Relativistic description of heavy tetraquarks

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Abstract

The masses of heavy tetraquarks with hidden charm and bottom are calculated in the framework of the relativistic quark model. The tetraquark is considered as the bound state of a heavy-light diquark and antidiquark. The light quark in a heavy-light diquark is treated completely relativistically. The internal structure of the diquark is taken into account by calculating the diquark-gluon form factor in terms of the diquark wave functions. New experimental data on charmonium-like states above open charm threshold are discussed. The obtained results indicate that the $X(3872)$ can be the tetraquark state with hidden charm. The masses of ground state tetraquarks with hidden bottom are found to be below the open bottom threshold.

Recently, significant progress in experimental investigations of charmonium spectroscopy has been achieved. Several new states $X(3872)$, $Z(3931)$, $Y(3943)$, $X(3943)$, $Y(4260)$ were observed [1] which provide challenge to the theory, since not all of them can be easily accommodated as the $c\bar{c}$-charmonium. The most natural conjecture is that these states are the multiquark composite systems considered long ago e.g. in [2]. Currently the best established state is the narrow $X(3872)$ which was originally discovered in B decays [3, 4] and later confirmed in $p\bar{p}$ collisions [5, 6]. Its mass and observed decays, which favour $J^{PC} = 1^{++}$ assignment, make a $c\bar{c}$ interpretation problematic [7]. Different theoretical interpretations of the $X(3872)$ state were put forward which use the near proximity of its mass to the $D^0\bar{D}^{*0}$ threshold. The most popular ones are: the $D^0 - \bar{D}^{*0}$ molecular state bound by pion and quark exchanges [8]; an s-wave cusp at $D^0\bar{D}^{*0}$ threshold [9] and the diquark-antidiquark $[cq][\bar{c}\bar{q}]$ tetraquark state [10] $(q = u, d)$.

Maiani et al. [10] in the framework of the phenomenological constituent quark model considered the masses of hidden charm diquark-antidiquark states in terms of the constituent diquark mass and spin-spin interactions. They identified the $X(3872)$ with the S-wave bound state of a spin one and spin zero diquark and antidiquark with the symmetric diquark-spin distribution $([cq]_{S=1}[\bar{c}\bar{q}]_{S=0} + [cq]_{S=0}[\bar{c}\bar{q}]_{S=1})$ and used its mass to fix the constituent diquark mass. Spin-spin couplings were fixed from the analysis of the observed meson and baryon masses. On this basis they predicted the existence of a 2^{++} state $[cq]_{S=1}[\bar{c}\bar{q}]_{S=1}$ that can be associated to the $Y(3943)$. They also argued [11] that $Y(4260)$ could be the first orbital excitation of the charm-strange diquark-antidiquark state $([cs]_{S=0}[\bar{c}\bar{s}]_{S=0})_{P-\text{wave}}$. In Ref. [12] it is pointed out that non-leptonic B decays provide a favourable environment for the production of hidden charm diquark-antidiquark bound states. In contrast it is argued [13] that the observed $X(3872)$ production in B decays and in high-energy $p\bar{p}$ collisions is too large for a loosely bound molecule (with binding energy of 1 MeV or less).

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In this paper we use the relativistic quark model [14, 15] based on the quasipotential approach to calculate the mass spectra of tetraquarks with hidden charm and bottom as the heavy-light diquark-antidiquark bound states ($[Qq][\bar{Q}\bar{q}]$, $Q = c, b$). Recently we considered the mass spectra of doubly heavy (QQq) [16] and heavy (qqQ) [17] baryons in the heavy-diquark–light-quark and light-diquark–heavy-quark approximations, respectively. The light quarks and light diquarks were treated completely relativistically. The internal structure of the light and heavy diquarks was taken into account by calculating diquark-gluon form factors on the basis of the determined diquark wave functions. The found good agreement [17] with available experimental data gives additional motivation for considering diquarks as reasonable building blocks of hadrons. It is important to note that all parameters of our model were determined from the previous considerations of meson mass spectra and decays, and we will keep them fixed in the following analysis of heavy tetraquarks.

In the quasipotential approach and diquark-antidiquark picture of heavy tetraquarks the interaction of two quarks in a diquark and the heavy diquark-antidiquark interaction in a tetraquark are described by the diquark wave function (Ψ_d) of the bound quark-quark state and by the tetraquark wave function (Ψ_T) of the bound diquark-antidiquark state respectively, which satisfy the quasipotential equation of the Schrödinger type [14]

$$\left(\frac{b^2(M)}{2\mu_R} - \frac{\mathbf{p}^2}{2\mu_R}\right)\Psi_{d,T}(\mathbf{p}) = \int \frac{d^3q}{(2\pi)^3} V(\mathbf{p},\mathbf{q};M)\Psi_{d,T}(\mathbf{q}), \tag{1}$$

where the relativistic reduced mass is

$$\mu_R = \frac{E_1 E_2}{E_1 + E_2} = \frac{M^4 - (m_1^2 - m_2^2)^2}{4M^3}, \tag{2}$$

and E_1, E_2 are given by

$$E_1 = \frac{M^2 - m_2^2 + m_1^2}{2M}, \quad E_2 = \frac{M^2 - m_1^2 + m_2^2}{2M}, \tag{3}$$

here $M = E_1 + E_2$ is the bound state mass (diquark or tetraquark), $m_{1,2}$ are the masses of quarks (q_1 and q_2) which form the diquark or of the diquark (d) and antiquark (d') which form the heavy tetraquark (T), and $\mathbf{p}$ is their relative momentum. In the center of mass system the relative momentum squared on mass shell reads

$$b^2(M) = \frac{[M^2 - (m_1 + m_2)^2][M^2 - (m_1 - m_2)^2]}{4M^2}. \tag{4}$$

The kernel $V(\mathbf{p},\mathbf{q};M)$ in Eq. (1) is the quasipotential operator of the quark-quark or diquark-antidiquark interaction. It is constructed with the help of the off-mass-shell scattering amplitude, projected onto the positive energy states. In the following analysis we closely follow the similar construction of the quark-antiquark interaction in mesons which were extensively studied in our relativistic quark model [14, 15]. For the quark-quark interaction in a diquark we use the relation $V_{qq} = V_{q\bar{q}}/2$ arising under the assumption about the octet structure of the interaction from the difference of the qq and $q\bar{q}$ colour states. An important role in this construction is played by the Lorentz-structure of the confining interaction. In our analysis of mesons while constructing the quasipotential of the quark-antiquark interaction, we adopted that the effective interaction is the sum of the usual one-gluon exchange term with the mixture of long-range vector and scalar linear confining potentials, where the vector confining potential contains the Pauli terms. We use the same conventions for the construction of the quark-quark and diquark-antidiquark interactions in the tetraquark. The quasipotential is then defined as follows [16, 15]

(a) for the quark-quark (Qq) interaction

$$V(\mathbf{p},\mathbf{q};M) = \bar{u}_1(p)\bar{u}_2(-p)\mathcal{V}(\mathbf{p},\mathbf{q};M)u_1(q)u_2(-q), \tag{5}$$

with

$$\mathcal{V}(\mathbf{p},\mathbf{q};M)=\frac{1}{2}\left[\frac{4}{3}\alpha_s D_{\mu\nu}(\mathbf{k})\gamma_1^{\mu}\gamma_2^{\nu}+V^V_{\rm conf}(\mathbf{k})\Gamma_1^{\mu}(\mathbf{k})\Gamma_{2;\mu}(-\mathbf{k})+V^S_{\rm conf}(\mathbf{k})\right],$$

Here α_s is the QCD coupling constant, $D_{\mu\nu}$ is the gluon propagator in the Coulomb gauge

$$D^{00}(\mathbf{k})=-\frac{4\pi}{\mathbf{k}^2},\quad D^{ij}(\mathbf{k})=-\frac{4\pi}{k^2}\left(\delta^{ij}-\frac{k^ik^j}{\mathbf{k}^2}\right),\quad D^{0i}=D^{i0}=0, \tag{6}$$

and $\mathbf{k}=\mathbf{p}-\mathbf{q}$; γ_μ and $u(p)$ are the Dirac matrices and spinors

$$u^\lambda(p)=\sqrt{\frac{\epsilon(p)+m}{2\epsilon(p)}}\begin{pmatrix}1\\ \dfrac{\boldsymbol{\sigma}\mathbf{p}}{\epsilon(p)+m}\end{pmatrix}\chi^\lambda, \tag{7}$$

with $\epsilon(p)=\sqrt{\mathbf{p}^2+m^2}$.

The effective long-range vector vertex of the quark is defined [15] by

$$\Gamma_\mu(\mathbf{k})=\gamma_\mu+\frac{i\kappa}{2m}\sigma_{\mu\nu}\tilde{k}^\nu,\qquad \tilde{k}=(0,\mathbf{k}), \tag{8}$$

where κ is the Pauli interaction constant characterizing the anomalous chromomagnetic moment of quarks. In the configuration space the vector and scalar confining potentials in the nonrelativistic limit reduce to

$$\begin{aligned} V^V_{\rm conf}(r) &= (1-\varepsilon)V_{\rm conf}(r),\\ V^S_{\rm conf}(r) &= \varepsilon V_{\rm conf}(r), \end{aligned} \tag{9}$$

with

$$V_{\rm conf}(r)=V^S_{\rm conf}(r)+V^V_{\rm conf}(r)=Ar+B, \tag{10}$$

where ε is the mixing coefficient.

(b) for diquark-antidiquark ($d\bar{d}'$) interaction

$$\begin{aligned} V(\mathbf{p},\mathbf{q};M) = {} & \frac{\langle d(P)|J_\mu|d(Q)\rangle}{2\sqrt{E_dE_d}}\frac{4}{3}\alpha_s D^{\mu\nu}(\mathbf{k})\frac{\langle d'(P')|J_\nu|d'(Q')\rangle}{2\sqrt{E_{d'}E_{d'}}}\\ & +\psi^*_d(P)\psi^*_{d'}(P')\left[J_{d;\mu}J^{\mu}_{d'}V^V_{\rm conf}(\mathbf{k})+V^S_{\rm conf}(\mathbf{k})\right]\psi_d(Q)\psi_{d'}(Q'), \end{aligned} \tag{11}$$

where $\langle d(P)|J_\mu|d(Q)\rangle$ is the vertex of the diquark-gluon interaction which takes into account the finite size of the diquark and is discussed below $\Big[P^{(\prime)}=(E_{d^{(\prime)}},\pm\mathbf{p})$ and $Q^{(\prime)}=(E_{d^{(\prime)}},\pm\mathbf{q})$, $E_d=(M^2-M_{d'}^2+M_d^2)/(2M)$ and $E_{d'}=(M^2-M_d^2+M_{d'}^2)/(2M)\Big]$.

The diquark state in the confining part of the diquark-antidiquark quasipotential (11) is described by the wave functions

$$\psi_d(p)=\begin{cases}1 & \text{for scalar diquark}\\ \varepsilon_d(p) & \text{for axial vector diquark}\end{cases}, \tag{12}$$

where the four vector

$$\varepsilon_d(p)=\left(\frac{(\boldsymbol{\varepsilon}_d\mathbf{p})}{M_d},\boldsymbol{\varepsilon}_d+\frac{(\boldsymbol{\varepsilon}_d\mathbf{p})\mathbf{p}}{M_d(E_d(p)+M_d)}\right),\qquad \varepsilon^\mu_d(p)p_\mu=0, \tag{13}$$

is the polarization vector of the axial vector diquark with momentum $\mathbf{p}$, $E_d(p)=\sqrt{\mathbf{p}^2+M_d^2}$ and $\varepsilon_d(0)=(0,\boldsymbol{\varepsilon}_d)$ is the polarization vector in the diquark rest frame. The effective long-range vector vertex of the diquark can be presented in the form

$$J_{d;\mu}=\begin{cases}\dfrac{(P+Q)_\mu}{2\sqrt{E_dE_d}} & \text{for scalar diquark}\\ -\dfrac{(P+Q)_\mu}{2\sqrt{E_dE_d}}+\dfrac{i\mu_d}{2M_d}\Sigma^\nu_\mu\tilde{k}_\nu & \text{for axial vector diquark}\end{cases}, \tag{14}$$

Table 1: Masses M and form factor parameters of heavy-light diquarks. S and A denote scalar and axial vector diquarks antisymmetric $[Q,q]$ and symmetric $\{Q,q\}$ in flavour, respectively.

Quark content	Diquark type	$Q=c$ M (MeV)	ξ (GeV)	ζ (GeV^2)	$Q=b$ M (MeV)	ξ (GeV)	ζ (GeV^2)
$[Q,q]$	S	1973	2.55	0.63	5359	6.10	0.55
$\{Q,q\}$	A	2036	2.51	0.45	5381	6.05	0.35
$[Q,s]$	S	2091	2.15	1.05	5462	5.70	0.35
$\{Q,s\}$	A	2158	2.12	0.99	5482	5.65	0.27

where $\tilde{k}=(0,\mathbf{k})$. Here the antisymmetric tensor

$$(\Sigma_{\rho\sigma})^{\nu}_{\mu}=-i(g_{\mu\rho}\delta^{\nu}_{\sigma}-g_{\mu\sigma}\delta^{\nu}_{\rho}) \tag{15}$$

and the axial vector diquark spin $\mathbf{S}_d$ is given by $(S_{d;k})_{il}=-i\varepsilon_{kil}$; μ_d is the total chromomagnetic moment of the axial vector diquark.

The constituent quark masses $m_b=4.88$ GeV, $m_c=1.55$ GeV, $m_u=m_d=0.33$ GeV, $m_s=0.5$ GeV and the parameters of the linear potential $A=0.18$ GeV^2 and $B=-0.3$ GeV have the usual values of quark models. The value of the mixing coefficient of vector and scalar confining potentials $\varepsilon=-1$ has been determined from the consideration of charmonium radiative decays [14] and the heavy quark expansion [18]. The universal Pauli interaction constant $\kappa=-1$ has been fixed from the analysis of the fine splitting of heavy quarkonia ^3P_J- states [14]. In this case the long-range chromomagnetic interaction of quarks vanishes in accord with the flux tube model. Finally, we choose the total chromomagnetic moment of the axial vector diquark $\mu_d=0$. Such a choice appears to be natural, since the long-range chromomagnetic interaction of diquarks proportional to μ_d then also vanishes in accord with the flux tube model.

At a first step, we calculate the masses and form factors of the heavy-light diquarks. As it is well known, the light quarks are highly relativistic, which makes the v/c expansion inapplicable and thus, a completely relativistic treatment is required. To achieve this goal in describing heavy-light diquarks, we closely follow our recent consideration of the spectra of light diquarks in heavy baryons and adopt the same procedure to make the relativistic quark potential local by replacing $\epsilon_{1,2}(p)\equiv\sqrt{m_{1,2}^2+\mathbf{p}^2}\to E_{1,2}$ (see discussion in Ref. [19]). The resulting light-quark–heavy-quark interaction potential is the same as the light quark-quark interaction [17] and is equal to 1/2 of the $Q\bar{q}$ interaction in the heavy-light meson.[1] We solve numerically the quasipotential equation with this complete relativistic potential which depends on the diquark mass in a complicated highly nonlinear way. The obtained ground-state masses of scalar and axial vector heavy-light diquarks are presented in Table 1.

In order to determine the diquark interaction with the gluon field, which takes into account the diquark structure, it is necessary to calculate the corresponding matrix element of the quark current between diquark states. This diagonal matrix element can be parameterized by the following set of elastic form factors

(a) scalar diquark (S)

$$\langle S(P)|J_\mu|S(Q)\rangle=h_+(k^2)(P+Q)_\mu, \tag{16}$$

(b) axial vector diquark (A)

$$\langle A(P)|J_\mu|A(Q)\rangle \;=\; -[\varepsilon_d^*(P)\cdot\varepsilon_d(Q)]h_1(k^2)(P+Q)_\mu$$

[1] The masses of the ground state heavy-light mesons are well reproduced with this $Q\bar{q}$ potential.

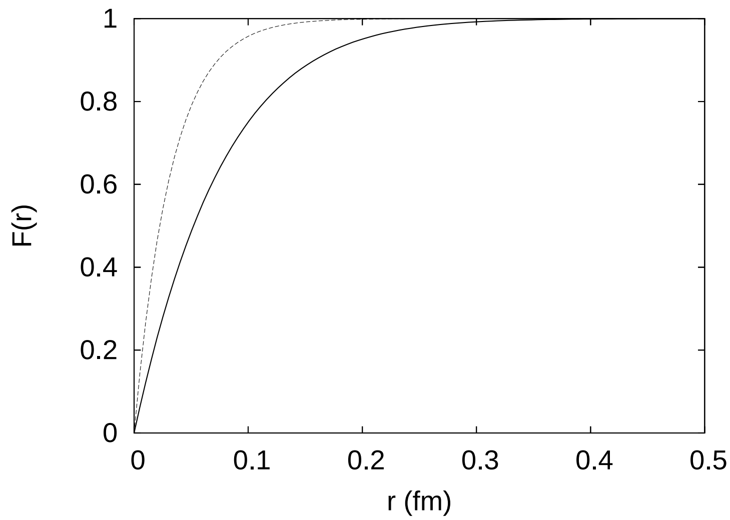

Figure 1: The form factors $F(r)$ for $\{c,q\}$ (solid line) and $\{b,q\}$ (dashed line) axial vector diquarks.

$$
\begin{aligned}
&+h_2(k^2)\left\{[\varepsilon_d^*(P)\cdot Q]\varepsilon_{d;\mu}(Q)+[\varepsilon_d(Q)\cdot P]\varepsilon^*_{d;\mu}(P)\right\}\\
&+h_3(k^2)\frac{1}{M_A^2}[\varepsilon_d^*(P)\cdot Q][\varepsilon_d(Q)\cdot P](P+Q)_\mu,
\end{aligned}
\tag{17}
$$

where $k=P-Q$ and $\varepsilon_d(P)$ is the polarization vector of the axial vector diquark (13).

Using the quasipotential approach with the impulse approximation for the vertex function of the quark-gluon interaction we find [17]

$$
\begin{aligned}
h_+(k^2) &= h_1(k^2)=h_2(k^2)=F(\mathbf{k}^2),\\
h_3(k^2) &= 0,
\end{aligned}
$$

$$
\begin{aligned}
F(\mathbf{k}^2) &= \frac{\sqrt{E_dM_d}}{E_d+M_d}\int\frac{d^3p}{(2\pi)^3}\bar\Psi_d\left(\mathbf{p}+\frac{2\epsilon_2(p)}{E_d+M_d}\mathbf{k}\right)\sqrt{\frac{\epsilon_1(p)+m_1}{\epsilon_1(p+k)+m_1}}\left[\frac{\epsilon_1(p+k)+\epsilon_1(p)}{2\sqrt{\epsilon_1(p+k)\epsilon_1(p)}}\right.\\
&\left.+\frac{\mathbf{pk}}{2\sqrt{\epsilon_1(p+k)\epsilon_1(p)}(\epsilon_1(p)+m_1)}\right]\Psi_d(\mathbf{p})+(1\leftrightarrow 2),
\end{aligned}
\tag{18}
$$

where Ψ_d are the diquark wave functions. We calculated the corresponding form factors $F(r)/r$ which are the Fourier transforms of $F(\mathbf{k}^2)/\mathbf{k}^2$ using the diquark wave functions found by numerical solving the quasipotential equation. Our estimates show that this form factor can be approximated with a high accuracy by the expression

$$
F(r)=1-e^{-\xi r-\zeta r^2},
\tag{19}
$$

which agrees with previously used approximations [16]. The values of parameters ξ and ζ for heavy-light scalar diquark $[Q,q]$ and axial vector diquark $\{Q,q\}$ ground states are given in Table 1. In Fig. 1 we plot the functions $F(r)$ for $\{Q,q\}$ axial vector diquarks.

At a second step, we calculate the masses of heavy tetraquarks considered as the bound state of a heavy-light diquark and antidiquark. For the potential of the heavy diquark-antidiquark interaction (11) we get

$$
V(r) = V_{\rm Coul}(r)+V_{\rm conf}(r)+\frac{1}{E_1E_2}\Bigg\{\mathbf{p}\left[V_{\rm Coul}(r)+V^V_{\rm conf}(r)\right]\mathbf{p}-\frac14\Delta V^V_{\rm conf}(r)+V'_{\rm Coul}(r)\frac{\mathbf{L}^2}{2r}
$$

Table 2: Masses of charm diquark-antidiquark states (in MeV). S and A denote scalar and axial vector diquarks.

State J^{PC}	Diquark content	Mass $cq\bar{c}\bar{q}$	$cs\bar{c}\bar{s}$	$cs\bar{c}\bar{q}/cq\bar{c}\bar{s}$
$1S$				
0^{++}	$S\bar{S}$	3812	4051	3922
$1^{+\pm}$	$(S\bar{A}\pm\bar{S}A)/\sqrt{2}$	3871	4113	3982
0^{++}	$A\bar{A}$	3852	4110	3967
1^{+-}	$A\bar{A}$	3890	4143	4004
2^{++}	$A\bar{A}$	3968	4209	4080
$1P$				
1^{--}	$S\bar{S}$	4244	4466	4350

Table 3: Masses of bottom diquark-antidiquark states (in MeV). S and A denote scalar and axial vector diquarks.

State J^{PC}	Diquark content	Mass $bq\bar{b}\bar{q}$	$bs\bar{b}\bar{s}$	$bs\bar{b}\bar{q}/bq\bar{b}\bar{s}$
$1S$				
0^{++}	$S\bar{S}$	10471	10662	10572
$1^{+\pm}$	$(S\bar{A}\pm\bar{S}A)/\sqrt{2}$	10492	10682	10593
0^{++}	$A\bar{A}$	10473	10671	10584
1^{+-}	$A\bar{A}$	10494	10686	10599
2^{++}	$A\bar{A}$	10534	10716	10628
$1P$				
1^{--}	$S\bar{S}$	10807	11002	10907

$$
\begin{aligned}
&+\frac{1}{r}\left[V'_{\rm Coul}(r)+\frac{\mu_d}{4}\left(\frac{E_1}{M_1}+\frac{E_2}{M_2}\right)V'^V_{\rm conf}(r)\right]\mathbf{L}(\mathbf{S}_1+\mathbf{S}_2)\\
&+\frac{\mu_d}{4}\left(\frac{E_1}{M_1}-\frac{E_2}{M_2}\right)\frac{V'^V_{\rm conf}(r)}{r}\mathbf{L}(\mathbf{S}_1-\mathbf{S}_2)\\
&+\frac{1}{3}\left[\frac{1}{r}V'_{\rm Coul}(r)-V''_{\rm Coul}(r)+\frac{\mu_d^2}{4}\frac{E_1E_2}{M_1M_2}\left(\frac{1}{r}V'^V_{\rm conf}(r)-V''^V_{\rm conf}(r)\right)\right]\left[\frac{3}{r^2}(\mathbf{S}_1\mathbf{r})(\mathbf{S}_2\mathbf{r})-\mathbf{S}_1\mathbf{S}_2\right]\\
&+\frac{2}{3}\left[\Delta V_{\rm Coul}(r)+\frac{\mu_d^2}{4}\frac{E_1E_2}{M_1M_2}\Delta V^V_{\rm conf}(r)\right]\mathbf{S}_1\mathbf{S}_2\Bigg\},
\end{aligned}
\tag{20}
$$

where

$$V_{\rm Coul}(r)=-\frac{4}{3}\alpha_s\frac{F_1(r)F_2(r)}{r}$$

is the Coulomb-like one-gluon exchange potential which takes into account the finite sizes of the diquark and antidiquark through corresponding form factors $F_{1,2}(r)$. Here $\mathbf{S}_{1,2}$ and $\mathbf{L}$ are spin operators of the diaquark and antidiquark and their orbital momentum.

The diquark-antidiquark model of heavy tetraquarks predicts [10, 12] the existence of a flavour $SU(3)$ nonet of states with hidden charm or beauty ($Q=c,b$): four tetraquarks ($[Qq][\bar{Q}\bar{q}]$, $q=u,d$) with neither open or hidden strangeness, which have electric charges 0 or ±1 and isospin 0 or 1; four tetraquarks ($[Qs][\bar{Q}\bar{q}]$ and $[Qq][\bar{Q}\bar{s}]$, $q=u,d$) with open strangeness ($S=\pm1$), which have electric charges 0 or ±1 and isospin $\frac{1}{2}$; one tetraquark ($[Qs][\bar{Q}\bar{s}]$) with

Table 4: Thresholds for open charm decays and nearby hidden-charm thresholds.

Channel	Threshold (MeV)	Channel	Threshold (MeV)	Channel	Threshold (MeV)
$D^0\bar{D}^0$	3729.4	$D_s^+D_s^-$	3936.2	$D^0D_s^\pm$	3832.9
D^+D^-	3738.8	$\eta' J/\psi$	4054.7	$D^\pm D_s^\mp$	3837.7
$D^0\bar{D}^{*0}$	3871.3	$D_s^\pm D_s^{*\mp}$	4080.0	$D^{*0}D_s^\pm$	3975.0
$\rho J/\psi$	3872.7	$\phi J/\psi$	4116.4	$D^0D_s^{*\pm}$	3976.7
$D^\pm D^{*\mp}$	3879.5	$D_s^{*+}D_s^{*-}$	4223.8	$K^{*\pm}J/\psi$	3988.6
$\omega J/\psi$	3879.6			$K^{*0}J/\psi$	3993.0
$D^{*0}\bar{D}^{*0}$	4013.6			$D^{*0}D_s^{*\pm}$	4118.8

Table 5: Thresholds for open bottom decays.

Channel	Threshold (MeV)	Channel	Threshold (MeV)	Channel	Threshold (MeV)
$B\bar{B}$	10558	$B_s^+B_s^-$	10739	BB_s	10649
$B\bar{B}^*$	10604	$B_s^\pm B_s^{*\mp}$	10786	B^*B_s	10695
$B^*\bar{B}^*$	10650	$B_s^{*+}B_s^{*-}$	10833	$B^*B_s^*$	10742

hidden strangeness and zero electric charge. Since in our model we neglect the mass difference of u and d quarks and electromagnetic interactions, corresponding tetraquarks will be degenerate in mass. A more detailed analysis [10] predicts that such mass differences can be of a few MeV so that the isospin invariance is broken for the $[Qq][\bar{Q}\bar{q}]$ mass eigenstates and thus in their strong decays. The (non)observation of such states will be a crucial test of the tetraquark model.

The calculated heavy tetraquark masses are presented in Tables 2 and 3. The corresponding open charm and bottom thresholds are given in Tables 4 and 5. We find that all S-wave tetraquarks with hidden bottom lie considerably below open bottom thresholds and thus they should be narrow states which can be observed experimentally. This prediction significantly differs from the molecular picture [8] where bound $B - \bar{B}^*$ states are expected to lie very close (only few MeV below) to the corresponding thresholds.

The situation in the hidden charm sector is considerably more complicated, since most of the tetraquark states are predicted to lie either above or only slightly below corresponding open charm thresholds. This difference is the consequence of the fact that the charm quark mass is substantially smaller than the bottom quark mass. As a result the binding energies in the charm sector are significantly smaller than those in the bottom sector.

In Table 6 and Fig. 2 we compare our results for the charm diquark-antidiquark bound states with the predictions of Ref. [10]. The differences in some of the mass values can be attributed to the substantial distinctions in the used approaches. We describe the diquarks as quark-quark bound systems dynamically and calculate their masses, while in Ref. [10] they are treated only phenomenologically. Then we consider the diquark-antidiquark interaction and the tetraqark as purely the diquark-antidiquark composite system. In distinction Maini et al. consider a hyperfine interaction between all quarks which, e.g., causes the splitting of 1^{++} and 1^{+-} states arising from the SA diquark-antidiquark compositions. From Table 6, where we also give possible experimental candidates for the neutral tetraquarks with hidden charm, we see that our calculation supports the assumption of Ref. [10] that $X(3872)$ can be the axial vector 1^{++} tetraquark state composed from scalar and axial vector diquark and antidiquark in the relative S-wave state. On the other hand, in our model the lightest scalar 0^{++} tetraquark is predicted to be above the open charm threshold $D\bar{D}$ and thus to be broad, while in the model

Table 6: Comparison of theoretical predictions for the masses of charm diquark-antidiquark states $cq\bar{c}\bar{q}$ (in MeV) and possible experimental candidates.

State	Theory		Experiment [1, 3, 4, 6, 5, 20, 21, 22, 23]	
J^{PC}	this work	[10]	state	mass
$1S$				
0^{++}	3812	3723		
1^{++}	3871	$3872^{\dagger}$	$X(3872)$	3871.2 ± 0.4
1^{+-}	3871	3754		
0^{++}	3852	3832		
1^{+-}	3890	3882		
2^{++}	3968	3952	$Y(3943)$	$3943 \pm 11 \pm 13$
$1P$				
1^{--}	4244		$Y(4260)$	$4259 \pm 8^{+2}_{-6}$

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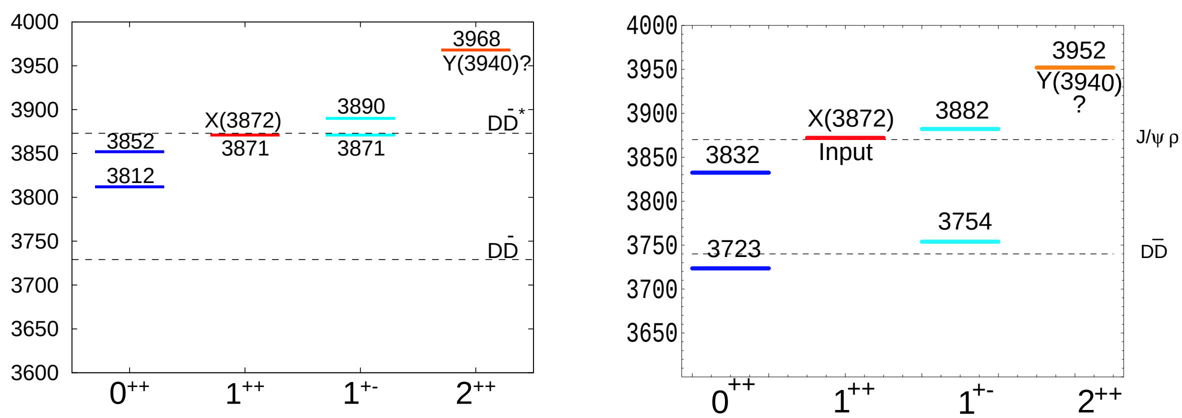

our

Maiani et al. [10]

Figure 2: Comparison of theoretical predictions for charmed tetraquarks.

of Ref. [10] it lies a few MeV below this threshold, and thus is predicted to be narrow. Our 2^{++} tetraquark also lies higher than the one in Ref. [10], thus making the interpretation of this state as $Y(3943)$ less probable. We also find that $Y(4260)$ cannot be interpreted as P-wave 1^{--} state of charm-strange diquark-antidiquark, since its mass is found to be ~ 200 MeV heavier (see Table 2). A more natural tetraquark interpretation could be the P-wave $([cq]_{S=0}[\bar{c}\bar{q}]_{S=0})_{P-\mathrm{wave}}$ state which mass is predicted in our model to be close to the mass of $Y(4260)$ (see Table 6). Then the dominant decay mode of $Y(4260)$ would be in $D\bar{D}$ pairs.

In summary, we calculated the masses of heavy tetraquarks with hidden charm and bottom in the diquark-antidiquark picture. In contrast to previous phenomenological treatments we used the dynamical approach based on the relativistic quark model. Both diquark and tetraquark masses were obtained by numerical solution of the quasipotential equation with the corresponding relativistic potentials. The diquark size was also taken into account with the help of the diquark-gluon form factor in terms of diquark wave functions. It is important to emphasize that, in our analysis, we did not introduce any free adjustable parameters but used their fixed values from our previous considerations of heavy and light meson properties. It was found that the $X(3872)$ can be the neutral charm tetraquark state. If it is really a tetraquark, one more neutral and two charged tetraquark states must exist with close masses. The ground states of bottom tetraquarks are predicted to have masses below the open bottom threshold and thus should be narrow. The (non)observation of these states will be an important test of the tetraquark model. This work was supported in part by the *Deutsche Forschungsgemeinschaft* under contract Eb 139/2-3 and by the *Russian Foundation for Basic Research* under Grant No.05-02-16243.

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