Ghost in the accelerating universe

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Abstract

It is know that Dvali-Gabadadze-Porrati braneworld model has self-acceleration branch. However, that cosmological solution contains a spin-2 ghost. We study the possibility of avoiding the appearance of the ghost by slightly modifying the model, introducing the second brane. First we consider a simple model without stabilization of the separation of the brane. By changing the separation between the branes, we find we can erase the spin-2 ghost. However, this can be done only at the expense of the appearance of a spin-0 ghost instead. We discuss why these two different types of ghosts are correlated. Then, we examine a model with stabilization of the brane separation. Even in this case, we find that the correlation between spin-0 and spin-2 ghosts remains. As a result we find we cannot avoid appearance of ghost by two-branes model. We also discuss whether spin-2 ghost is really harmful or not.

1 Introduction

Various recent cosmological observations, such as the Type-Ia supernovae, indicate the present-day accelerated expansion of the universe. One approach to this phenomenon is to introduce the dark energy which assists the accelerated expansion of the universe. One can say that this is a modification of the right hand side of the Einstein equations. As an alternative way, we can modify the left hand side of the Einstein equations, which means modification of the theory of gravity. To explain the accelerated expansion of the universe in this scheme, it will be naturally required to introduce the corresponding mass scale into the theory. A standard way is to introduce a scalar field which carries this mass scale. But an interesting alternative possibility is to give a mass to the graviton itself.

Here we consider the DGP model [1], which is a five-dimensional brane world model with the induced gravity term on the brane. This model admits a solution showing an accelerated expansion of the universe without introducing the dark energy [2]. By studying perturbations around this solution, the mass of graviton is found, in fact, to be non-zero.

On the Minkowski background the form of a covariant mass term is uniquely determined up to the quadratic order in the action by the requirement that the ghost is absent [3]. That form of mass term is known as the Fierz-Pauli mass term. The action generalized to a general background $g^{(0)}_{\mu\nu}$ is given by

$$S = \int d^4x\sqrt{-g}\left[R - \frac{m^2}{4}(h^{\mu\nu}h_{\mu\nu} - h^2) + O(h^3)\right], \tag{1}$$

where $h_{\mu\nu} = g_{\mu\nu} - g^{(0)}_{\mu\nu}$, is the metric perturbation. This action is manifestly breaks general covariance. Basically, DGP model also takes a similar effective action at the quadratic order. But a significant difference is that the theory is covariantly defined.

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However, there is an argument that the model defined by the action (1) possesses a ghost-like mode in spin-2 excitations when we adopt de Sitter space as the background spacetime, if the mass of the graviton is small [4]. If we wish to explain the accelerated expansion of the universe by a certain modification of gravity, it seems that the graviton mass is necessarily as small as the Hubble parameter. If so, the appearance of a ghost seems to be unavoidable. In fact, it was explicitly shown that a similar ghost-like mode appears in the DGP model [5]. Here we consider a variant of the DGP model by introducing another boundary brane, searching for a model coping with both the accelerated expansion of the universe and the ghost-free condition. We will find that such a model cannot be constructed in the two-branes extension.

2 Background spacetime in the DGP model

In this section we consider the unperturbed background of the model of the self-accelerating universe [2]. The five dimensional action of the DGP model is given by [1]

$$S = -M_5^3 \int d^5X \sqrt{-\tilde{g}}\, \tilde{R} - M_4^2 \int d^4x \sqrt{-g}\, R + \int d^4x \sqrt{-g}\, L_m, \tag{2}$$

where $\tilde{g}_{\mu\nu}$ and $\tilde{R}$ are the five-dimensional metric and Ricci scalar. $g_{\mu\nu}$, R are the metric and the Ricci scalar induced on the brane, respectively. The distinctive feature of this model is the presence of the four-dimensional Einstein-Hilbert term localized on the brane. We assume the Z_2 symmetry across the brane.

This model has a critical length scale determined by the ratio between four and five-dimensional gravitational coupling, $r_c = M_4^2/2M_5^3$. Below the critical length scale, four dimensional term becomes dominant, so recovery of four dimensional gravity is expected. The truth is a little more complicated. At the linear level, even at a short wavelength the gravity significantly deviates from the four dimensional general relativity. This deviation is cured only after taking into account the non-linear effect [6].

The evolution of the homogeneous and isotropic universe is easily derived from the junction conditions assuming that the brane is embedded in five dimensional Minkowski bulk. The derived equation becomes

$$\frac{1}{3M_4^2}\rho = H^2 - \epsilon r_c H, \tag{3}$$

where ρ is the matter energy density and H is the expansion rate of the universe. $\epsilon = \pm 1$ determines which side of the Minkowski space is taken as the bulk. When $\epsilon = +1$, the bulk contains the spatial infinity of the Minkowski bulk.

In the early universe, where $H \gg r_c^{-1}$, cosmic expansion is standard in both branches. But the late time behavior for (+)-branch shows self-acceleration, i.e., $H \to r_c^{-1}$ in the limit $\rho \to 0$.

3 Ghost in self-acceleration branch of DGP model and a simple extension to two-branes model

In the self-acceleration branch, the lowest mass of the Kaluza-Klein gravitons is $m^2 = 2H^2$. It is known that the graviton with $m^2 < 2H^2$ contains a ghost mode in the theory with Fierz-Pauli mass term [4]. Thus the DGP self-acceleration is the marginal case. A detailed analysis shows that there is a ghost mode in this model, too. However, since this is the marginal case, a question arises how serious this ghost is. "Can we erase this ghost by slight modification of the model?"

Our attempt tried here is to put the second (−)-brane as a regulator in the bulk. Our naive idea is the following. If the separation between two branes gets closer, the KK mass of gravitons will increase. Then the lowest mass escapes the dangerous zone, $m^2 < 2H^2$. Then the ghost will disappears.

In fact, this ghost disappears as soon as the second brane is introduced. If we add some vacuum energy localized on the brane, the expansion rate becomes faster. In that case, initially the lowest KK graviton mass is smaller than the critical value, $m^2 < 2H^2$. Even if we introduce the second brane, spin-2 ghost does not disappear immediately. Only when the second brane gets close enough, this ghost disappears. However, this is not the end of the story unfortunately. At the point where the spin-2 ghost disappears, spin-0 ghost appears instead [7]. Hence, the number of ghost degrees of freedom is conserved in total.

But, why these two different types of ghosts can be synchronized with each other. It looks strange. Usually, spin-2 and spin-0 modes are completely decoupled at the level of linear perturbation. The trick is as follows. Let Y be a scalar function, i.e., a spin-0 mode. One can construct a traceless tensor from Y as

$$Y_{\mu\nu} = \left(\nabla_\mu \nabla_\nu - \frac{1}{4}\Box^{(4)}\right) Y. \tag{4}$$

This traceless tensor is not transverse in general. In de Sitter background, we have, in fact, the relation

$$\nabla^\mu \left(\nabla_\mu \nabla_\nu - \frac{1}{4}\Box^{(4)}\right) Y = \frac{3}{4}\nabla^\mu \left(\Box^{(4)} + 4H^2\right) Y. \tag{5}$$

But if $\left(\Box^{(4)} + 4H^2\right) Y = 0$, i.e., if $m^2 + 4H^2 = 0$, this tensor satisfies transverse conditions. Therefore this tensor has the properties of both spin-0 and spin-2 modes, and hence it is quite special.

This mode has the mass squared $m^2 = -4H^2$ in the language of spin-0 mode. However, the tensor equation satisfied by the above tensor is $(\Box^{(4)} - 4H^2)Y_{\mu\nu} = 0$. Namely the mass squared in the notation of spin-2 mode is $2H^2$, and it exactly agrees with the critical mass for the existence of spin-2 ghost. This is the mechanism that explains the correlation between spin-0 and spin-2 modes.

But what is spin-0 mode in this model? In the present model the bulk gravitational perturbations are all transverse traceless in the four dimensional sense. The only possible origin of spin-0 mode (in vacuum case) is the brane bending mode. We denote this mode by X. The equation of motion for this mode is given by

$$\left(\Box^{(4)} + 4H^2\right) X = 0. \tag{6}$$

Hence the mass of this mode perfectly agrees with the critical mass.

4 model with a stabilized modulus

We have identified the mixing spin-0 mode is the brane bending. In the single brane case, brane bending is a spurious mode, which can be erased by a gauge transformation. In this sense, it is more appropriate to call it as the oscillation mode of the brane separation. Hence, if we fix the brane separation by hand, this mode disappears. However, fixing by hand will not be acceptable choice. Here we introduce Goldberger-Wise mechanism [8] into our two-branes model. Then our naive expectation is that all the spin-0 spectrum becomes highly massive. Therefore they become unable to couple with the spin-2 mode.

However, we found that the correlation between spin-0 and spin-2 ghosts does not disappear even if such a stabilization mechanism is introduced. The reason is as follows. Stabilization removes physical spin-0 mode from $m_{s=0}^2 = -4H^2$ in general. However, when the spin-2 mode crosses the critical mass, the one of the mass of spin-0 mode also necessarily crosses the critical value, and it turns into a ghost. But why? The equation for the spin-2 mode takes the following form:

$$\left[(\partial_y^2+\cdots)+\frac{\Box^{(4)}_{H=1}-2}{y^2}\right]h^{(s=2)}_{\mu\nu}=-\frac{2}{M_5^3}\Sigma_{\mu\nu}\delta(y-y_{brane}), \tag{7}$$

with

$$\Sigma_{\mu\nu}=T_{\mu\nu}+2M_5^3\nabla_\mu\nabla_\nu X-(\text{trace part}). \tag{8}$$

The trace part of $\Sigma_{\mu\nu}$ is determined so that $\Sigma_{\mu\nu}$ becomes traceless. From this expression, we can compute the effective action obtained after tracing out the gravitational degrees of freedom. The correction to the matter Lagrangian will be given by $T^{\mu\nu}\times h_{\mu\nu}$. Then the contribution due to the spin-2 perturbation contains the term related to the trace of $T_{\mu\nu}$ since the brane bending X enters as a source of $h^{(s=2)}_{\mu\nu}$. Hence, the contribution to $T^{\mu\nu}\times h_{\mu\nu}$ from $h^{(s=2)}_{\mu\nu}$ also contains a pole at $\Box^{(4)}=-4H^2$ concerning the trace of $T_{\mu\nu}$ as

$$\approx T\frac{1}{(m_i^2-2H^2)(\Box^{(4)}+4H^2)}T, \tag{9}$$

where m_i is one of KK graviton masses. This pole is originally from X, but X itself does not diverge when m_i^2 crosses $2H^2$. Although the effective Lagrangian of the matter looks divergent, this divergence must be cancelled in total. The only possible way to cancel this divergence is that the spin-0 contribution has the same (but opposite in sign) divergence simultaneously. This requires the divergence of the spin-0 propagator and hence the vanishing kinetic term. Vanishing kinetic term naturally results in the flip of its signature.

Let's summarize the results up to here. The self-accelerating branch of DGP model has a spin-2 ghost. Introducing the second brane erases the spin-2 ghost for a sufficiently small brane separation. However, spin-0 ghost appears at the time when the spin-2 ghost disappears. Even if stabilization mechanism is introduced, the connection between spin-0 and spin-2 ghosts does not disappear.

5 Do we really need to be afraid of spin-2 ghost

We could not succeed in erasing ghost. But we want to ask whether this spin-2 ghost is really a serious problem, since it is harmless in the flat background. Usually the ghost is problematic because of its bad behavior in the ultra violet regime. But in UV limit, the field will not sense the presence of background curvature. If this speculation is correct, spontaneous pair production of ghost and normal particles may not be divergent.

We estimated the pair production rate by using a simple toy model composed of a massive graviton and a conformal coupled scalar field [9]. Only the helicity-0 mode of the massive graviton behaves as a ghost. The effective action for the helicity-0 mode is given by [4]

$$S_{ghost}=\sum_k\int d\eta\frac{M_4^2m^2(m^2-2H^2)}{k^4\eta^2}s_k\left[\Box_{flat}+\frac{2}{\eta}\partial_\eta-\frac{m^2}{H^2\eta^2}\right]s_k\,d\eta. \tag{10}$$

For $m^2<2H^2$, the signature of this action becomes negative and the mode becomes a ghost. From this expression, we can read that the expectation value of the amplitude of fluctuations $\langle s_k^2\rangle$ becomes very large for large k. This means that the model becomes strongly coupled at a relatively long wavelength. The metric perturbation can be reconstructed from s_k as

$$h_{\mu\nu}=\begin{pmatrix} s_k & -\frac{\nabla_i}{k^2}\left(\partial_\eta-\frac{2}{\eta}\right)s_k \\ * & \cdots \end{pmatrix}. \tag{11}$$

We computed the interaction term by $S_{int}=-\frac{1}{2}\int d^4x\sqrt{-g}h^{\mu\nu}T_{\mu\nu}$, where $T_{\mu\nu}$ is the energy momentum tensor of the scalar field.

Let's consider the diagram for spontaneous creation of two scalar particles and one ghost graviton. The large amplitude of $\langle s_k^2 \rangle$ in UV is a bad signal. However, coupling to the matter field is also suppressed in the UV limit. In the UV limit, the graviton mass becomes more and more irrelevant. In the massless limit, the ghost graviton is a pure gauge mode. Therefore its coupling to the ordinary matter field also vanishes.

After lengthy calculation, we obtained the following preliminary result. The total energy density of the created scalar particles is $\rho \approx H^2\Lambda^4/M_4^2$, where Λ is the UV cutoff scale. As this expression tells, the resulting energy density is found to be divergent if we do not introduce a cutoff. However, even if we set Λ to the Planck mass M_4, the resulting particle creation is comparable to the value of the critical density $\rho_{crit} = H^2M_4^2$. The massive graviton model becomes strongly coupled at much lower scale than the Planck scale. If we cut off the theory at this strong coupling scale, particle production rate is extremely suppressed.

6 Summary and conclusions

We found that ghost is hard to kill. But probably and hopefully the real problem is just how to tame the problem of strong coupling. A more detailed explanation about the results present here will be found in our forthcoming paper [10].

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