Generalized Haag's theorem in $SO(1,1)$ and $SO(1,3)$ invariant quantum field theory

M. Chaichian[a*], M. N. Mnatsakanova [b†], A. Tureanu[a‡], Yu. S. Vernov[c§]

[a] *High Energy Physics Division, Department of Physical Sciences, University of Helsinki*
and
Helsinki Institute of Physics
P.O. Box 64, Helsinki, Finland

[b] *Skobeltsyn Institute of Nuclear Physics, Moscow State University*
Moscow 119992, Russia

[c] *Institute for Nuclear Research of Russian Academy of Sciences*
INR, 60-the October Anniversary prospect 7a, Moscow, 117312, Russia

Abstract

One of the most important results of the axiomatic quantum field theory - the generalized Haag theorem - is proven in $SO(1,1)$ invariant quantum field theory, of which an important example is noncommutative quantum field theory. In $SO(1,3)$ invariant theory new consequences of generalized Haag's theorem are obtained: it is proven that the equality of four-point Wightman functions in two theories leads to the equality of elastic scattering amplitudes and thus total cross-sections in these theories.

1 Introduction

Quantum field theory (QFT) as a mathematically rigorous and consistent theory was formulated in the framework of the axiomatic approach in the works of Wightman, Jost, Bogolyubov, Haag and others ([1] - [5]).

Within the framework of this theory, on the basis of most general principles such as Poincaré invariance, local commutativity and spectrality, a number of fundamental physical results, for example, the CPT-theorem and the spin-statistics theorem were proven, analytical properties of scattering amplitudes in energy and angular variables were established and a number of rigorous bounds on high energy behavior of such amplitudes were obtained [1] - [3], [6]. Haag's theorem [7, 8] (see also [1, 3]) is one of the most novel results in the axiomatic approach in quantum field theory (QFT). During the recent years the different generalizations of the standard QFT have been widely considered.

Noncommutative quantum field theory (NC QFT) is one of these generalizations of standard QFT which has been intensively developed during the past years (for reviews, see [9, 10]). The idea of such a generalization of QFT ascends to Heisenberg and it was initially developed in Snyders work [11]. The present development in this direction is connected with the construction of noncommutative geometry [12] and new physical arguments in favour of such a generalization of QFT [13]. Essential interest in NC QFT is also due to the fact that in some cases it is obtained as a low-energy limit of string theory [14]. The simplest and at the same time most studied

* **e-mail**:chaichia@cc.helsinki.fi
† **e-mail**: mnatsak@theory.sinp.msu.ru
‡ **e-mail**:atureanu@cc.helsinki.fi
§ **e-mail**: vernov@ms2.inr.ac.ru

version of noncommutative field theory is based on the following Heisenberg-like commutation relations between coordinates:

$$[\hat{x}_\mu, \hat{x}_\nu] = i\,\theta_{\mu\nu}, \tag{1}$$

where $\theta_{\mu\nu}$ is a constant antisymmetric matrix.

It is known that the construction of NC QFT in a general case ($\theta_{0i} \neq 0$) meets serious difficulties with unitarity and causality [16] - [19]. For this reason the version with $\theta_{0i} = 0$ (space-space noncommutativity), in which there are no such difficulties and which is a low-energy limit of the string theory, draws special attention. Then always there is a system of coordinates, in which only $\theta_{12} = -\theta_{21} \neq 0$. Thus, when $\theta_{0i} = 0$, without loss of generality it is possible to choose the coordinates x_0 and x_3 as commutative and the coordinates x_1 and x_2 as noncommutative ones.

The relation (1) breaks the Lorentz invariance of the theory, while the symmetry under the $SO\,(1,1) \otimes SO\,(2)$ subgroup of the Lorentz group survives [18]. Translational invariance is still valid. Remark that when $\theta_{0i} = 0$, the group of symmetry of the theory is $O\,(1,1) \otimes SO\,(2)$. However, in order to obtain the results of this paper it is sufficient to use only the weaker requirement of $SO\,(1,1)$ invariance, though we consider the case when $\theta_{0i} = 0$. Below we shall consider the theory to be $SO\,(1,1)$ invariant with respect to the coordinates x_0 and x_3. Besides these classical groups of symmetry, in the paper [20] it was shown that the noncommutative field theory with the commutation relation (1) of the coordinates, and built according to the Weyl-Moyal correspondence, has also a quantum symmetry, namely the twisted Poincaré invariance.

In the works [21] - [23], in which $\theta_{0i} = 0$, the Wightman approach was formulated for NC QFT. For scalar fields the CPT theorem and the spin-statistics theorem were proven.

In [21] it was proposed that Wightman functions in the noncommutative case can be written down in the standard form

$$W\,(x_1, \ldots, x_n) = \langle\, \Psi_0, \varphi(x_1) \ldots \varphi(x_n)\, \Psi_0\, \rangle, \tag{2}$$

where Ψ_0 is the vacuum state. However, unlike the commutative case, these Wightman functions are only $SO\,(1,1) \otimes SO\,(2)$ invariant.

In [22] it was proposed that in the noncommutative case the usual product of operators in the Wightman functions be replaced by the Moyal-type product (see also [10]). Such a product of operators is compatible with the twisted Poincaré invariance of the theory [24] and also reflects the natural physical assumption, that noncommutativity should change the product of operators not only in coinciding points, but also in different ones. This follows also from another interpretation of NC QFT in terms of a quantum shift operator [25]. In [23] it was shown that for the derivation of some axiomatic results, the concrete type of product of operators in various points is insignificant. It is essential only that from the appropriate spectral condition (see formula (7)), the analyticity of Wightman functions with respect to the commutative variables x^0 and x^3 follows, while x^1 and x^2 do not need to be complexified. The Wightman functions can be written down as follows [23]:

$$W\,(x_1, \ldots, x_n) = \langle \Psi_0, \varphi\,(x_1)\,\tilde{\star} \cdots \tilde{\star}\, \varphi\,(x_n)\Psi_0 \rangle. \tag{3}$$

The meaning of $\tilde{\star}$ depends on the considered case, and in particular

$$\varphi\,(x)\tilde{\star}\varphi\,(y) = \varphi\,(x)\varphi\,(y) \text{ according to [21]} \qquad \text{or}$$

$$\varphi\,(x)\tilde{\star}\varphi\,(y) = \varphi\,(x) \exp\left(\frac{i}{2}\,\theta_{\mu\nu}\,\frac{\overleftarrow{\partial}}{\partial x_\mu}\,\frac{\overrightarrow{\partial}}{\partial y_\nu}\right)\varphi\,(y) \text{ according to [22].}$$

In [23] it was shown that, besides the above-mentioned theorems, in NC QFT with ($\theta_{0i} = 0$) a number of other classical results of the axiomatic theory, such as the equivalence of local commutativity condition and symmetry of Wightman functions with respect to the rearrangement of the arguments in the domain of analyticity, remain valid. In [24] on the basis of the twisted Poincaré invariance of the theory the Haag's theorem was obtained [7, 8] (see also [1] and references in it).

In the present work, we consider generalized Haag's theorem (see Theorem 4.17 in [1] or Theorem 5.4.2 in [3]) in both $SO(1,3)$ and $SO(1,1)$ invariant theories. An important example of the latter case is the noncommutative theory. In the $SO(1,3)$ invariant theory new consequences of Haag's theorem are found, without analogues in NC QFT.

The analysis of Haag's theorem reveals essential distinctions between commutative and noncommutative cases, more precisely between the $SO(1,3)$ and $SO(1,1)$ invariant theories.

At the same time it is proven that the basic physical conclusion of Haag's theorem is valid also in the $SO(1,1)$ invariant theory. Namely, if we consider two theories, in which field operators are connected by a unitary condition at equal times, and one of these theories is a trivial one, that is the corresponding S-matrix is equal to unity, the other is trivial as well. To derive this result it is sufficient to assume that spectrality, local commutativity condition and translational invariance are fulfilled only for the transformations concerning the commutative coordinates.

In the commutative case, the conditions which relate the field operators by a unitary transformation at equal times (see Section 3), whose consequence is Haag's theorem, lead to the equality of Wightman functions in the two theories up to four-point ones. In the present paper it is shown that in the $SO(1,1)$ invariant theory, unlike the commutative case, only two-point Wightman functions are equal. It is proven that from the equality of four-point Wightman functions in the two theories, the equality of their elastic scattering amplitudes follows and, owing to the optical theorem, the equality of total cross sections. In derivation of this result local coomutativity condition (LCC) is not used.

Note that actually field operators are the smoothed operators

$$\varphi_f \equiv \int \varphi(x)\, f(x)\, dx, \tag{4}$$

where $f(x)$ are test functions.

Taking $\theta_{0i} = 0$, we consider a frame in which noncommutativity does not affect the variables x_0 and x_3 and it is possible to assume that after smearing over noncommutative variables the Wightman functions are standard generalized functions (tempered distribution) with respect to the commuting coordinates, which leads to the necessary analytical properties of Wightman functions in these variables. It is natural to assume, as it is done in various versions of nonlocal theory [26] - [28], that with respect to the noncommutative variables Wightman functions belong to one of Gel'fand-Shilov spaces [29]. The concrete choice of the Gel'fand-Shilov space is not important in the derivation of our results.

As the consideration given below for formal operator $\varphi(x)$ coincides with the consideration for operator φ_f, in order not to complicate formulas, we shall deal with the field operators as if they were given at a point.

2 Basic Properties of Wightman Functions

In Wightman's approach cyclicity of the vacuum vector is assumed. In the noncommutative case this means that any vector of the space under consideration J can be approximated with arbitrary accuracy by vectors of the type

$$\varphi(x_1)\,\tilde{\star}\,\varphi(x_2)\,\tilde{\star}...\tilde{\star}\,\varphi(x_n)\,\Psi_0. \tag{5}$$

For simplicity we consider the case of a real field, however, the results are easily extended to a complex field. For reasons given below, it is important only that the scalar product of any two vectors $\langle\,\Phi, \Psi\,\rangle$ can be approximated by linear combination of Wightman functions $W\,(x_1, \ldots, x_n)$ with arbitrary precision. For the results obtained below, translational invariance only in commuting coordinates is essential, therefore we write down the Wightman functions as:

$$W\,(x_1, \ldots, x_n) = W\,(\xi_1, \ldots, \xi_{n-1}, X), \tag{6}$$

where X designates the set of noncommutative variables $x_i^1, x_i^2,\ i = 1, \ldots n$, and $\xi_i = \{\xi_i^0, \xi_i^3\}$, where $\xi_i^0 = x_i^0 - x_{i+1}^0, \xi_i^3 = x_i^3 - x_{i+1}^3$.

Let us formulate now the spectral condition. We assume that complete vector system in p space consists only of time-like vectors with respect to momentum components P_n^0 and P_n^3, i.e. that

$$P_n^0 \geq |P_n^3|. \tag{7}$$

The condition (7) is conveniently written as $P_n \in \bar{V}_2^+$, where $\bar{V}_2^+$ is the set of the vectors satisfying the condition $x^0 \geq |x^3|$. Recall that the usual spectral condition is $P_n \in \bar{V}^+$, i.e. $P_n^0 \geq |\vec{P}_n|$. From the condition (7) and the completeness of the system of basis vectors Ψ_{P_n}:

$$\langle\Phi, \Psi\rangle = \sum_n \int d\,P_n \langle\Phi, \Psi_{P_n}\rangle\langle\Psi_{P_n}, \Psi\rangle$$

it follows that

$$\int d\,a\,e^{-i\,p\,a}\,\langle\Phi, U\,(a)\,\Psi\rangle = 0, \quad \text{if} \quad p \notin \bar{V}_2^+, \tag{8}$$

where $a = \{a^0, a^3\}$ is a two-dimensional vector, $U\,(a)$ is a translation in the plane p^0, p^3, and Φ and Ψ are arbitrary vectors. The equality (8) is similar to the corresponding equality in the standard case ([1], Chap. 2.6). The direct consequence of the equality (8) is the spectral property of Wightman functions:

$$W\,(P_1..., P_{n-1}, X) = \frac{1}{(2\pi)^{n-1}} \int e^{i\,P_j\,\xi_j}\,W\,(\xi_1..., \xi_{n-1}, X)\,d\,\xi_1...d\,\xi_{n-1} = 0, \tag{9}$$

if $P_j \notin \bar{V}_2^+$. The proof of the equality (9) is similar to the proof of the spectral condition in the commutative case [1], [3]. Recall that in the latter case the equality (9) is valid, if $P_j \notin \bar{V}^+$. Having written down $W\,(\xi_1, \ldots, \xi_{n-1}, X)$ as

$$W\,(\xi_1..., \xi_{n-1}, X) = \frac{1}{(2\pi)^{n-1}} \int e^{-i\,P_j\,\xi_j}\,W\,(P_1..., P_{n-1}, X)\,d\,P_1...d\,P_{n-1}, \tag{10}$$

we obtain that, due to the condition (9), $W\,(\nu_1, \ldots, \nu_{n-1}, X)$ is analytical in the "tube" T_n^-:

$$\nu_i \in T_n^-, \quad \text{if} \quad \nu_i = \xi_i - i\,\eta_i,\ \eta_i \in V_2^+,\ \eta_i = \{\eta_i^0, \eta_i^3\}. \tag{11}$$

It should be stressed that the noncommutative coordinates $x_i^1,\ x_i^2$ are always real.

The $SO\,(1,1)$ invariance of the theory allows to expand the domain of analyticity. This expansion is similar to the transition from tubes to expanded tubes in the commutative case [1] - [3]. According to the Bargmann-Hall-Wightman theorem, $W\,(\nu_1, \ldots, \nu_{n-1}, X)$ is analytical in the domain T_n

$$T_n = \cup_{\Lambda_c}\,\Lambda_c\,T_n^-, \tag{12}$$

where $\Lambda_c \in S_c\,O\,(1,1)$ is the two-dimensional analogue of the complex Lorentz group. Just as in the commutative case, the expanded domain of analyticity contains real points x_i, which are noncommutative Jost points, satisfying the condition $x_i \sim x_j$, which means that

$$\left(x_i^0 - x_j^0\right)^2 - \left(x_i^3 - x_j^3\right)^2 < 0, \quad \forall\, i, j. \tag{13}$$

It should be emphasized that the noncommutative Jost points are a subset of the set of Jost points of the commutative case, when

$$(x_i - x_j)^2 < 0 \qquad \forall\, i, j. \tag{14}$$

LCC has the same form as in the local theory at the Jost points (recall that we consider a scalar field):

$$W\,(x_1, \ldots, x_i, x_{i+1}, \ldots, x_n) = W\,(x_1, \ldots, x_{i+1}, x_i, \ldots, x_n), \tag{15}$$

However, in (15) x_i are noncommutative Jost points, i.e. they satisfy the condition (13).

3 Generalized Haags Theorem

Recall the formulation of the generalized Haags theorem in the commutative case ([1], Theorem 4.17):

Let $\varphi_1\,(t, \vec{x})$ and $\varphi_2\,(t, \vec{x})$ be two irreducible sets of operators, for which the vacuum vectors Ψ_0^1 and Ψ_0^2 are cyclic. Further, let the corresponding Wightman functions be analytical in the domain T_n[1]*.*
Then the two-, three- and four-point Wightman functions coincide in the two theories if there is a unitary operator V, such that

$$1) \quad \varphi_2\,(t, \vec{x}) = V\,\varphi_1\,(t, \vec{x})\,V^*, \tag{16}$$

$$2) \qquad \Psi_0^2 = C\,V\,\Psi_0^1, \quad C \in \mathbb{C}, \quad |C| = 1. \tag{17}$$

It should be emphasized that actually the condition 2) is a consequence of condition 1) with rather general assumptions (see **Statement**). In the formulation of Haags theorem it is assumed that the operators $\varphi_i\,(t, \vec{x})$ can be smeared only on the spatial variables. This assumption is natural if $\theta_{0i} = 0$.

Let us consider Haags theorem in the $SO(1,1)$ invariant field theory and show that the corresponding equality is true only for two-point Wightman functions.

For the proof we first note that in the noncommutative case, just as in the commutative one, from conditions 1) and 2) it follows that the Wightman functions in the two theories coincide at equal times

$$\langle \Psi_0^1, \varphi_1\,(t, \vec{x_1})\,\tilde{\star} \cdots \tilde{\star}\,\varphi_1\,(t, \vec{x_n})\,\Psi_0^1 \rangle = \langle \Psi_0^2, \varphi_2\,(t, \vec{x_1})\,\tilde{\star} \cdots \tilde{\star}\,\varphi_2\,(t, \vec{x_n})\,\Psi_0^2 \rangle. \tag{18}$$

Having written down the two-point Wightman functions $W_i\,(x_1, x_2),\ i = 1, 2$ as $W_i\,(u_1, v_1, u_2, v_2)$, where $u_i = \{x_i^0, x_i^3\},\ v_i = \{x_i^1, x_i^2\}$ we can write for them equality (18) as:

$$W_1\,(0, \xi^3, v_1, v_2) = W_2\,(0, \xi^3, v_1, v_2), \tag{19}$$

where $\xi = u_1 - u_2,\ v_1$ and v_2 are arbitrary vectors. Now we notice that, due to the $SO(1,1)$ invariance,

$$W_i\,(0, \xi^3, v_1, v_2) = W_i\,(\tilde{\xi}, v_1, v_2) \tag{20}$$

hence,

$$W_1\,(\tilde{\xi}, v_1, v_2) = W_2\,(\tilde{\xi}, v_1, v_2), \tag{21}$$

where $\tilde{\xi}$ is any Jost point. Due to the analyticity of the Wightman functions in the commuting variables they are completely determined by their values at the Jost points. Thus at any ξ from the equality (21), it follows that

$$W_1\,(\xi, v_1, v_2) = W_2\,(\xi, v_1, v_2). \tag{22}$$

[1]Remark that the required analyticity of the Wightman functions follows only from the spectral condition and the $SO(1,3)$ invariance of the theory.

As v_1 and v_2 are arbitrary, the formula (22) means the equality of two-point Wightman functions at all values of their arguments.

Thus, for the equality of the two-point Wightman functions in two theories related by the conditions (16) and (17), the $SO(1,1)$ invariance of the theory and corresponding spectral condition are sufficient.

It is impossible to extend this proof to three-point Wightman functions. Indeed, let us write down $W_i(x_1, x_2, x_3)$ as $W_i(u_1, u_2, u_3, v_1, v_2, v_3)$, where vectors u_i and v_i are determined as before. Equality (19) means that

$$W_1(0, \xi_1^3, 0, \xi_2^3, v_1, v_2, v_3) = W_2(0, \xi_1^3, 0, \xi_2^3, v_1, v_2, v_3), \tag{23}$$

v_1, v_2, v_3 are arbitrary. In order to have equality of the three-point Wightman functions in the two theories from the $SO(1,1)$ invariance, the existence of transformations $\Lambda \in SO(1,1)$ connecting the points $(0, \xi_1^3)$ and $(0, \xi_2^3)$ with an open vicinity of Jost points is necessary. It would be possible, if there existed two-dimensional vectors $\tilde{\xi}_1$ and $\tilde{\xi}_2$, $(\tilde{\xi}_i = \Lambda(0, \xi_i^3))$, satisfying the inequalities:

$$(\tilde{\xi}_1)^2 < 0, \quad (\tilde{\xi}_2)^2 < 0, \quad |(\tilde{\xi}_1, \tilde{\xi}_2)| < \sqrt{(\tilde{\xi}_1)^2 (\tilde{\xi}_2)^2}.$$

These inequalities are similar to the corresponding inequalities in the commutative case (see equation (4.87) in [1]). However, it is easy to check up, that the latter from these inequalities can not be fulfilled when the first two are fulfilled.

Let us show now that the condition (17) actually is a consequence of a condition (16).

Statement *Condition (17) is fulfilled, if the vacuum vectors* Ψ_0^i *are unique, normalized, translationally invariant vectors with respect to translations* $U_i(a)$ *along the axis* x_3*.*

It is easy to see that the operator $U_1^{-1}(a) V^{-1} U_2(a) V$ commutes with operators $\varphi_1(t, \vec{x})$ and, owing to the irreducibility of the set of these operators, it is proportional to the identity operator. Having considered the limit $a = 0$, we see that

$$U_1^{-1}(a) V^{-1} U_2(a) V = \mathbb{I}. \tag{24}$$

From the equality (24) it follows directly that if

$$U_1(a) \Psi_0^1 = \Psi_0^1, \tag{25}$$

then

$$U_2(a) V \Psi_0^1 = V \Psi_0^1, \tag{26}$$

i.e. the condition (17) is fulfilled. If the theory is translationally invariant in all variables, the equality (26) is true, if the vacuum vector is unique, normalized, translationally invariant in the spatial coordinates.

The most important consequence of the generalized Haag theorem is the following statement: if one of the two fields related by conditions (16) and (17) is a free field, the other is also free. In deriving this result the equality of the two-point Wightman functions in the two theories and LCC are used. In [24] it is proven that this result is valid also in the noncommutative theory, if $\theta_{0i} = 0$.

We show below that from the equality of the four-point Wightman functions for the fields $\varphi_1(x)$ and $\varphi_2(x)$, related by the conditions (16) and (17), which takes place in the commutative theory, an essential physical consequence follows. Namely, for such fields the elastic scattering amplitudes of the corresponding theories coincide, hence, due to the optical theorem, the total cross-sections coincide as well. In particular, if one of these fields, for example, $\varphi_1(x)$ is a trivial field, i.e. its corresponding S matrix is equal to unit, also the field $\varphi_2(x)$ is trivial. In the derivation of this result the local commutativity condition is not used. The statement follows directly from the Lehmann-Symanzik-Zimmermann reduction formulas [30].

Let $< p_3, p_4|p_1, p_2 >_i$, $i = 1, 2$ be an elastic scattering amplitudes for the fields $\varphi_1(x)$ and $\varphi_2(x)$ respectively. Owing to the reduction formulas,

$$< p_3, p_4|p_1, p_2 >_i \sim \int d\,x_1 \cdots d\,x_4\, e^{i\,(-p_1\,x_1 - p_2\,x_2 + p_3\,x_3 + p_4\,x_4)}\,.$$

$$\prod_{j=1}^{4} (\Box_j + m^2) \;< 0|T\,\varphi_i\,(x_1) \;\cdots\; \varphi_i\,(x_4)|0 >, \tag{27}$$

where $T\,\varphi_i\,(x_1) \;\cdots\; \varphi_i\,(x_4)$ is the chronological product of operators. From the equality

$$W_2\,(x_1, \ldots, x_4) = W_1\,(x_1, \ldots, x_4)$$

it follows that

$$< p_3, p_4|p_1, p_2 >_1 = < p_3, p_4|p_1, p_2 >_2 \tag{28}$$

at any p_i. Having applied this equality for the forward elastic scattering amplitudes, we obtain that, according to the optical theorem, the total cross-sections for the fields $\varphi_1(x)$ and $\varphi_2(x)$ coincide. If now the S-matrix for the field $\varphi_1(x)$ is unity, then it is also unity for the field $\varphi_2(x)$. We stress that the conclusions following from the equality of the four-point Wightman functions in the two theories are valid in the commutative field theory.

Let us proceed now to the $SO(1,1)$ symmetric theory. In this case, using the equality of the two-point Wightman functions in the two theories, we come to the conclusion that if LCC is fulfilled (15) and the current in one of the theories is equal to zero, for example, $j_1(x) = 0$, then $j_2(x) = 0$ as well. Recall that $j_i(x) = (\Box + m^2)\,\varphi_i(x)$. Indeed as $W_1\,(x_1, x_2) = W_2\,(x_1, x_2)$,

$$< 0|j_1\,(x_1) j_1\,(x_2)|0 > = < 0|j_2\,(x_1) j_2\,(x_2)|0 > = 0, \quad \forall\, x_1, x_2, \tag{29}$$

since $j_1\,(x) = 0$. Hence,

$$j_2\,(x)|0 > = 0.$$

In order to prove that the latter condition implies that $j_2\,(x) = 0$ we can use arguments similar with the ones in the standard case [22].

Let us point out that in the proof the spectral conditions and translational invariance only with respect to the commutating coordinates were used. If the theory is translationally invariant with respect to all variables and the standard spectral condition is fulfilled then if $\varphi_1(x)$ is a free field, $\varphi_2(x)$ is a free field as well [24].

4 Conclusions

It has been shown that in two $SO(1,1)$ invariant theories, whose field operators are connected by a unitary transformation at equal times, the two-point Wightman functions coincide. This leads to the conclusion that if one of the theories is a trivial one, then the other is trivial as well. In the derivation of this statement the assumptions of spectrality, local commutativity and translation invariance were used, which are weaker than standard.

In case two $SO(1,3)$ invariant theory are connected by the same condition new consequences of Haag's theorem have been obtained. It is proven that the equality of four-point Wightman functions, which is fulfilled only in $SO(1,3)$ invariant theory, leads to the equality of elastic scattering amplitudes and total cross-sections in the two theories.

Acknowledgements Yu.V. is supported by the grant of the President of the Russian Federation NS-7293.2006.2 (government contract 02.445.11.7370).

References

[1] R. F. Streater and A. S. Wightman, *PCT, Spin and Statistics and All That*, Benjamin, NewYork (1964).

[2] R. Jost, *The General Theory of Quantum Fields*, Amer. Math. Soc., Providence, R.I. (1965).

[3] N. N. Bogoliubov, A. A. Logunov, and I. T. Todorov, *Introduction to Axiomatic Quantum Field Theory*, Benjamin, Reading, Mass (1975).

[4] N. N. Bogoliubov, A. A. Logunov, A.I. Oksak and I. T. Todorov, *General Principles of Quantum Field Theory*, Kluwer, Dordrecht (1990).

[5] R. Haag, *Local Quantum Physics*, Springer, Berlin (1996).

[6] A. A. Logunov, M.A. Mestvirishvili, O.A. Khrustalev, *Nucl. and Part.* **3** 3 (1972) [in Russian].

[7] R.Haag, *Dan. Mat. Fys.* **29** 12 (1955).

[8] P.G. Federbush and K.A. Johnson, *Phys. Rev.* **120** 1926 (1960); R. Jost, *Properties of Wightman Functions*, edited by E.R. Caianiello, Lectures on Field Theory and Many-Body Problem, Academic Press, New Jork, (1961).

[9] M. R. Douglas and N. A. Nekrasov, *Rev. Mod. Phys.* **73** 977 (2001), hep-th/0106048.

[10] R. J. Szabo, *Phys. Rept.* **378** 207 (2003), hep-th/0109162.

[11] H. S. Snyder, *Phys. Rev.* **71** 38 (1947).

[12] A. Connes, *Noncommutative Geometry*, Academic Press, New York (1994).

[13] S. Doplicher, K. Fredenhagen and J. E. Roberts, *Phys. Lett. B* **331** 39 (1994); *Comm. Math. Phys.*, **172**, 187 (1995).

[14] N. Seiberg and E. Witten, *JHEP* **9909** 32 (1999), hep-th/9908142.

[15] L. Álvarez-Gaumé, J. L. F. Barbon and R. Zwicky, *JHEP* **0105** 057 (2001), hep-th/0103069.

[16] J. Gomis and T. Mehen, *Nucl. Phys. B* **591** 265 (2000), hep-th/0005129.

[17] N. Seiberg, L. Susskind and N. Toumbas, *JHEP* **0006** 044 (2000), hep-th/0005015.

[18] L. Álvarez-Gaumé and J. L. F. Barbon, *Int. J. Mod. Phys. A*, **16** 1123 (2001), hep-th/0006209.

[19] M. Chaichian, K. Nishijima and A. Tureanu, *Phys. Lett. B* **568** 146 (2003), hep-th/0209006.

[20] M. Chaichian, P.P. Kulish, K. Nishijima and A. Tureanu, *Phys. Lett.* **B 604** 98, hep-th/0408069.

[21] L. Álvarez-Gaumé and M. A. Vázquez-Mozo, *Nucl. Phys. B*, **668** 293 (2003), hep-th/0305093.

[22] M. Chaichian, M. N. Mnatsakanova, K. Nishijima, A. Tureanu and Yu. S. Vernov, hep-th/0402212.

[23] Yu.S. Vernov, M.N. Mnatsakanova, *Theor. Math. Phys.* **142** 337 (2005).

[24] M. Chaichian, P. Prešnajder and A. Tureanu, *Phys. Rev. Lett.*, **94** 151602 (2005), hep-th/0409096.

[25] M. Chaichian, K. Nishijima and A. Tureanu, *Phys. Lett.*, **B 633** 129 (2006), hep-th/0511094.

[26] W. Lücke, *J. Math. Phys* **27** 1901 (1985).

[27] M. A. Solov'ev, *Theor. Math.Phys.* **121** 1377 (1999).

[28] E. Brüning and S. Nagamachi, *J. Math. Phys.* **30** 2340 (1989).

[29] Gel'fand and Shilov, *Generalized Functions*, V.2, Chapter IV, Academic Press Inc., New York (1968).

[30] H. Lehmann, K. Symanzik, W. Zimmermann, *Nuovo Cim.* **1** 205 (1955); *Nuovo Cim.*, **6**, 319 (1957).

[31] A.S. Wightman, *J. Indian Math. Soc.*, **24** 625 (1960-61);
D. Ya. Petrina, *Ukr. Mat. Zh.* **13**, 109 (1961);
V.S. Vladimirov, *Sov. Math. Dokl.* **1** (1960) 1039; *Methods of the Theory of Functions of Several Complex Variables*, Cambridge, Massachusetts, MIT Press, 1966.