Selected applications of covariant perturbation theory

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Abstract

Large proper time asymptotic limit of quark Green's functions in confining background fields is discussed. Possible applications to B-physics problems are outlined.

There have been recently a revival of interest to computing nonperturbative (np) low energy matrix elements of hadron currents, whose uncertainty limits to a large extent our ability to search for the New physics (NP) hidden in the short distance coefficients of the Wilson expansion

$$H_{eff} = \frac{G_F}{\sqrt{2}} \sum_q V_{qb} V^*_{qQ} \sum_i C_i(\mu) O_i(\mu)$$

Quantitative understanding of np GeV - scale QCD physics of $\langle O_i(\mu)\rangle$ is needed for reliable claims concerning TeV - range NP physics hidden in $C_i(\mu)$. There are two basic strategies: either to minimize effects of np working with inclusive modes, factorizable ratios, using different symmetries etc, or, on the contrary, to study cases where np effects are most pronounced.

One has three basic ways of computing $\langle O_i(\mu)\rangle$: the constituent models, the sum rules and numerical simulations on the lattice. Let us consider them in more details. Speaking about the lattice one has to stress that it is not a model, but numerical simulation of QCD (moreover, we have no operational definition of QCD beyond perturbation theory other than lattice at the moment). Tremendous progress has been achieved in recent years, and still more is ahead, but different well known problems of this approach are still here and not going to disappear suddenly.

Constituent models physically are transparent, but it is not clear in most cases how to control approximations being made. Last, but not least, it is hard to incorporate confinement and chiral symmetry breaking simultaneously (”Where does QCD string go to in the pion?”)

One usually considers n-point correlators of hadron currents, in the simplest $n = 2$ case

$$G_{\mu\nu}(x-y) = \langle j_\mu(x) j_\nu(y)^\dagger \rangle$$

where we confine our attention below to heavy-light axial-vector currents

$$j_\mu(x) = \bar{Q}(x)\gamma_\mu\gamma_5 q(x)$$

with $Q = b, c\,;\ q = u, d, s$.

At small distances it reads as: $G^{SD}(x-y) = \{\text{Pert. theory}\} + \{\text{power corrections}\}$, while at large distances one has $G^{LD}(x-y) = \{\text{Lowest resonance}\} + \{\text{all the rest}\}$. Since this is one and the same quantity we must have sum rules [1]:

$$\boxed{G^{SD}(x-y) = G^{LD}(x-y) \equiv G(x-y)}$$

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It is crucial that nonperturbative vacuum is parameterized by local universal quantities, e.g.

$$\langle G^a_{\mu\nu} G^a_{\mu\nu} \rangle \ , \ \langle \bar\psi\psi \rangle$$

Some important issues here are:

- Structure of power corrections (divergencies, subseries summation, λ^2 - term etc.)
- {all the rest} is generally unknown - threshold scale s_0 comes
- existence of the *working window*

One must have $G(x-y)/G_0(x-y) \to 1$ if $x \to y$. Short distance interpretation of this statement is *trivial*: asymptotic freedom, while large distance interpretation is *nontrivial*: quark-hadron duality, the whole spectral density contributes.

There is an alternative idea here: to choose some long-distance, nonlocal but still universal quantities for parametrization of npQCD vacuum. The simplest case is light scalar in the field of infinitely heavy source

$$G^{(2)}(x-y) = \langle q^\dagger(x)\Phi(x,y)q(y)\rangle$$

We have

$$G^{(2)}(x-y) = \int\limits_0^\infty ds\, \frac{1}{(4\pi s)^{d/2}}\, \exp\left(-m^2 s - \frac{(x-y)^2}{4s}\right) \cdot w(s, x-y)$$

where $w(s,x-y) = \langle\langle \mathrm{Tr}\, W[A, z(\sigma)]\rangle_A\rangle_z$ with

$$\langle \mathcal{O}[z]\rangle_z = (4\pi s)^{d/2} \int\limits_{z(0)=z(s)=0} \mathcal{D}z_\mu \exp\left(-\frac{1}{4}\int\limits_0^s \dot z^2_\mu(\sigma)d\sigma\right) \mathcal{O}[z]$$

Then all dynamical information is fully encoded in $w(s,x-y)$. Small s (corresponding to large m) expansion is nothing but the standard operator product expansion in powers of fields and derivatives, corresponding to the well known Schwinger - DeWitt expansion. The typical term looks like $c \cdot \langle G^k D^m G^n\rangle \cdot s^{k+n+\frac{m}{2}}$. We are interested in the opposite, large s regime. World-line integration technique is convenient to sum up all terms of the kind $GDD..DG$ in classical backgrounds G (see, e.g. [2]). The result is essentially the gauge field counterpart of what is called covariant perturbation theory (Schwinger-DeWitt subseries summation) [3, 4].

It is convenient to parameterize $2\mu s = T$ where $T^2 = (x-y)^2$, then one distinguishes

$$w(\mu, T) \sim \exp\left(-\frac{\Delta}{2\mu}T\right)$$

regime, which corresponds to perimeter law, and

$$w(\mu, T) \sim \exp\left(-\frac{c_0}{\mu^\delta}T\right)$$

with $0 < \delta < 1$ for confinement ($\delta = 1/3$ corresponds to area law). The short distance (but still large T) perturbative contribution goes as

$$w(\mu, T) \sim \exp\left(-c[\alpha_s]\mu\, T\right)$$

Thus universal (i.e. m and T independent) information about nonperturbative dynamics is encoded in Δ and c_0

It can be shown that for spinor particle in general case for $T = 0$

$$w(s, 0) \sim \exp\Big[-\Delta \cdot s + \gamma \cdot \log s + \mathcal{O}\left((1/s)\cdot \log s\right)\Big]$$

with $\Delta = 0$ and $\gamma_{np} = 0$ in the deconfinement phase. To get chiral symmetry breaking in the confinement phase one should have $\Delta = 0$ and $\gamma_{np} = 3/2$ (in $d = 3+1$). It is to be compared with the value $\gamma = 1$ from Heisenberg-Euler - type Lagrangians for constant fields and from general analysis for classical backgrounds.

Going back to heavy-light system (for definiteness we chose $T = y_4 - x_4$) we have:

$$G_{44}(T) = N_c T^2 \int_0^\infty \frac{d\mu_1}{2\mu_1^2} \int_0^\infty \frac{d\mu_2}{2\mu_2^2} \exp\left[-\frac{T}{2}\left(\frac{m_1^2}{\mu_1} + \frac{m_2^2}{\mu_2} + \mu_1 + \mu_2\right)\right] \cdot$$

$$\left\langle\left\langle 4(m_1 m_2 + \mu_1\mu_2 + p_4^{(1)}p_4^{(2)} - p_i^{(1)}p_i^{(2)}) \mathrm{Tr}\, W[A, Z_1, Z_2]\right\rangle_A\right\rangle_Z$$

where average goes over bosonic and fermionic coordinates $Z_\mu(s,\theta) = z_\mu(s) + \theta\psi_\mu(s)$ with free world-line action given by

$$S_0 = \sum_{i=1}^{2} \int_0^{s_i} d\sigma_i \left(\frac{1}{4}\dot{z}(\sigma_i)\dot{z}(\sigma_i) + \frac{1}{2}\psi(\sigma_i)\dot{\psi}(\sigma_i)\right)$$

Since $m_2 \gtrsim \Lambda_{QCD} \gg m_1$, we need large μ_2 - small μ_1 asymptotic

Thus we choose in the leading order in $1/m_b$

$$w(\mu_1, \mu_2, T) \sim \exp\left(-\frac{c_0}{\mu_1^{1/3}} T\right)$$

where universal parameter c_0 is to be fitted e.g. from the Regge slope, numerically $c_0^{3/4} \approx 0.65\, GeV$. Using the standard values for the current quark and B, B_s masses we get

$$G_{44} \xrightarrow{T\to\infty} \frac{f_M^2 \cdot M}{2} \exp(-MT) \quad ; \quad \xi = \frac{f(B_s)}{f(B)} \approx 1.18$$

to be compared with $\xi = 1.15$ from [5] and with $\xi = 1.21$ from [6] and with $\xi = 1.16$ from sum rules analysis [7]. All approaches show strong deviation from naive $f_M \sim 1/\sqrt{M}$ law.

The main conclusions from the above can be summarized as follows:

- Parametrization of quark Green's functions large proper time dependence can be chosen in universal way (i.e. independent on quark masses and meson quantum numbers).
- This approach is close in spirit to einbein fields formalism, however no Hamiltonians have been written and no approximations like instantaneous interaction have been made
- More interesting to apply to 3-point form-factors: the results will not necessarily be better, but the method is definitely alternative (which means - may be complementary) to the good old sum rules approach

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