch-Gordan coefficients in SO(10) [21, 22, 23, 24], the Higgs doublet mass matrix [22], the running of the gauge couplings at two loops together with threshold corrections [23], and the study of proton decay [25].

What if this model turns out to be wrong? There are other models on the market. The easiest idea is to add a 120 dimensional Higgs. In this reaspect there are three different ways of doing that considered in the literature: a) take 120 as a small, nonleading, contribution, perturbation, to the previous formualae [27]; b) consider 120 on an equal footing as 10 and $\overline{126}$, but assume real parameters in the superpotential, breaking CP spontaneously [28] and allowing small the dangerous $d = 5$ proton decay modes; c) assume small $\overline{126}$ contributions to the charged fermion masses [29].

Another limit is to forget the 10_H altogether, as has been proposed for nonsupersymmetric theories [30]. The two generation study predicts a top large ratio $m_b/m_\tau \approx .3$, instead of the value 0.5 that one gets by straight running. The idea is that this could get large corrections due to Dirac neutrino Yukawas [31] and the effect of finite second generation masses, as well as the inclusion of the first generation and CP violating phases.

3 Radiative $\overline{\mathbf{126}}_{\mathbf{H}}$

The original idea [3] is that there is no $\overline{126}_H$ representation in the theory, but the same operator is generated by loop corrections. The representation that breaks the rank of SO(10) is now 16_H, which vev let us call M_R. Generically there is a contribution to the righthanded neutrino mass at two loops:

$$M_{\nu_R} \approx \left(\frac{\alpha}{4\pi}\right)^2 \frac{M_R^2}{M_{GUT}} \frac{M_{SUSY}}{M_{GUT}} , \tag{11}$$

and is thus too small in low-energy supersymmetry (low breaking scale M_{SUSY}) as well as nonsupersymmetric theories (low intermediate scale M_R, required by gauge coupling unification). The only exception, proposed in [32], could be split supersymmetry [33, 34].

In the absence of $\overline{126}_H$, the charged fermion masses must be given by only 10_H and 120_H or by two 10_H's [32], together with radiative corrections. The simplest analysis of the tree order two generation case gives three interesting predictions-relations [35]: 1) almost exact $b-\tau$ unification;

2) large atmospheric mixing angle related to small quark mixing angle; 3) degenerate neutrinos. For a serious numerical analysis one needs to use the RGE for the case of split supersymmetry, taking a very small $\tan\beta < 1$ to get an approximate $b-\tau$ unification [34]. Also one needs some fine-tuning of the parameters to account for the small ratio $M_{SUSY}/M_{GUT} \leq 10^{-(3-4)}$ required in realistic models to have gluinos decay fast enough [36]. The three generation case is on the way [20].

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