Color bosonization: definition of effective action and decomposition of the QCD gauge field

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Abstract

We consider two main points in the color bosonization approach to the infrared QCD: (a) the definition of the effective action and (b) decomposition of the QCD gauge field, which includes quark chiral papameters.

1 Introduction

Due to the phenomenon of Chiral Anomaly [1], the total color space unifies gauge and chiral (anomalous) sectors in the unique sector with topological properties, but with the same number of field variables as in the gauge sector. To understand the infrared dynamics in the total color space is essential for solution of the confinement problem and comparing different scenarios of infrared behaviour, and , in particular, for verification of the monopole condensation scenario [2]. In this talk we discuss two main points in the color bosonization approach to infrared QCD [3]: definition of an effective action and decomposition of the QCD gauge field.

A search for new types of the gauge field decompositions or configurations with topological properties, which could be essential for the infrared QCD, has now quite a long history (n-field by Faddeev [4], the Cho decomposition [5], two Faddeev-Niemi decompositions into magnetic and electric-like variables [6] and related research [7, 8, 9, 10, 11]. The philosophy of the color bosonization approach differs from that of these studies only in relation to the color Chiral Anomaly: the Anomaly is taken into account explicitly in color bosonization by considering quark chiral parameters, while in other gauge field decompositions the Anomaly was present implicitly via topological variables. The main problem of the color bosonization is how to merge both types of variables (gauge field and Anomaly ones) into unique set. Definition of an effective bosonization action depends on situation with Anomaly variables: if they are external to the gauge field variables, the effective action requires a flavor-like definition, if they are separated from quarks and dissolved in the gauge field variables, the action is given by direct integration over quarks.

We develop the theory at example of the color SU(2) group. An extension to the color SU(3) will use many points of the SU(2) case [12].

2 Bosonization in the flavor case

Let us review the bosonization approach to the effective chiral action in the case of chiral flavor [13]. We consider massless fermions in external vector and axial vector fields V_μ, A_μ. The path integral of fermions $Z_\psi[V, A]$ is a functional of V, A:

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$$Z_\psi[V,A] = \int d\mu_\psi \exp i \int dx \overline{\psi}\ \not{D}(V,A)\,\psi \tag{1}$$

where $/D = i\gamma^\mu(\partial_\mu + V_\mu + i\gamma_5 A_\mu)$ is the Dirac operator. The chiral transformation of fermions is given by

$$\psi'_L = \xi_L\psi_L, \psi'_R = \xi_R\psi_R, \psi = \psi_L + \psi_R \tag{2}$$

where $\xi_L(x)$ and $\xi_R(x)$ are local chiral phase factors of left and right quarks ψ_L and ψ_R, represented by unitary matrices in defining representations of left $SU(N)_L$ and right $SU(N)_R$ subgroups of the chiral group $G_{LR} = SU(N)_L \times SU(N)_R$. For $\psi_L = \frac{1}{2}(1+\gamma_5)\psi, \psi_R = \frac{1}{2}(1-\gamma_5)\psi$, generators t_{La} and t_{Ra} of left and right subgroups of G_{LR} can be written as $t_{La} = \frac{1}{4}(1+\gamma_5)\tau_a, t_{Ra} = \frac{1}{4}(1-\gamma_5)\tau_a, [t_{La,}t_{Rb}] = 0$, where $\tau_a, a = 1,2,3$ are the Pauli matrices. Then quark left and right chiral phase factors ξ_L, ξ_R arise from application of operators $\hat{\xi}_L = \exp(-it_{La}\omega_{La}), \hat{\xi}_R = \exp(-it_{Ra}\omega_{Ra})$ to left and right quarks ψ_L and ψ_R. Vector gauge transformations $g(x)$ are associated with $t_a = t_{La} + t_{Ra} = \tau_a/2$, i.e. $g(x)$ has properties of the product $\hat{\xi}_L(x)\hat{\xi}_R(x)$ of identical left and right rotations, $\omega_L = \omega_R = \alpha$. The generator of purely chiral transformations $g_5(x)$ is $t_{5a} = \gamma_5\tau_a/2 = t_{La} - t_{Ra}$; thus, $g_5(x)$ has properties of $\hat{\xi}_L(x)\hat{\xi}_R^+(x)$ for $\omega_L = \omega_R = \Theta$. Infinitesimally, the Dirac operator is transformed according to

$$\delta\ \not{D} = [i\frac{1}{2}\alpha_a\tau_a, \not{D}] + \{i\frac{1}{2}\gamma_5\Theta_a\tau_a, \not{D}\} \tag{3}$$

Commutation relations for t_a, t_{5a} are given by

$$[t_a, t_b] = i\varepsilon_{abc}t_c, [t_a, t_{5b}] = i\varepsilon_{abc}t_{5c}, [t_{5a}, t_{5b}] = i\varepsilon_{abc}t_c \tag{4}$$

Instead of phases ξ_L and ξ_R one can work with the chiral field $U = \xi_R^+\xi_L$, which describes rotation of only left quark leaving right quark in peace $\psi_L \to \psi'_L = U\psi_{L,}\psi \to \psi'_R = \psi_R$. The same result can be obtained by the chiral transformation $\psi_L \to \xi_L\psi_L, \psi_R \to \xi_R\psi_R$, followed by a vector gauge transformation with a gauge function ξ_R^+. The usual chiral gauge choice is $\xi_R = \xi_L^+$, then the chiral field is taken as squared left chiral phase: $U = \xi_L^2$.

The chiral tranformation of fermions in the Dirac action is equivalent to the following change of the Dirac operator

$$\begin{aligned} \bar{\psi}'\ \not{D}(V,A)\,\psi' &= \bar{\psi}D\left(V^U, A^U\right)\psi \\ V^U &= \frac{1}{2}[U^+(\partial + V + A)U + (V - A)] \\ A^U &= \frac{1}{2}[U^+(\partial + V + A)U - (V - A)] \end{aligned} \tag{5}$$

A transformed fermionic path integral Z'_ψ because of $\psi \to \psi'$ in the Dirac action is equal to an original path integral as a functional of transformed fields $Z'_\psi[V,A] = Z_\psi[V',A']$. Z_ψ is invariant under vector gauge transformations of fermions, but undergoes changes under chiral tranformations, because of non-invariance of the fermionic measure $d\bar{\psi}d\psi$ [14]: chiral transformations are anomalous. The chiral anomaly $\mathcal{A}$ is defined by an infinitesimal change of $\ln Z_\psi$ due to an infinitesimal chiral transformation $\delta g_5 = i\theta_a\tau_a \equiv \Theta$.

We put $g_5(s) = \exp\gamma_5\Theta s$ and write the anomaly $\mathcal{A}(x,\Theta)$ at a chiral angle Θ

$$\mathcal{A}(x,\Theta) = \frac{1}{i}\frac{\delta\ln Z_\psi(\exp\Theta s)}{\delta s}_{s=1} \tag{6}$$

The usual way [13] to calculate effective chiral action W_{eff} is to find the anomaly and integrate it over s up to $g_5 = \exp\gamma_5\Theta$.

$$W_{eff} = -\int d^4x \int_0^1 ds A\left(x; s\Theta\right)\Theta\left(x\right) = \int d^4x L_{eff} - W_{WZW} \tag{7}$$

where the Wess-Zumino-Witten term W_{WZW} describes topological properties of g_5 (and U) and is represented by a five-dimensional integral with $x_5 = s$. It is the analogue of W_{eff} for color that we are interested in.

3 Color bosonization

The basic color fields are the Yang-Mills field $V_\mu\left(x\right)$ and the quark field $\bar{\psi}\left(x\right), \psi\left(x\right)$ with the Lagrangian

$$L_\psi = \bar{\psi}\ \not{D}\left(V\right)\psi, \tag{8}$$

where $\not{D}\left(V\right)$ is the Dirac operator for massless quarks with the Yang-Mills field V_μ. There are no dynamical axial vector field: $A_\mu = 0$.

We consider the vacuum functional Z for the system quarks + gluons as being always in the presence of the color chiral field $U\left(x\right)$ describing local chiral degrees of freedom of quarks $\bar{\psi}, \psi$ and resulting in replacement of $Z_\psi\left[V\right]$ by $Z_\psi\left[V^U, A^U\right]$, where V^U and A^U are vector and axial vector fields arising in the Dirac operator $\not{D}\left(V\right) \rightarrow \not{D}\left(V^U, A^U\right)$ from the gluonic field V_μ due to chiral rotation

$$Z = \int d\mu_V \{\exp i \int dx L_{YM}\left(V\right)\} Z_\psi\left[V^U, A^U\right] \tag{9}$$

The vacuum functional Z depends on color degrees of freedom of quarks and gluons. The gluon measure $d\mu_V$ includes only vector color degrees, while the quark functional Z_ψ contains both vector and chiral color degrees in the quark measure $d\bar{\psi}d\psi$. Explicitly $Z_\psi[V]$ depends only on gluonic field V_λ. Under transformations of the color gauge group $SU\left(2\right)_c = SU\left(2\right)_{L+R}$ the vacuum functional is invariant,$\delta Z=0$. The existence of chiral anomaly means that $\delta Z \neq 0$ under chiral transformations belonging to the coset $G_{LR}/SU\left(2\right)_{L+R}$.

While in the flavor case V_μ and U are always independent variables, in the color case two types of questions are possible:

(a) what is an action for the chiral field in a given gluon field. For example, what is an action for chiral soliton in color vacuum field [15]. Total number of variables should not exceed that of dynamical gluon field. This question is of the same type as in the flavor case, and an action is given by an expression (7) for flavor one.

(b) What is an anomalous action for color variables taking into account that a pair $\left(V^U, A^U\right)$ should contain the same number of variables as V. Then an initial path integral is Z_U in (9) with all color variables $\left(V^U, A^U\right)$ of left-right group shown explicitly. In this case, an anomalous action is defined by the expression, which formally is of opposite sign compared with the flavor case (7). The "bosonized", or anomalous action is defined by the expression

$$W_{bose}\left[V^U, A^U\right] = -i(\ln Z_\psi\left[V^U, A^U\right] - \ln Z_\psi\left[V, 0\right]), \tag{10}$$

which includes in general two different actions: a topological W_{WZW} and a non-topological W_{an} ones. For SU(2) $W_{WZW} = 0$. For SU(3) it is the most interesting part.

We consider one loop approximation for gluons in the background gauge in absence of external vector fields. Then V_μ will be a classical (background) field for gluons. There is no background axial vector field A_μ. After quark color chiral transformation with the chiral field U , we get from V_μ a vector matrix W_μ^U containing both V_μ^U and A_μ^U

$$W_\mu^U = V_{\mu a}^U t_a + A_{\mu a}^U t_{5a} V_\mu^U = \frac{1}{2}\left(U^+ V_\mu U + V_\mu + U^+ \partial_\mu U\right),$$

$$A_\mu^U = \frac{1}{2} U^+ D_\mu U, \tag{11}$$

where D_μ is defined with the background field V_μ. When $U = 1$ we return to the case, when $A^U = 0$ and $W_\mu^U = V_\mu$. Note that a chirally rotated gauge field is just an extension of an initial gauge field by an induced axial vector field: $V_\mu^U = V_\mu + A_\mu^U$.

Thus, in the color bosonization approach, there are three vectors $V_\mu, V_\nu^U, A_\lambda^U$ living in the common color space of gluons and chiral color of quarks. They include two gauge fields V_μ and V_μ^U, belonging to different vector-type subgroups of left-right chiral group. V_μ transforms with $L_a + R_a$ color generators, while V_μ^U transforms with generators $L_a^U + R_a$, where the left generator is additionally rotated. We remind that the chiral field U belongs to the anomalous channel Θ.

Because of chiral transformations, asymptopic constraints imposed on gluonic field V_μ lead to constraints for V_μ^U, A_μ^U. It is usually required for the dynamical V_μ that

$$\int d^3 x tr V_\mu V_\mu \prec \infty \tag{12}$$

We assume that this property is preserved by chiral transformations. In view of orthogonality of t_{5a} and t_b, we have

$$\int d^3 x \{ tr A_\mu^U A_\mu^U \} \prec \infty \tag{13}$$

It means that asymptotically

$$U^+ D_\mu U \to 0, r \to \infty$$

Consider the chiral field $U = \xi_L^2$, where ξ_L is an $SU(2)_L$ rotation in the fundamental representation with generators $\tau_a/2$

$$\begin{gathered} \xi_L(x) = \exp(i\hat{n}F/2), 0 \le F \le 2\pi \\ U = \exp 2(i\hat{n}F/2), \hat{n} = n_a \tau_a, n_a n_a = 1 \\ U^+ D_\mu U = i\hat{n}\partial_\mu F + i\frac{1}{2} D_\mu \hat{n} \sin 2F + [\hat{n}, D_\mu \hat{n}] \frac{1}{2} \sin^2 F \end{gathered} \tag{14}$$

Then asymptotically at $r \to \infty$

$$\partial_\mu F \to 0, D_\mu \hat{n} \to 0$$

Within the chiral left-right group gauge fields V_μ and V_μ^U are associated with different SU(2)-subgroups. One may conjecture that there is a finite region in the total color space, where both fields are equivalent. Such a region should correspond to restricted number of color degrees of freedom. A boundary of this region, where number of variables is changed, will be reflected in behaviour of the determinant $\det[1 + R(U)]$.

Consider a change of fields $A_\mu(x) \to A'_\mu(x) = \frac{1}{2}(U^+(x) A_\mu(x) U(x) + A_\mu(x))$, or

$$A'_{\mu a} = \frac{1}{2}(\delta_{ab} + R_{ab}(U)) A_{\mu b}, R_{ab}(U) = \frac{1}{2} tr \left(\lambda_a U^+ \lambda_b U\right) \tag{15}$$

where $R_{ab}(U)$ is the transformation U in adjoint representation. We get 3×3 matrix $R(U) = R\left(\xi_L^2\right)$ by replacing $SU(2)$ generators $\tau_a/2$ with $SO(3)$ hermitian generators O_a

$$R(U) = \exp i O_a n_a 2F = 1 + i\hat{N} \sin 2F + \hat{N}^2 (\cos 2F - 1) \tag{16}$$

The 3×3 matrix $\hat{N} = O_a n_a$ has a property $\hat{N}^3 = \hat{N}$, so that eigenvalues of $\hat{N}$ are equal to +1,-1,0. It follows that $\det \frac{1}{2}(1 + R(U)) = \frac{1}{2}(1 + \cos 2F)$. Thus, we have singularities at points x_s, where $F(x_s) = \pi/2$, $U_s \equiv U(x_s) = i\hat{n}(x_s)$ and $R(U_s) = 1 - 2\{\hat{N}(x_s)\}^2$. At singularity two eigenvalues of the matrix $R(U)$ coincide. In the chiral color space, these singular points x_s constitute a spherical surface of radius $F(x_s)$ in the anomalous channel (i.e. which is gauge equivalent to a region of $\gamma_5\theta$ -parameter of the left-right group), where a general gluonic field V_μ cannot be expressed in terms of chirally dependent field V_μ^U. The transformation $V_\mu[x] \to \frac{1}{2}[1 + R(U)]V_\mu(x)$ may be induced by a global chiral rotation, $\partial_\mu U = 0$; thus, already a global chiral rotation leads to a singular determinant.

Explicitly, using an expression $V_\mu^U = V_\mu + A_\mu^U$ we get in terms of color vectors $\vec{n}, \vec{V}_\mu$

$$\vec{V}_\mu^U = \vec{V}_\mu \cos 2F - \frac{1}{2}\left[\vec{V}_\mu, \vec{n}\right] \sin 2F - \vec{n}(\vec{V}_\mu, \vec{n}) \sin^2 F +$$

$$i\vec{n}\partial_\mu F - \frac{1}{2}\partial_\mu \vec{n} \sin 2F - [\vec{n}, \partial_\mu \vec{n}] \sin^2 F \tag{17}$$

so that at $F = \pi/2$ the field V_μ^U looses structures represented by $\partial_\mu \vec{n}$ and $\left[\vec{V}_\mu, \vec{n}\right]$:

$$V_\mu^U = \frac{1}{2}\left(\hat{n}\partial_\mu\hat{n} + \hat{n}\hat{V}_\mu\hat{n} + V_\mu\right), F = \frac{\pi}{2} \tag{18}$$

while the matrix $\Delta = \frac{1}{2}(1 + R(U))$ reduces to $\Delta^0 = 1 - \hat{N}^2$ with matrix elements $\Delta_{ab}^0 = n_a n_b$, and determinant $\det \Delta^0 = 0$. The matrix $1 - N^2$ is a projector on eigenvalue $\hat{N}' = 0$. Thus, at $F = \pi/2$, only fields with $N' = 0$ are essential for construction of singular free connections in color space including chiral degrees of freedom introduced by U. In fact, this conclusion follows directly from properties of Δ

$$(\Delta)_{ab}\, n_b = n_a, (\Delta^{-1})_{ab} n_b = (1 - i\hat{N}\tan F)_{ab} n_b = n_a \tag{19}$$

It reflects (by construction of the chiral field $U = \exp i\hat{n}F$) the fact that U commutes locally with a function of $\hat{n}$.

Let us demonstrate, that it is possible to find such a gluon field V_μ and such a chiral field U, that we have an invariance relation $V_\mu = V_\mu^U$. We represent V_μ in the form $V_\mu = C_\mu\hat{n} + \frac{1}{2}U\partial_\mu U^+$, where C_μ is an abelian gauge field and check the relation $V_\mu^U = \frac{1}{2}(UV_\mu U^+ + V_\mu + U\partial_\mu U^+) = V_\mu$. It can be satisfied if $U = i\hat{n}$ or $F = \pi/2$. We denote this special field on the sphere $\Omega(\pi/2)$ by V_μ^Ω

$$V_\mu^\Omega = C_\mu\hat{n} + \frac{1}{2}\hat{n}\partial_\mu\hat{n},$$

$$A_\mu^\Omega = \frac{1}{2}\left(UV_\mu^\Omega U^+ - V_\mu + U\partial_\mu U^+\right) = 0 \tag{20}$$

The chiral field U and the gauge field V_μ contain now the same unit color vector $\hat{n}$. Second term in V_μ^Ω satisfies separately the equivalence relation $V_\mu^U = V_\mu$. The field V_μ^Ω is a basic field in the color bosonization approach, because then an axial vector $A_\mu^U(\Omega) = 0$; both the Yang-Mills action $I_{YM}\left(V^\Omega\right)$ and the quark integral $Z_\psi\left[V^U, A^U\right]$, which is in Ω just $Z_\psi\left[V^\Omega, 0\right]$, depend on the same V^Ω only. There is no color chiral anomaly -neither a topological one (for SU(3)), nor of a non-topological type. The field V_μ^Ω can be obtained by chiral transformation from the simplest vector field, namely, from an abelian field $V_\mu^0 = C_\mu\hat{n}$. Also, V_μ^Ω is invariant under the gauge transformation with the chiral field $U = i\hat{n}$ and under chiral transformation

$$V_\mu^\Omega = (V_\mu^0)^U = \frac{1}{2}(\hat{n}V_\mu^0\hat{n} + V_\mu^0 + \hat{n}\partial\hat{n}), V_\mu^\Omega = \hat{n}V_\mu^\Omega\hat{n} + \hat{n}\partial_\mu\hat{n}$$

The corresponding Yang-Mills field strength $V_{\mu\nu}^\Omega$ is

$$V_{\mu\nu}^\Omega = C_{\mu\nu}\hat{n} + \frac{1}{4}\left[\partial_\mu\hat{n}, \partial_\nu\hat{n}\right] \tag{21}$$

The field V_μ^Ω depends on four degrees of freedom, instead of required 6 degrees in the case of SU(2). The field V_μ^Ω was introduced as a starting point of $n-$model [4]and "Restricted gauge theory" [5].

Fixing $det\Delta = 1$ corresponds to excluding the Cartan mode $\exp i\tau_3 F$ from the chiral field $U = \exp i\hat{n}F$. In terms of $SU(2)_L \times SU(2)_R$ generators $\tau_a/2, \gamma_5\tau_b/2$, it means that we rotate $\hat{n}$ to τ_3 and then fix $\gamma_5\tau_3-$ parameter F.

4 QCD-SU$(2)_c$ at low energies: gauge field and the effective gluonic action

The field V_μ^Ω is a common part of initial gauge field V_μ and chirally rotated version V_μ^U. In left-right group without dynamical A_μ these gauge fields are interrelated by $V_\mu = V_\mu^\Omega - A_\mu^U$. Chiral rotation of quarks $i\hat{n}$ transforms $Z\psi[V^\Omega - A^U, 0]$ into $Z_\mu[V^\Omega, A^U]$, acting as a shift operator. A_μ^U should anticommute with $\hat{n}$. Denoting $A_\mu^U = -X_\mu$ we come to the decomposition for the QCD gauge field

$$V_\mu = V_\mu^\Omega + X_\mu \tag{22}$$

This decomposition for the QCD gauge field was discussed by Cho [5] in different approach.

For color bosonization approach anticommutativity relation $\{\hat{n}, X_\mu\} = 0$ is essential. Due to this property of X_μ, a chirally rotated gauge field ${V_\mu}^U = (V_\mu^\Omega + X_\mu)^U = V_\mu^\Omega$ is independent of X_μ, while the axial field A_μ^U picks up the value $A_\mu^U = -X_\mu$. Thus, this expression for a gauge field V_μ is, at the same time, a decomposition of a gauge field into chirally rotated vector part $V_\mu^U = V_\mu^\Omega$ and chirally rotated axial vector part $A_\mu^U = -X_\mu$. In this form, it is explicitly seen that the gluon sector and the quark sector are built on the same color variables, and theYang-Mills action $I_{YM}(V)$ and bosonization part $Z_\psi\left[V^\Omega, -X\right]$ contain the same set of background fields (V_μ^Ω, X_μ). In the SU(2) case, an axial field $A_\mu^U = -X_\mu$ leads to a non-topological chiral anomaly of the quark integral Z_ψ. In the case SU(3) the chiral anomaly will include also a topological term. Thus, in this decomposition of V_μ, the field X_μ is responsible for the chiral anomaly, and consequently, for a bosonization action. Such an action together with the Yang-Mills action and kinetic term will determine low energy color dynamics.

The color bosonization action W_{an} can be written in analogy with the flavor case [13]. In our notations, the non-topological part of W_{an} corresponds to the following Lagrangian $L_{an} = L_+\left(V_\mu^\Omega, -X_\mu\right) - L_+\left(V_\mu^\Omega + X_\mu, 0\right)$ in the Minkowski space

$$\begin{aligned} L_{an} = \frac{\Lambda^2}{4\pi^2} tr X_\mu^2 - \frac{1}{12\pi^2} tr\{\frac{1}{4}\left(V_{\mu\nu}^\Omega\right)^2 + X_\mu V_{\mu\nu}^\Omega X_\nu - \\ \frac{1}{2}\left[D_\mu^\Omega, X_\mu\right]^2 - \frac{1}{4}\left[X_\mu, X_\nu\right]^2 + \left(X_\mu^2\right)^2\} + \frac{1}{48\pi^2} tr\left(V^\Omega + X\right)_{\mu\nu}^2 \end{aligned} \tag{23}$$

where D^Ω contains the field V_μ^Ω, while

$$V_{\mu\nu}^\Omega = C_{\mu\nu}\hat{n} + \frac{1}{4}\left[\partial_\mu\hat{n}, \partial_\nu\hat{n}\right]$$

is the field strength of V_μ^Ω.

The Yang-Mills Lagrangian for $V_\mu = V_\mu^\Omega + X_\mu$ is given by

$$L_{YM} = L_{YM}^\Omega + \frac{1}{2g^2} tr\{\left(D_\mu^\Omega X_\nu - D_\nu^\Omega X_\mu\right)^2 + \left[X_\mu, X_\nu\right]^2 + 2V_{\mu\nu}^\Omega\left[X_\mu, X_\nu\right]\} \tag{24}$$

The effective $SU(2)$ gluonic Lagrangian in variables (V_μ^Ω, X_ν) is

$$L = L_{an} + L_{YM} = L_{YM}^{\Omega} + \frac{\Lambda^2}{4\pi^2} tr(\partial_\mu \hat{n})^2 +$$

$$T + P + (\frac{1}{24\pi^2} + \frac{1}{g^2}) tr\{V_{\mu\nu}^{\Omega} [X_\mu, X_\nu]\} \tag{25}$$

where T is the kinetic term for X_μ and $(-P)$ is the potential

$$T = (\frac{1}{48\pi^2} + \frac{1}{2g^2}) tr\{\left(D_\mu^\Omega X_\nu - D_\nu^\Omega X_\mu\right)^2\} + \frac{1}{24\pi^2} tr\left[D_\mu^\Omega, X_\mu\right]^2 \tag{26}$$

$$P = \frac{\Lambda^2}{4\pi^2} tr X_\mu^2 - \frac{1}{12\pi^2} tr(X_\mu^2)^2 + (\frac{1}{24\pi^2} + \frac{1}{2g^2}) tr\left[X_\mu, X_\nu\right]^2 \tag{27}$$

We do not calculate the kinetic term $tr(\partial_\mu \hat{n})^2$, because it comes from next approximation [3]. It follows that $tr X_\mu^2$ can form a gauge invariant condensate $\left(tr X_\mu^2\right)_0 = -g^2\sigma/2$ as a minimum of $-P$, and a mass appears. Denote a hermitian vacuum field by ϕ_μ^a, so that $\sigma = (\phi_\mu^a \phi_\mu^a)$ and $X_\mu = (\phi_\mu^a + Y_\mu^a)\tau_a/2i$. Then $\sigma = -9\Lambda^2/(7g^2 + 48\pi^2)$ and $m_Y^2 = -\frac{1}{3}\sigma b_3$, where $b_3 = (g^2 + 12\pi^2)/24\pi^2 g^2$. The condensate σ is negative, and the vacuum field ϕ_μ^a is space-like.

The last term in T should be analysed together with the gauge condition for V_μ. In the flavor case, the term $(-P)$ without $tr\{(X_\mu^2)^2$ corresponds to the Skyrme Lagrangian. The lagrangian for the n-field [4] is contained in the first two terms of L.

The Lagrangian of [5] is $L_{YM}(V^\Omega + X)$. New terms are contained in L_{an}; they are partly built on structures already existing in L_{YM}, but with different coefficients. Note, that an expression $tr\left[D_\mu^\Omega, X_\mu\right]^2$ in color L_{an} looks as a standard gauge condition term, while in the flavor case $[D_\mu A_\mu]^2$ leads to ghosts. Last term in T and first two terms in P are quite new; they are specific for bosonization. To get more insight into meaning of L_{an} we need to assume a definite representation for X_μ. Investigation of the effective Lagrangian is the next step of bosonization approach.

5 Discussion

We have studied the case of SU(2) color dynamics in the complete color space including the color Chiral Anomaly, when not only color degrees of freedom of gluons, but also color chiral degrees of freedom of quarks are taken into account. Usually it is admitted, that total number of gluonic color degrees is the same as the number of gluonic plus quark chiral degrees, so that finally the role of chiral degrees is to introduce topological structures, but not additional degrees. From this viewpoint, there are different ways to investigate dynamics in the complete color space according to different ways to incorporate topological properties in gluonic variables. Use of chiral anomaly is one of such ways.The central role of the chiral anomaly in mass generation was recently emphasized [16].

In order to develop color dynamics in presence of the color chiral field U of the Anomaly, one should know, how it is related to gluonic field V_μ. We have shown, how to express chirally rotated fields V_μ^U and A_μ^U in terms of the decomposition components of gluonic field V_μ, and, consequently, how to write down the Anomaly as a function of induced axial vector field A_μ^U. Also, we have shown that, the definition of bosonization action should be changed compared with the flavor Anomaly case, because the chiral field does not introduce new variables.

It was shown that in the SU(2) color case, generation of mass of axial vector field A_μ^U and formation of bilinear condensate $\langle A^U A^U \rangle$ is due to potential term in bosonization action. A necessity to have a bilinear condensate in the infrared QCD was find in [17]. To get a gauge invariant bilinear condensate of the gauge field requires a special treatment [18]. In the color bosonization approach the bilinear condensate is composed of axial vector components and is gauge invariant by construction.

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