Total resummaion of leading logarithms vs standard description of the Polarized DIS

B. I. Ermolaev[a*], M. Greco[b], S.I. Troyan[c]

[a] *Ioffe Physico-Technical Institute, 194021 St.Petersburg, Russia*

[b] *Department of Physics and INFN, University Rome III, Rome, Italy*

[c] *St.Petersburg Institute of Nuclear Physics, 188300 Gatchina, Russia*

Abstract

Total resummation of leading logarithms of x contributing to the spin-dependent structure function g_1 ensures its steep rise at small x. DGLAP lacks such a resummation. Instead, it is complemented by special phenomenological fits for the initial parton densities. The singular factors $x^{-\alpha}$ in the fits mimic the resummation and also ensure the steep (power-like) rise of g_1 at the small-x region. Furthermore, DGLAP by definition cannot describe the region of small Q^2 whereas our approach can do it.

1 Introduction

The Standard Approach (SA) for theoretical investigation of DIS structure function $g_1(x, Q^2)$ involves DGLAP[1] and Standard fits[2] for the initial parton densities. In the SA framework, g_1^{DGLAP} is a convolution of the coefficient functions C_{DGLAP} and evolved (with respect to Q^2) parton distributions which are also expressed as a convolution of the splitting functions P_{DGLAP} and initial parton densities. The latter are found from experimental data at large x, $x \sim 1$ and $Q^2 \sim 1$ GeV^2. As a result, SA accounts for the Q^2 -evolution through the DGLAP evolution equations whereas the x -evolution is accounted for through the fits which are found from phenomenological considerations. The reason for such asymmetric treating the Q^2 and x -evolutions in SA is that DGLAP was originally constructed for operating at large x where x- contributions from higher loops were small and could be neglected. In other words, the x -evolution can be neglected at large x. However, in the small-x region the situation looks opposite: logarithms of x are becoming quite sizable and should be accounted to all orders in α_s. The total resummation of leading logarithms of x was first done in Refs. [3] in the double-logarithmic approximation so that α_s was kept fixed at an unknown scale and later in Refs. [4] where the running α_s effects were accounted for. Contrary to DGLAP where

$$\alpha_s^{DGLAP} = \alpha_s(Q^2), \tag{1}$$

Ref. [4] used the parametrization of α_s suggested in Ref. [5] because the DGLAP parametrization of Eq. (1) cannot be used at small x. The parametrization of Ref. [5] is universally good for both small x and large x. It converge to the DGLAP- parametrization at large x but differs from it at small x.

Nevertheless, it is known that, despite DGLAP lacks the total resummation of $\ln x$, it successfully operates at $x \ll 1$. As a result, the common opinion was formed that not only the total resummation of DL contributions in Refs. [3] but also the much more accurate calculations performed in Refs. [4] should be out of use at available x and might be of some importance in

*e-mail: ermolaev@mail.cern.ch

a distant future at extremely small x. In Ref. [6] we argued against such a point of view and explained why SA can be so successful at small x: in order to be able to describe the available experimental data, SA uses the singular fits of Refs. [2] for the initial parton densities. Singular factors x^{-a} in the fits mimic the total resummaton of Refs. [4]. Using the results of Ref [4] allows to simplify the rather sophisticated structure of the standard fits.

Another essential difference between SA and our description of g_1 is the obvious fact that DGLAP works at the kinematic regions of large Q^2 whereas our approach is valid for large and small Q^2. The latter is important in particular for theoretical explanation of the COMPASS collaboration results. In Ref. [7] we showed that g_1 practically does not depend on x at small x, even at $x \ll 1$. Instead, it depends on the total invariant energy $2pq$. Experimental investigation of this dependence is extremely interesting because according to our results g_1, being positive at small $2pq$, can turn negative at greater values of this variable. The position of the turning point is sensitive to the ratio between the initial quark and gluon densities, so its experimental detection would enable to estimate this ratio.

2 Difference between DGLAP and our approach

In DGLAP, g_1 is expressed through convolutions of the coefficient functions and evolved parton distributions. As convolutions look simpler in terms of integral transforms, it is convenient to represent g_1 in the form of the Mellin integral. For example, the non-singlet component of g_1 can be represented as follows:

$$g_1^{NS}{}_{DGLAP}(x,Q^2) = (e_q^2/2)\int_{-\imath\infty}^{\imath\infty}\frac{d\omega}{2\imath\pi}(1/x)^{\omega}C_{DGLAP}(\omega)\delta q(\omega)\exp\Big[\int_{\mu^2}^{Q^2}\frac{dk_\perp^2}{k_\perp^2}\gamma_{DGLAP}(\omega,\alpha_s(k_\perp^2))\Big] \tag{2}$$

with $C_{DGLAP}(\omega)$ being the non-singlet coefficient functions, $\gamma_{DGLAP}(\omega,\alpha_s)$ the non-singlet anomalous dimensions and $\delta q(\omega)$ the initial non-singlet quark densities in the Mellin (momentum) space. The expression for the singlet g_1 is similar, though more involved. Both γ_{DGLAP} and C_{DGLAP} are known in first two orders of the perturbative QCD. Technically, it is simpler to calculate them at integer values of $\omega = n$. In this case, the integrand of Eq. (2) is called the n-th momentum of g_1^{NS}. When the moments for different n are known, g^{NS} at arbitrary values of ω is obtained with interpolation of the moments. Expressions for the initial quark densities are defined from phenomenological consideration, with fitting experimental data at $x \sim 1$. Eq. (2) shows that γ_{DGLAP} govern the Q^2- evolution whereas C_{DGLAP} evolve $\delta q(\omega)$ in the x-space from $x \sim 1$ into the small$x$ region. When, at the x-space, the initial parton distributions $\delta q(x)$ are regular in x, i.e. do not $\to \infty$ when $x \to 0$, the small-x asymptotics of $g_1{}_{DGLAP}$ is given by the well-known expression:

$$g_1^{NS}{}_{DGLAP},\quad g_1^{S}{}_{DGLAP} \sim \exp\Big[\sqrt{\ln(1/x)\ln\Big(\ln(Q^2/\mu^2)/\ln(\mu^2/\Lambda_{QCD}^2)\Big)}\,\Big]. \tag{3}$$

On the contrary, when the total resummation of the double-logarithms (DL) and single- logarithms of x is done[5], the Mellin representation for g_1^{NS} is

$$g_1^{NS}(x,Q^2) = (e_q^2/2)\int_{-\imath\infty}^{\imath\infty}\frac{d\omega}{2\pi\imath}(1/x)^{\omega}C_{NS}(\omega)\delta q(\omega)\exp\big(H_{NS}(\omega)\ln(Q^2/\mu^2)\big)\ , \tag{4}$$

with new coefficient functions C_{NS},

$$C_{NS}(\omega) = \frac{\omega}{\omega - H_{NS}^{(\pm)}(\omega)} \tag{5}$$

and anomalous dimensions H_{NS},

$$H_{NS} = (1/2)\Big[\omega - \sqrt{\omega^2 - B(\omega)}\Big] \tag{6}$$

where

$$B(\omega) = (4\pi C_F(1+\omega/2)A(\omega) + D(\omega))/(2\pi^2)\ . \tag{7}$$

$D(\omega)$ and $A(\omega)$ in Eq. (7) are expressed in terms of $\rho = \ln(1/x)$, $\eta = \ln(\mu^2/\Lambda^2_{QCD})$, $b = (33-2n_f)/12\pi$ and the color factors $C_F = 4/3$, $N = 3$:

$$D(\omega) = \frac{2C_F}{b^2 N}\int_0^\infty d\rho e^{-\omega\rho}\ln\big(\frac{\rho+\eta}{\eta}\big)\Big[\frac{\rho+\eta}{(\rho+\eta)^2+\pi^2} \mp \frac{1}{\eta}\Big]\ , \tag{8}$$

$$A(\omega) = \frac{1}{b}\Big[\frac{\eta}{\eta^2+\pi^2} - \int_0^\infty \frac{d\rho e^{-\omega\rho}}{(\rho+\eta)^2+\pi^2}\Big]. \tag{9}$$

H_S and C_{NS} account for DL and SL contributions to all orders in α_s. When $x \to 0$,

$$g_1^{NS} \sim \big(x^2/Q^2\big)^{\Delta_{NS}/2},\ g_1^S \sim \big(x^2/Q^2\big)^{\Delta_S/2} \tag{10}$$

where the non-singlet and singlet intercepts are $\Delta_{NS} = 0.42$, $\Delta_S = 0.86$. The x- behavior of Eq. (10) is much steeper than the one of Eq. (3). Obviously, the total resummation of logarithms of x leads to the faster growth of g_1 when x decreasing compared to the one predicted by DGLAP, providing the input δq in Eq. (2) is a regular function of ω at $\omega \to 0$.

3 Structure of the standard DGLAP fits

Although there are different fits for $\delta q(x)$ in literature, all available fits include both regular and singular factors when $x \to 0$ (see e.g. Refs. [**?**] for detail). For example, the typical expression is

$$\delta q(x) = N\eta x^{-\alpha}\Big[(1-x)^\beta(1+\gamma x^\delta)\Big], \tag{11}$$

with N, η being a normalization, $\alpha = 0.576$, $\beta = 2.67$, $\gamma = 34.36$ and $\delta = 0.75$. In the ω -space Eq. (11) is a sum of pole contributions:

$$\delta q(\omega) = N\eta\Big[(\omega-\alpha)^{-1} + \sum m_k(\omega+\lambda_k)^{-1}\Big], \tag{12}$$

with $\lambda_k > 0$, so that the first term in Eq. (12) corresponds to the singular factor $x^{-\alpha}$ of Eq. (11). When the fit Eq. (11) is substituted in Eq. (2), the singular factor $x^{-\alpha}$ affects the small -x behavior of g_1 and changes its asymptotics Eq. (3) for g_1 for the Regge asymptotics. Indeed, the small- x asymptotics is governed by the leading singularity $\omega = \alpha$, so

$$g_{1\ DGLAP} \sim C(\alpha)(1/x)^\alpha\Big((\ln(Q^2/\Lambda^2))/(\ln(\mu^2/\Lambda^2))\Big)^{\gamma(\alpha)}. \tag{13}$$

Obviously, the actual DGLAP asympotics of Eq. (13) is of the Regge type, it differs a lot from the conventional DGLAP asympotics of Eq. (3) and looks similar to our asymptotics given by Eq. (10): incorporating the singular factors into DGLAP fits ensures the steep rise of g_1^{DGLAP} at small x and thereby leads to the success of DGLAP at small x. Ref. [6] demonstrates that without the singular factor $x^{-\alpha}$ in the fit of Eq. (11), DGLAP would not be able to operate successfully at $x \le 0.05$. In other words, the singular factors in DGLAP fits mimic the total resummation of logarithms of x of Eqs. (4,10). Although both (13) and (10) predict the Regge asymptotics for g_1, there is a certain difference between them: Eq. (13) predicts that the intercept of g_1^{NS} should be $\alpha = 0.57$. As α is greater than the non-singlet intercept $\Delta_{NS} = 0.42$, the non-singlet g_1^{DGLAP} grows, when $x \to 0$, faster than our predictions. However, such a rise

is too steep. It contradicts the results obtained in Refs. [4] and confirmed by several groups fitting HERMES data. Usually, the DGLAP equations for the non-singlets are written in the x-space as convolutions of splitting functions P_{qq} with evolved parton distributions Δq and the latter are written as another convolution:

$$\Delta q(x) = C_q(x, y) \otimes \delta q(y), \tag{14}$$

with C_q being the coefficient function. Written in this way, Δq is sometimes believed to be less singular than δq because of the evolution. However applying the Mellin transform to Eq. (14) immediately disproves it.

4 g_1 at small x and small Q^2

The COMPASS experiment measures the singlet g_1 at $x \sim 10^{-3}$ and $Q^2 \ll 1$ GeV^2, i.e. in the kinematic region where it is impossible to use DGLAP. Although formulae for singlet and non-singlet g_1 are different, with formulae for the singlet being much more complicated, we can explain the essence of our approach, using Eq. **??**gnsint) for the non-singlet.

In the COMPASS experiment $Q^2 \ll \mu^2$. The expression for g_1 at such small Q^2 with logarithmic accuracy is given by Eq. (4) where the Q^2 -dependence is dropped and x is replaced by $\mu^2/2pq$, so

$$g_1^{NS}(x, Q^2 \lesssim \mu^2) = (e_q^2/2) \int_{-\imath\infty}^{\imath\infty} \frac{d\omega}{2\pi\imath} (\mu^2/2pq)^\omega C_{NS}^q(\omega)\delta q(\omega) \ . \tag{15}$$

The expression for the g_1 singlet looks similar, though e_q^2 should be replaced by the averaged charge $< e_q^2 >$ and $C_{NS}(\omega)\delta q(\omega)$ should be replaced by the sum

$$C_S^q(\omega)\delta q(\omega) + C_S^g(\omega)\delta g(\omega) \tag{16}$$

so that $\delta q(\omega)$ and $\delta g(\omega)$ are the initial quark and gluon densities respectively and $C_S^{q,g}$ are the singlet coefficient functions. Explicit expressions for $C_S^{q,g}$ are given in Ref. [7]. The standard fits for δq and δg contain singular factors $\sim x^{-a}$ which mimic the total resummation of leading logarithms of x. Such a resummation leads to the expressions for the coefficient functions different from the DGLAP ones. After that the singular factors in the fits can be dropped and the initial parton densities can be approximated by constants:

$$\delta q \approx N_q \quad \delta g \approx N_g \ , \tag{17}$$

so, one can write

$$g_1(Q^2 \ll \mu^2) \approx (< e_q^2 > /2) N_q G_1(z) \tag{18}$$

with

$$G_1 = \int_{-\imath\infty}^{\imath\infty} \frac{d\omega}{2\pi\imath} (1/z)^\omega \left[C_S^q + (N_g/N_q C_g) \right] \tag{19}$$

where $z = \mu^2/2pq$. Obviously, G_1 depends on the ratio N_g/N_q. The results for different values of the ratio $r = N_g/N_q$, G_1 are plotted in Fig. 1. When the gluon density is neglected, i.e. $N_g = 0$ (curve 1), G_1 being positive at $x \sim 1$, is getting negative very soon, at $z < 0.5$ and falls fast with decreasing z. When $N_g/N_q = -5$ (curve 2), G_1 remains positive and not large until $z \sim 10^{-1}$, turns negative at $z \sim 0.03$ and falls afterwards rapidly with decreasing z . This turning point where G_1 changes its sign is very sensitive to the magnitude of the ratio r . For instance, at $N_g/N_q = -8$ (curve 3), G_1 passes through zero at $z \sim 10^{-3}$. When $N_g/N_q < -10$, G_1 is positive at any experimentally reachable z (curve 4) . Therefore, the experimental measurement of the turning point would allow to draw conclusions on the interplay between the initial quark and gluon densities.

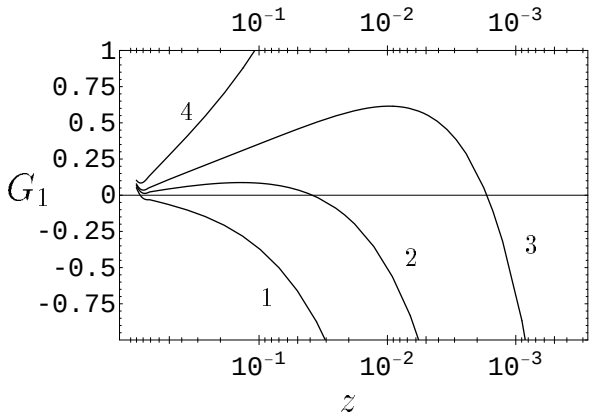

Figure 1: G_1 evolution with decreasing $z = \mu^2/2(pq)$ for different values of ratio $r = \delta g/\delta q$: curve 1 - for $r = 0$, curve 2 - for $r = -5$, curve 3 -for $r = -8$ and curve 4 -for $r = -15$.

5 Conclusion

Comparison of Eqs. (3) and (13) shows explicitly that the singular factor $x^{-\alpha}$ in the Eq. (11) for the initial quark density converts the exponential DGLAP-asympotics into the Regge one. On the other hand, comparison of Eqs. (10) and (13) demonstrates that the singular factors in the DGLAP fits mimic the total resummation of logarithms of x. These factors can be dropped when the total resummation of logarithms of x performed in Ref. [4] is taken into account. The remaining, regular x-terms of the DGLAP fits (the terms in squared brackets in Eq. (11)) can obviously be simplified or even dropped at small x so that the rather complicated DGLAP fits can be replaced by constants. It immediately leads to an interesting conclusion: the DGLAP fits for δq have been commonly believed to represent non-perturbative QCD effects but they actually mimic the contributions of the perturbative QCD, so the whole impact of the non-perturbative QCD on g_1 at small x is not large and can be approximated by normalization constants. We have used the latter for studying the g_1 singlet at small Q^2 because this kinematic is presently investigated by the COMPASS collaboration. It turns out that g_1 in this region depends on $z = \mu^2/2pq$ only and practically does not depend on x. Numerical calculations show that the sign of g_1 is positive at z close to 1 and can remain positive or become negative at smaller z, depending on the ratio between δg and δq. It is plotted in Fig. 1 for different values of $\delta g/\delta q$. Fig. 1 demonstrates that the position of the sign change point is sensitive to the ratio $\delta g/\delta q$, so the experimental measurement of this point would enable to estimate the impact of δg.

References

[1] G. Altarelli and G. Parisi, Nucl. Phys.B126 (1977) 297; V.N. Gribov and L.N. Lipatov, Sov. J. Nucl. Phys. 15 (1972) 438; L.N.Lipatov, Sov. J. Nucl. Phys. 20 (1972) 95; Yu.L. Dokshitzer, Sov. Phys. JETP 46 (1977) 641.

[2] G. Altarelli, R.D. Ball, S. Forte and G. Ridolfi, Nucl. Phys. B496 (1997) 337; Acta Phys. Polon. B29(1998)1145; E. Leader, A.V. Sidorov and D.B. Stamenov. Phys. Rev. D73 (2006) 034023; J. Blumlein, H. Botcher. Nucl. Phys. B636 (2002) 225; M. Hirai at al. Phys. Rev. D69 (2004) 054021.

[3] B.I. Ermolaev, S.I. Manaenkov and M.G. Ryskin. Z.Phys.C69(1996)259; J. Bartels, B.I. Ermolaev and M.G. Ryskin. Z.Phys.C 70(1996)273; J. Bartels, B.I. Ermolaev and M.G. Ryskin. Z.Phys.C 72(1996)627.

[4] B.I. Ermolaev, M. Greco and S.I. Troyan. Nucl.Phys.B 594 (2001)71; ibid 571(2000)137; Phys.Lett.B579(321),2004.

[5] B.I. Ermolaev, M. Greco and S.I. Troyan. Phys.Lett.B 522(2001)57.

[6] B.I. Ermolaev, M. Greco and S.I. Troyan. Phys. Lett. B622 (2005) 93 (hep-ph/0503019).

[7] B.I. Ermolaev, M. Greco, S.I. Troyan. hep-ph/0605133.