Pentaquarks as chiral solitons and as multiquarks states

A. M. Shunderuk*
Institute for Nuclear Research of RAS,
Moscow 117312, Russia

Abstract

Chiral soliton approach is used to describe spectrum of exotic and nonexotic baryons. Strangeness contents of baryons are calculated within rigid rotator model for arbitrary number of colors. Comparison of different variants of the model (rigid rotator and rigid oscillator) at large number of colors is performed. The behavior of strangeness content as function of N_c reveals a problem of extrapolation from large N_c to $N_c = 3$. Results of chiral soliton approach are compared with simple quark model.

1 Introduction

Present talk is based on the work [1]. Within chiral soliton model the spectrum of exotic and nonexotic baryon states can be obtained by means of quantization of the collective motion of starting classical field configuration. Explicit calculation of the strangeness contents of exotic and nonexotic baryon states at arbitrary number of colors allows to perform comparison of different quantization schemes: rigid rotator and rigid oscillator models. Another question is comparison of the results of the chiral soliton approach and simple quark model.

2 Rigid rotator model

The Lagrangian of the chiral soliton model in the simplest case has the following form:

$$\mathcal{L} = -\frac{F_\pi^2}{16} Tr L_\mu^2 + \frac{1}{32e^2} Tr[L_\mu L_\nu]^2 + \frac{F_\pi^2 m_\pi^2}{16} Tr(U + U^\dagger - 2), \tag{1}$$

where U is unitary matrix incorporating the chiral fields, $L_\mu = \partial_\mu U U^\dagger$, m_π and F_π are pion mass and decay constant, e is the Skyrme parameter defining the weight of the 4-th order term.

In the collective coordinates quantization procedure [2], the angular velocities of rotation of soliton in the $SU(3)$ configuration space are introduced:

$$U \to A(t) U A(t)^\dagger, \qquad A^\dagger(t)\dot{A}(t) = -i\omega_a \lambda_a/2, \tag{2}$$

λ_a being Gell-Mann matrices, $a = 1, \dots, 8$. The rotational part of the lagrangian is quadratic in angular velocities with two coefficients, isotopical moment of inertia (Θ_π) and flavor moment of inertia (Θ_K):

$$\mathcal{L}_{rot} = \frac{1}{2}\Theta_\pi(\omega_1^2 + \omega_2^2 + \omega_3^2) + \frac{1}{2}\Theta_K(\omega_4^2 + ... + \omega_7^2) - \frac{N_c B}{2\sqrt{3}}\omega_8. \tag{3}$$

* **e-mail**: a.shunderuk@umail.ru

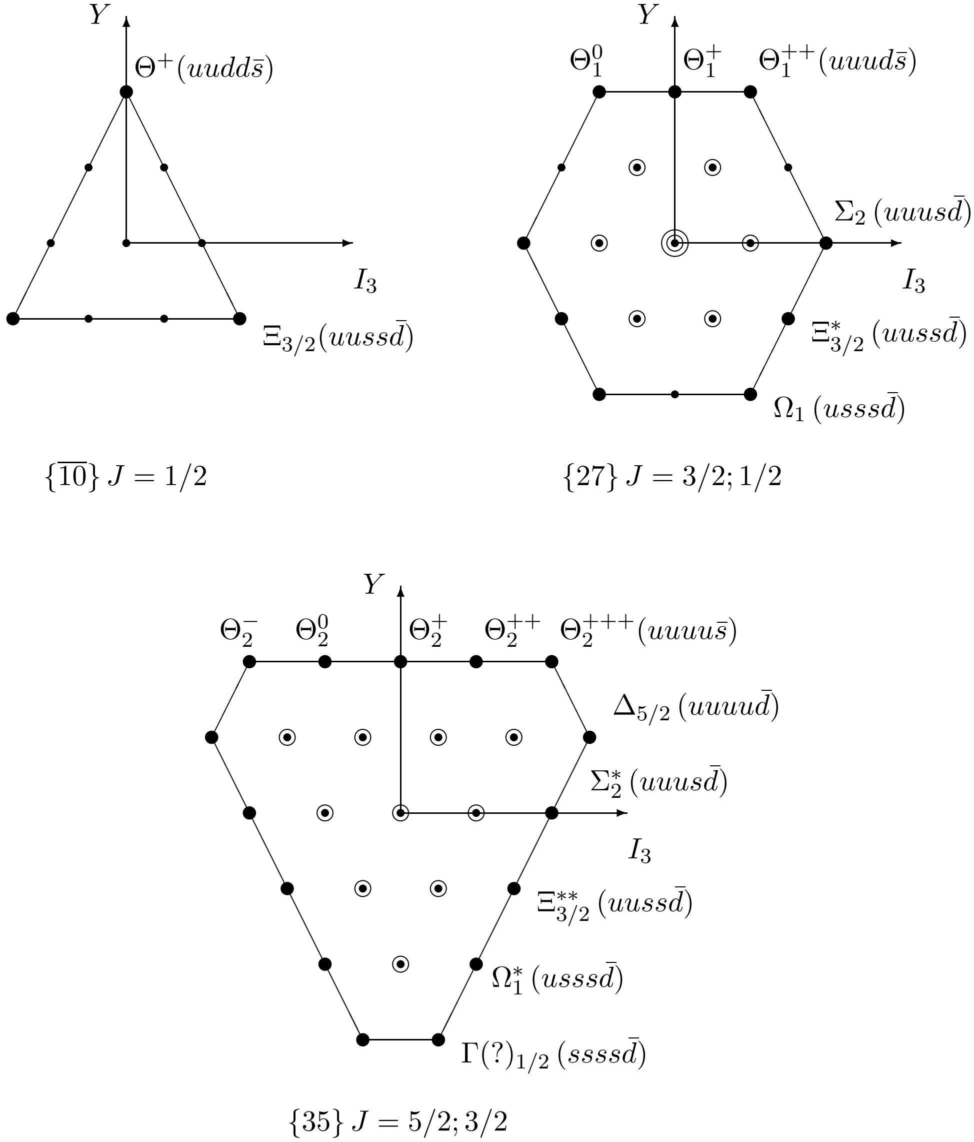

Figure 1: Antidecuplet, 27-plet and 35-plet of baryons. Large full circles show the exotic states, small circles show the cryptoexotic states.

Linear in angular velocity term originates from the Wess-Zumino-Witten term in the action of the model [3]. The hamiltonian of the model can be obtained by means of canonical quantization procedure,

$$H = M_{cl} + \frac{1}{2\Theta_\pi}\mathbf{R}^2 + \frac{1}{2\Theta_K}\left[C_2(SU_3) - \mathbf{R}^2 - \frac{N_c^2 B^2}{12}\right], \tag{4}$$

where operators $R_a = \partial\mathcal{L}/\partial\omega_a$, the second order Casimir operator for the $SU(3)$ group $C_2(SU_3) = \sum_{a=1}^{8} R_a^2$, the second order C. o. for the $SU(2)$ group $C_2(SU_2) = \mathbf{R}^2 = \sum_{a=1}^{3} R_a^2$. For the (p,q)-multiplet, $C_2(SU_3) = (p^2+q^2+pq)/3 + p + q$, $C_2(SU_2) = J(J+1)$, where J is so called right or body fixed isospin, which for one-baryon configurations is equal to the spin of baryon. In addition to usual octet and decuplet of baryons, it is possible to consider multiplets of pentaquark baryons, antidecuplet, 27-plet and 35-plet (see fig. 1).

The mass splitting inside of $SU(3)$-multiplets of baryons is defined by the flavor symmetry breaking term in the lagrangian. In the first order in flavor symmetry breaking mass m_K this term has the following form:

$$\mathcal{L}_{FSB} = \frac{m_K^2 - m_\pi^2}{24} Tr(1-\sqrt{3}\lambda_8)(U + U^\dagger - 2), \tag{5}$$

and resulting mass formula for the quantized states in the rigid rotator approximation is

$$M = M_{cl} + \frac{J(J+1)}{2\Theta_\pi} + \frac{1}{2\Theta_K}\left[\left(\frac{p^2+q^2+pq}{3} + p + q\right) - J(J+1) - \frac{N_c^2}{12}\right] + \Delta M_{FSB}, \tag{6}$$

$$\Delta M_{FSB} = \Gamma(m_K^2 - m_\pi^2) C_S.$$

C_S here is strangeness content of baryon. If we parametrize the matrix of collective coordinates $A \in SU(3)$ as

$$A = A_1(SU_2) e^{i\nu\lambda_4} A_2(SU_2) e^{i\rho\lambda_8/\sqrt{3}}, \tag{7}$$

so that the only flavor changing parameter within this parametrization is ν, the angle of rotation in "strange" direction, then strangeness content

$$C_S = \frac{1}{2}\left\langle \Psi(\nu)|sin^2\nu|\Psi(\nu)\right\rangle. \tag{8}$$

Strangeness content for states of the octet, decuplet, antidecuplet, 27-plet and 35-plet of baryons at arbitrary number of colors is given in tables 1, 2. It can be seen from the tables that for the fixed value of strangeness, C_S decreases as $1/N_c$ with increasing N_c — in agreement with the fact that fixed number of quarks are strange, whereas total number of constituent quarks is N_c, or $N_c + 2$ for pentaquarks. Also it can be seen that the parameter for $1/N_c$ expansion of strangeness content is $7/N_c$ for octet, $9/N_c$ for decuplet and antidecuplet, $11/N_c$ for 27-plet, and so on, so the expansion parameter is large for $N_c = 3$. From this it follows that $1/N_c$ expansion methods for exotic and nonexotic baryons are questionable, because there is a problem of extrapolation from the large N_c limit to the real value $N_c = 3$.

3 Rigid oscillator model

In the rigid oscillator model [4] different parametrization of matrix A is used:

$$A(t) = A_{SU(2)}(t) S(t), \tag{9}$$

where matrix $S(t) = e^{i\mathcal{D}}$, $\mathcal{D} = \sum_{a=4}^{7} d_a\lambda_a$. Two-component spinor

$$D = \frac{1}{\sqrt{2}}\begin{pmatrix} d_4 - id_5 \\ d_6 - id_7 \end{pmatrix} \tag{10}$$

is deviation of starting $SU(2)$ soliton into "strange" direction, which is believed to be small. The hamiltonian is of the oscillator type. For one-baryon systems

$$H = M_{cl} + 4\Theta_K \Pi^\dagger\Pi + \left(\Gamma m_K^2 + \frac{N_c^2}{16\Theta_K}\right) D^\dagger D, \tag{11}$$

where Π is momentum canonically conjugate to variable D. After diagonalization the hamiltonian can be written in terms of flavor and antiflavor creation operators and flavor and antiflavor excitation energies:

$$H = M_{cl} + \omega_- a^\dagger a + \omega_+ b^\dagger b, \tag{12}$$

$$\omega_- = \frac{N_c}{8\Theta_K}(\mu - 1), \qquad \omega_+ = \frac{N_c}{8\Theta_K}(\mu + 1), \qquad \mu = \sqrt{1 + \frac{16\Theta_K\Gamma m_K^2}{N_c^2}}.$$

$[p, q]$	$C_S(N)$	$C_S(N=3)$
$[1, (N-1)/2]$		
$S=0,\ I=1/2$	$2(N+4)/[(N+3)(N+7)]$	7/30
$S=-1, I=1$	$(3N+13)/[(N+3)(N+7)]$	11/30
$S=-1,\ I=0$	$3/(N+7)$	9/30
$S=-2, I=1/2$	$4/(N+7)$	12/30
$[3, (N-3)/2]$		
$S=0,\ I=3/2$	$2(N+4)/[(N+1)(N+9)]$	7/24
$S=-1,\ I=1$	$(3N+7)/[(N+1)(N+9)]$	8/24
$S=-2,\ I=1/2$	$(4N+6)/[(N+1)(N+9)]$	9/24
$S=-3, I=0$	$5/(N+9)$	10/24
$[0, (N+3)/2]$		
$S=+1,\ I=0$	$3/(N+9)$	6/24
$S=0,\ I=1/2$	$(4N+9)/[(N+3)(N+9)]$	7/24
$S=-1,\ I=1$	$(5N+9)/[(N+3)(N+9)]$	8/24
$S=-2,\ I=3/2$	$(6N+9)/[(N+3)(N+9)]$	9/24

Table 1: Strangeness contents of states of the octet, decuplet and antidecuplet of baryons at arbitrary number of colors. Table from paper [1].

$[p, q]$	$C_S(N)$	$C_S(N=3)$
$[2, (N+1)/2]$		
$S=+1,\ I=1$	$(3N+23)/[(N+5)(N+11)]$	32/112
$S=0,\ I=3/2$	$(4N^2+65N/2-3/2)/[(N+1)(N+5)(N+11)]$	33/112
$S=0,\ I=1/2$	$(4N+24)/[(N+5)(N+11)]$	36/112
$S=-1,\ I=2$	$(5N^2+39N-26)/[(N+1)(N+5)(N+11)]$	34/112
$S=-1,\ I=1$	$(5N^2+33N+8)/[(N+1)(N+5)(N+11)]$	38/112
$S=-1,\ I=0$	$5/(N+11)$	5/14
$S=-2,\ I=3/2$	$(6N^2+38N-8)/[(N+1)(N+5)(N+11)]$	40/112
$S=-2,\ I=1/2$	$(6N+7/2)/[(N+1)(N+11)]$	43/112
$S=-3,\ I=1$	$(7N+2)/[(N+1)(N+11)]$	46/112
$[4, (N-1)/2]$		
$S=+1,\ I=2$	$(3N+25)/[(N+3)(N+13)]$	34/96
$S=0,\ I=5/2$	$(4N^2+85N/3-79)/[(N-1)(N+3)(N+13)]$	21/96
$S=0,\ I=3/2$	$(4N+24)/[(N+3)(N+13)]$	36/96
$S=-1,\ I=2$	$(5N^2+74N/3-67)/[(N-1)(N+3)(N+13)]$	26/96
$S=-1,\ I=1$	$(5N+23)/[(N+3)(N+13)]$	38/96
$S=-2,\ I=3/2$	$(6N^2+21N-55)/[(N-1)(N+3)(N+13)]$	31/96
$S=-2,\ I=1/2$	$(6N+22)/[(N+3)(N+13)]$	40/96
$S=-3,\ I=1$	$(7N^2+52N/3-43)/[(N-1)(N+3)(N+13)]$	36/96
$S=-3,\ I=0$	$7/(N+13)$	42/96
$S=-4,\ I=1/2$	$(8N-31/3)/[(N-1)(N+13)]$	41/96

Table 2: Strangeness contents of states of the 27-plet ($J=3/2$) and 35-plet ($J=5/2$) of baryons at arbitrary number of colors. Table from paper [1].

Also there is $O(1/N_c)$ contribution, which is expressed in terms of the hyperfine splitting constants, different for flavor and antiflavor:

$$\Delta E = \frac{J(J+1)}{2\Theta_\pi} + \frac{1}{2\Theta_\pi}\Big\{(c-1)[J(J+1) - I(I+1)] + (\bar{c} - c)I_S(I_S+1)\Big\}, \tag{13}$$

J is the spin of baryon, I — isospin, $I_S = |S|/2$. For flavor

$$c = 1 - \frac{4\Theta_\pi \Gamma m_K^2}{N_c^2} + O(m_K^4), \qquad \bar{c} = 1 - \frac{8\Theta_\pi \Gamma m_K^2}{N_c^2} + O(m_K^4), \tag{14}$$

for antiflavor

$$c = 1 - \frac{\Theta_\pi}{\Theta_K} + \frac{4\Theta_\pi \Gamma m_K^2}{N_c^2} + O(m_K^4), \qquad \bar{c} = 1 + \frac{2\Theta_\pi}{\Theta_K} - \frac{24\Theta_\pi \Gamma m_K^2}{N_c^2} + O(m_K^4). \tag{15}$$

4 Comparison of RR and RO models in large N_c limit

We can compare results of the rigid rotator and rigid oscillator models in the limit of large number of colors and small kaonic mass. Let us consider the decuplet of baryons ($J = 3/2$, $(p, q) = (3, (N_c - 3)/2)$). Contributions to mass formula linear in the square of flavor symmetry breaking mass m_K are found to be

$$\begin{aligned}
\delta M_\Delta^{RR} &\simeq \left(\frac{2}{N_c} - \frac{12}{N_c^2}\right) m_K^2 \Gamma, & \delta M_\Delta^{RO} &= \frac{2}{N_c} m_K^2 \Gamma, \\
\delta M_{\Sigma^*}^{RR} &\simeq \left(\frac{3}{N_c} - \frac{23}{N_c^2}\right) m_K^2 \Gamma, & \delta M_{\Sigma^*}^{RO} &= \left(\frac{3}{N_c} - \frac{5}{N_c^2}\right) m_K^2 \Gamma, \\
\delta M_{\Xi^*}^{RR} &\simeq \left(\frac{4}{N_c} - \frac{34}{N_c^2}\right) m_K^2 \Gamma, & \delta M_{\Xi^*}^{RO} &= \left(\frac{4}{N_c} - \frac{10}{N_c^2}\right) m_K^2 \Gamma, \\
\delta M_\Omega^{RR} &\simeq \left(\frac{5}{N_c} - \frac{45}{N_c^2}\right) m_K^2 \Gamma, & \delta M_\Omega^{RO} &= \left(\frac{5}{N_c} - \frac{15}{N_c^2}\right) m_K^2 \Gamma.
\end{aligned} \tag{16}$$

As it is known, in the limit of large number of colors both rigid rotator and rigid oscillator apporaches coincide [5]. But it can be seen that there is difference in the next to leading order corrections in the $1/N_c$ expansion. Possible way to remove disagreement is the following. The rigid oscillator calculation involves normal-ordering ambiguities, so if we assume that the normal ordering corrections change the mass formula by an extra additive term of order $O(1/N_c^2)$,

$$\Delta M(norm.ord.) = -\frac{6\,(2 + |S|)}{N_c^2} m_K^2 \Gamma, \tag{17}$$

then the difference between two variants of the model will be removed.

In the same way we can consider, for example, different pentaquark states, with strangeness $+1$ and isospin 0, 1 or 2:

$$\begin{aligned}
\delta M_{\Theta_0}^{RR} &\simeq \left(\frac{3}{N_c} - \frac{27}{N_c^2}\right) m_K^2 \Gamma, & \delta M_{\Theta_0}^{RO} &= \left(\frac{3}{N_c} - \frac{9}{N_c^2}\right) m_K^2 \Gamma, \\
\delta M_{\Theta_1}^{RR} &\simeq \left(\frac{3}{N_c} - \frac{25}{N_c^2}\right) m_K^2 \Gamma, & \delta M_{\Theta_1}^{RO} &= \left(\frac{3}{N_c} - \frac{7}{N_c^2}\right) m_K^2 \Gamma, \\
\delta M_{\Theta_2}^{RR} &\simeq \left(\frac{3}{N_c} - \frac{23}{N_c^2}\right) m_K^2 \Gamma, & \delta M_{\Theta_2}^{RO} &\simeq \left(\frac{3}{N_c} - \frac{5}{N_c^2}\right) m_K^2 \Gamma.
\end{aligned} \tag{18}$$

The difference between results of rigid rotator and rigid oscillator models in this case too is described by expression (17).

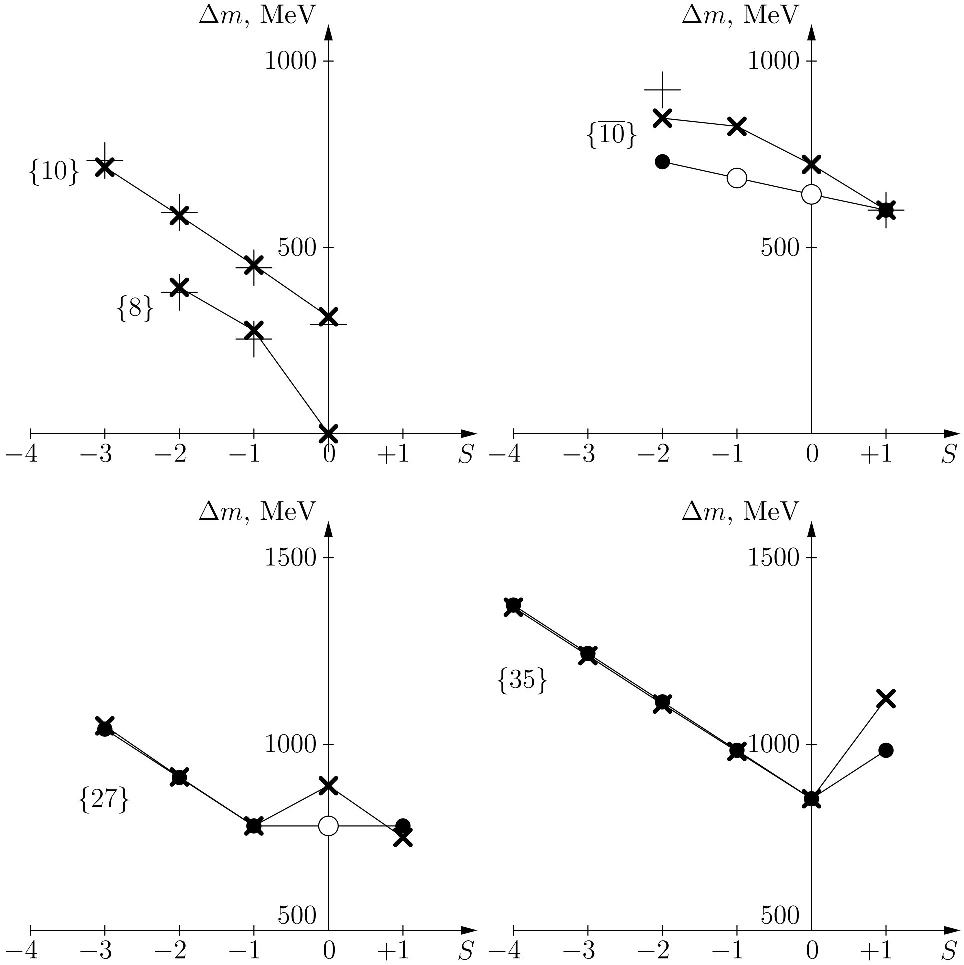

Figure 2: Schematic picture of the mass splittings within chiral soliton model. The upper left figure corresponds to the nonexotic octet and decuplet, the upper right one — to exotic antidecuplet, the lower — to 27-plet ($J = 3/2$) and to 35-plet ($J = 5/2$) of exotic baryons. Experimental data are shown by direct crosses +, position of states obtained within CSA with configuration mixing is marked by incline crosses ×, the circles show position of states within simplistic quark model with difference between strange and nonstrange quark masses $\Delta m_s \simeq 130 MeV$. Full circles show manifestly exotic states and empty circles — cryptoexotic states. For antidecuplet the fit was made for the state with $S = +1$.

5 Comparison with simple quark model

One important property of strangeness contents of baryons given in tables 1, 2 is that they satisfy (at arbitrary number of colors) Gell-Mann — Okubo relations:

$$C_S(p,q,Y',I) = a(p,q)Y' + b(p,q)\left[\frac{Y'^2}{4} - I(I+1)\right] + c(p,q), \qquad (19)$$

$$Y' = S + 1.$$

Here a, b and c are constant within any $SU(3)$ multiplet. For octet

$$a(\{8\}) = -\frac{N_c+2}{(N_c+3)(N_c+7)}, \quad b(\{8\}) = -\frac{2}{(N_c+3)(N_c+7)}, \quad c(\{8\}) = \frac{3}{(N_c+7)}, \qquad (20)$$

for decuplet

$$a(\{10\}) = -\frac{N_c+2}{(N_c+1)(N_c+9)}, \quad b(\{10\}) = -\frac{2}{(N_c+1)(N_c+9)}, \quad c(\{10\}) = \frac{3}{(N_c+9)}, \qquad (21)$$

and so on. From (19), in particular, follows linear behaviour of masses of any chain of states with $I = \pm Y'/2 + const.$ All states within definite multiplet, satisfying such relation, are equidistant.

Baryon spectrum obtained within the chiral soliton approach (for details of CSA calculation see [6, 1]) can be compared with quark model (fig. 2). For comparison the most simple variant of the quark model was used, where strange quark and antiquark masses are equal, as well as they are equal in different $SU(3)$ multiplets. The result of chiral soliton model calculation is in rough agreement with the mass splitting given by the quark model. It is important that better agreement is achieved for manifestly exotic states, which have unique quark contents, while the quark contents of cryptoexotic states $qqq(\alpha\, s\bar{s} + \beta\, u\bar{u} + \gamma\, d\bar{d})$ are model-dependent.

6 Conclusions

Rigid rotator and rigid oscillator variants coincide in the limit of large N_c, however, already subleading terms are different. The difference between results of this two models can be described by expression of special form for normal-ordering correction. Explicit calculation of strangeness contents at arbitrary number of colors shows that chiral soliton model mimics the quark model due to Gell-Mann — Okubo relations. Numerically, results of quark model are mainly reproduced in CSA, expessially for manifestly exotic states.

References

[1] V.B.Kopeliovich and A.M.Shunderuk, Phys. Rev. D73, 094018 (2006); hep-ph/0602125.

[2] G.Adkins, C.Nappi and E.Witten, Nucl. Phys. B228, 552 (1983);
E.Guadagnini, Nucl. Phys. B236, 35 (1984).

[3] E.Witten, Nucl. Phys. B223, 433 (1983).

[4] C.G.Callan and I.R.Klebanov, Nucl. Phys. B262, 365 (1985);
K.Westerberg and I.Klebanov, Phys. Rev. D50, 5834 (1994).

[5] N.Itzhaki, I.R.Klebanov, P.Ouyang and L.Rastelli, Nucl. Phys. B684, 264 (2004);
H.Walliser and H.Weigel, Eur. Phys. J. A26, 361 (2005); hep-ph/0510055.

[6] H.Walliser and V.B.Kopeliovich, JETP 97, 433 (2003); hep-ph/0304058.